Private vs Public Currency

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Abstract

We examine the general equilibrium impact of a private currency. If the government is predatory and has a monopoly over money issuance, it issues too much money, in order to finance excessive public expenditures. In this context, what are the consequences of introducing a private currency in limited supply but exposed to crash risk? When crash risk is not too large, competition from the private currency constrains the government's monetary policy and lowers inflation. If the government is very predatory, this improves citizens welfare, but if the government is rather benevolent, competition from the private currency can lower citizens' welfare.

1 Introduction

Large inflation is often attributed to excessive money creation to fund large public spending. An extreme form of this phenomenon is hyperinflation.[1] It was documented by Lopez and Mitchener (2020) in the case of Europe after World War 1.[2] Pittaluga, Seghezza and Morelli (2021) also offer evidence consistent with that view for Venezuela in the 2010s.[3] Maybe the most infamous example is Zimbabwe in the 2000s, when the government printed such vast amounts of money, to fund the war in Congo, that the value of the currency basically went to 0.

In his essay entitled "Denationalisation of Money", Hayek (1976, point 5 in the abstract) argued that these problems could be avoided thanks to

> "the replacement of the government monopoly of money by competition in currency supplied by private issuers, who, to preserve public confidence, will limit the quantity of their paper issue and thus maintain its value."

As recognized by Hayek himself (see Hayek (1976) page 14), private currencies in the past have often proved unreliable or inconvenient. But the case for private currencies has recently been restated with innovations in digital technologies, in particular blockchains and cryptocurrencies. Incorporating the issuance policy in the blockchain protocol, as successfully implemented by Bitcoin, enables to commit to bounded currency creation. Can such commitment in turn enable private currencies to effectively compete with the official public currency? And if so, what will be the impact of this competition on monetary and fiscal policy? Hayek (1976, page 17) argues that competing private currencies would

> "impose upon existing monetary and financial agencies a very much needed discipline... "

Can we expect such discipline to arise in equilibrium and increase citizens' welfare?

To study these questions, we use a simple general equilibrium model, featuring a government and a continuum of agents operating technologies with i.i.d. productivity shocks. We assume agents are risk-averse (for tractability we assume they have logarithmic utilities). Because of risk-aversion, the agents seek to reduce their exposure to their random productivity shocks. However, we assume that the market is incomplete and agents cannot buy insurance against their productivity shocks. Although we take this incompleteness as given in

[1] See, e.g., the seminal analysis of Cagan (1956).

[2] On page 450 of their article, Lopez and Mitchener (2020, page 450) write "Why do hyperinflations begin? In a mechanical sense, economists have known the answer to this question at least since the monetarist revolution: money is printed in response to unsustainable fiscal policy"

[3] Pittaluga, Seghezza and Morelli (2021) write (pages 337 and 338): "The financing of the existing level of public spending no longer could be sustained by domestic and oil-related taxes, inflationary financing was adopted, and hyperinflation ensued. "

the present paper, it can be microfounded by assuming that agents privately observe their productivity shocks, as in Biais, Gersbach, Rochet, von Thadden, and Villeneuve (2025).[4] In this context, agents value the opportunity to hold money, to buffer idiosyncratic shocks, in line with Bewley (1977). Thus, agents make portfolio choices, deciding which fraction of their wealth to invest in the risky production technology and which fraction to invest in money, in line with Merton (1969). The government chooses how much money to issue, and also how much agents' wealth should be taxed, as well as the level of public spending. The budget constraint of the government is that public spending equals tax proceeds plus seigneurage revenues. There is a conflict of interest between the government and the agents, because the government's preferences put weight on public spendings that are useless for the agents.

First, as a benchmark, we consider the case in which there is no private currency, so that the government has a monopoly over money issuance. In that case, we show that a predatory government relies on seigneurage to fund excessive public spending. Correspondingly, monetary creation and inflation are larger than what would be optimal for agents.[5] When the government is highly predatory, this leads to hyperinflation, which in our theoretical framework is the situation in which agents are unwilling to hold the official currency, the value of which goes to zero.

Second, we turn to the situation in which there is a private currency. In practice this could be a cryptocurrency or a stablecoin denominated in another currency than the domestic currency. Appendix C offers a discussion of institutional aspects of cryptocurrencies and stablecoins captured by our model, and also discusses the link with dollarization. The two major characteristics of the private currency in our model are in line with stylized facts on cryptocurrencies: First, there is a predetermined and limited rate of creation of the private currency. Second, the private currency is risky. More precisely, we assume there is a risk of crash driving the value of the private currency to zero. This is in line with analyses where money becomes valueless when a sunspot realizes (see, e.g., Trejos and Wright, 1995), recently revisited by equilibrium models of cryptocurrency prices (see, e.g., Garratt and Wallace (2019) and Biais, Bisière, Bouvard, Casamatta, and Menkveld (2023)). In this context, we show that two cases can arise:

- If the private currency is very risky, its presence does not change the outcomes. This is because when the private currency is too risky it does

[4] Appendix B briefly sketches this microfoundation, in which the government does not have the skills to operate the risky technology and delegates it to agents who privately observe their output. We show that the optimal dynamic mechanism analyzed in Biais, Gersbach, Rochet, Von Thadden, and Villeneuve (2025) is implemented by the monetary equilibrium analyzed in the present paper.

[5] This does not stem from lack of commitment, but from differences between the preferences of the agents and those of the government. In our framework, the policy that is optimal for the government is time consistent. This is because, like the agents, the government solves a Merton type of problem. With logarithmic utility, in a stationary environment the optimal policy rates are constant functions of the exogenous parameters.

not offer an attractive store of value, so the government effectively keeps its monopoly over money issuance.

- In contrast, if the private currency is not too risky, it places a cap on the issuance of public currency and thus on inflation. If the government tried to go above this cap, agents would not want to hold the public currency and would only hold the private currency. This disciplining effect of the private currency is in line with Hayek (1976).[6] In this case, if the government is predatory the private currency increases citizens' welfare. If the government is rather benevolent, however, the constraint imposed by the private currency reduces the welfare of citizens.

Our paper is related to the literature providing microfoundations for the usefulness of money (dating back to the seminal papers of Allais, 1947, Samuelson, 1958, Tirole, 1985, Weil, 1987, Kiyotaki and Wright, 1989 and 1993, and Lagos and Wright, 2005) and to the literature extending monetary theory to cryptocurrencies (see, e.g., Garatt and Wallace, 2018, Schilling and Uhlig, 2019, Benigno, Schilling, and Uhlig, 2022, and d'Avernas, Vandeweyer and Maurin, 2023). The papers to which our analysis is closest are those studying hyperinflation and those studying competition between currencies. Rocheteau (2024) offers an insightful analysis of equilibria in which the value of the currency progressively declines until it reaches zero. While in Rocheteau (2024) hyperinflation corresponds to a progressive erosion of the value of money due to the self-fulfilling beliefs of the agents, in our analysis hyperinflation corresponds to an instantaneous erosion of the value of the money due to the unsustainability of the government policy. Kareken and Wallace (1981), Garatt and Wallace (2018), Schilling and Uhlig (2019), Fernandez-Villaverde and Sanches (2019), Biais, Bisière, Bouvard, Casamatta and Menkveld (2023), Rocheteau (2024) and Arifovic, Salle and Schilling (2025) study competition between currencies. The main contribution of the present paper, relative to that literature, is to endogenize the differences in usefulness between private and public currencies, driven by endogenous monetary and fiscal policies, themselves reflecting conflicts of interest between the agents and the government.

Our theoretical analysis is also related to empirical analyses of cryptocurrency markets. Luckner, Reinhart, and Rogoff (2023) provide empirical evidence that cryptocurrencies are used to conduct transactions and store value, not unlike in our model. In our model, as long as there is no crash, the demand for cryptocurrency tends to increase faster than its supply, so that the price of the cryptocurrency increases. This is not unlike the mechanism econometrically analyzed by Jermann (2021).

In the next section, we analyze the benchmark case in which there is no private currency. Section 3 then extends the analysis to the case in which there is a private currency. Section 4 concludes. Proofs not given in the main

[6] In our model like in many countries, there is no independent central bank. If there was an independent and benevolent central bank, there would be no need for a private currency to discipline public currency issuance.

text are in Appendix A. Appendix B offers a microfoundation for the market incompleteness in our model. Appendix C offers a discussion of institutional aspects of cryptocurrencies and stablecoins captured by our model, and also discusses the link with dollarization.

2 Market equilibrium and government policy without private currency

In this section, we build a simple dynamic general equilibrium model with incomplete markets, in the tradition of Bewley (1977).[7] We assume that a self-interested government has a monopoly over money creation, and exploits it to extract rents from citizens. In the next section, we study how competition from the private currency can constrain the government and whether this increases the welfare of citizens.

2.1 Technology

Time is continuous and indexed by $t \in (0, \infty)$. There is a government and a mass 1 population of agents, indexed by i. There is only one good, which is produced by agents with a constant return to scale technology, and can be used for private consumption, public spending, or investment. At date t, agent i operates k_t^i units of capital. We first consider the case in which there are N agents, each with mass $\frac{1}{N}$. The output of agent i on date t is

$$\frac{k_t^i}{N}(\mu dt + \sigma dZ_t^i),$$

where Z_t^i are N independent Brownian motions that represent idiosyncratic risks. The total capital is denoted by K_t^N where

$$K_t^N = \sum_{i=1}^{N} \frac{1}{N} k_t^i,$$

and

$$dK_t^N = \sum_{i=1}^{N} \frac{1}{N} k_t^i \mu \, dt + \sum_{i=1}^{N} \frac{1}{N} k_t^i \sigma dZ_t^i$$

Then, for tractability we take the mean-field limit and consider the case of a continuum of agents, i.e., $N \to \infty$.[8] The total capital K_t^N converges to a deterministic process K_t. In this continuum of agents limit, under mild regularity

[7] As mentioned above, and clarified in Appendix B, the market incompleteness we consider can be micro-founded following the dynamic mechanism design approach of Biais, Gersbach, Rochet, von Thadden, and Villeneuve, 2025.

[8] For a more rigorous characterization of the mean-field limit, see Biais, Gersbach, Rochet, von Thadden, and Villeneuve (2025).

assumptions agent's idiosyncratic risks wash away in aggregate, so that total output (GDP) flow is μK_t.[9]

In the absence of frictions, it would be optimal to eliminate idiosyncratic risks by diversification. However, we assume that individual output is not publicly observable. Agents can secretly divert a fraction of their output and secretly consume it. This prevents agents from fully diversifying risks through financial instruments such as equity or insurance contracts. However, they can partially insure their risks by holding money whose supply is entirely controlled by the government; for a microfoundation see Biais, Gersbach, Rochet, von Thadden, and Villeneuve (2025).

2.2 Preferences

We denote by c_t^i the consumption flow of agent i and by G_t the flow of public spending. The preferences of the citizens are as follows.

$$U^i \equiv \mathbb{E}\int_0^\infty e^{-\rho t}\log c_t^i dt, \tag{1}$$

for agent $i \in (0,1)$. The preferences of the government are as follows.

$$U_G \equiv \int_0^1 U^i di + \beta\mathbb{E}\int_0^\infty e^{-\rho t}\log G_t dt, \tag{2}$$

where $\beta \geq 0$. Equation (2) states that the government maximizes the sum of agents' utilities from consumption plus its own utility from public spending, putting weight β on the latter. If $\beta = 0$, the government is fully benevolent. The larger β, the less benevolent the government.[10]

2.3 Government Policy

The government issues M_0 units of fiat money on date $t = 0$ and distributes them equally to agents, as well as K_0 units of capital.[11] Money supply at time t is denoted by M_t. The only market is a spot market in which the physical good is exchanged for money at an endogenous price p_t.

Since preferences and technology are constant and utility is logarithmic, there exist balanced-growth stationary equilibria in which all aggregate quantities grow at a constant rate.[12] In such equilibria, money-supply growth is

$$\frac{dM_t}{dt} = g_m M_t,$$

[9] A condition under which idiosyncratic risk washes away is that at each date t, the mapping $i \mapsto k_t^i$ is square-integrable.

[10] For simplicity, we assume here that agents don't derive any utility from public spending. We checked that, when agents derive some utility from public spending, our qualitative results are unchanged, as long as agents value public spending less than the government.

[11] This is optimal when the government puts equal welfare weights on all agents.

[12] Biais, Gersbach, Rochet, von Thadden and Villeneuve (2025) offer a micro-foundation in which the monetary equilibrium on which we focus in the present paper implements second-best allocations.

where g_m is a constant, and public spending is a constant fraction of aggregate capital K_t:

$$G_t = \gamma K_t.$$

The government finances its expenditures by a combination of taxes and seigneurage, the revenue from the issuance of money $\frac{dM_t}{dt}$. We assume that taxes are proportional to agents' wealth.[13] Because growth is balanced, the tax rate τ is constant[14]

To analyze equilibrium outcomes in our model, we take the following three steps:

- First, we characterize the optimal policies of competitive agents taking as given the government policy and market equilibrium.
- Second, relying on the characterization of individual policies, we analyze market equilibrium for a given government policy.
- Third, we study the optimal policy set by the government to maximize its expected utility, rationally anticipating how market equilibrium will reflect government policy.

2.4 Individual behavior

Agents form rational expectations about the price p_t of the good at all future dates and choose their consumption c_t, money holdings m_t and investment k_t to maximize their expected utility from consumption, given in (1).

On a balanced growth path, the inflation rate π and the capital growth rate g are constant:

$$p_t = p_0 e^{\pi t}, K_t = K_0 e^{gt}.$$

Agents' real wealth is the sum of their capital holdings and real balances:

$$w_t = k_t + \frac{m_t}{p_t}.$$

Since there are no transaction costs, agents can immediately and costlessly adjust the composition of their wealth at any time. Thus, w_t is the single state variable for each agent. The dynamics of w_t is given by the state equation:

$$dw_t = k_t(\mu dt + \sigma dZ_t) - (\tau w_t + c_t + \pi(w_t - k_t))\, dt. \tag{3}$$

Equation (3) states that the change in real wealth is output, minus taxes, consumption, and depreciation due to inflation. Denoting by $u(w)$ the value function of the agent, we can write the Bellman equation.

$$\rho u(w) = \max_{c,k \leq w} \Big(\log c + u'(w)[\mu k - \tau w - c - \pi(w-k)] + \frac{\sigma^2 k^2}{2} u''(w) \Big), \tag{4}$$

[13] For a micro-foundation see Biais, Gersbach, Rochet, von Thadden, and Villeneuve (2025).

[14] We allow τ to be negative, corresponding to the case in which the government transfers wealth to the agents.

where the maximum is subject to the constraint that money holdings cannot be negative, which is equivalent to $k \leq w$. As in Merton (1969), the homogeneity of the agent's program and the logarithmic utility of consumption imply that the value function of the agent, $u(w)$, is also logarithmic:

$$u(w) = \frac{\log w}{\rho} + u(1). \tag{5}$$

Substituting (5) into (4) and taking first order conditions, we obtain our first proposition:

Proposition 1 ***Individual choice:*** *From the point of view of an agent taking government policy and prices as given:*

- *optimal consumption is a constant fraction ρ of wealth*

$$c_t = \rho w_t. \tag{6}$$

- *optimal investment is a constant fraction x of wealth such that*

$$x = \min\left[\frac{\mu + \pi}{\sigma^2}, 1\right] \tag{7}$$

- *the agent's value function is such that*

$$\rho u(w) = \log(\rho w) - \tau - \rho + max_x \left[\mu x - \pi(1-x) - \frac{\sigma^2}{2}x^2\right], \tag{8}$$

  so that it is (weakly) decreasing in inflation π.

By (7), when $\pi < \sigma^2 - \mu$, the capital investment share x chosen by the agent is smaller than 1. In that case,

$$\sigma^2 x = \mu + \pi. \tag{9}$$

The left-hand side of (9) is the cost of capital investment, namely the productivity risk.[15] The higher this risk, the lower the propensity of agents to invest in capital. The right-hand side of (9) is the benefit of capital investment, equal to the expected return μ plus the benefit of being protected from inflation. In this context, investment in the risky asset increases with inflation, while money holdings decrease with inflation.

Higher inflation increases the cost of money holdings and therefore worsens the opportunity set faced by an agent. Hence, an agent's utility decreases with inflation.

[15] Note that the agents' portfolio choice, x, does not depend on the tax rate τ because taxes are proportional to total wealth.

2.5 Equilibrium

Having characterized individual behavior as a function of government policy, and in particular inflation π, we now study equilibrium in the markets for goods and money for a given government policy.

2.5.1 Goods:

Since there is no depreciation of capital, the growth of aggregate capital is equal to aggregate investment. Market clearing implies that aggregate investment is equal to aggregate savings, that is, output μK_t minus agents' consumption C_t, and government spending γK_t. That is

$$\frac{dK_t}{dt} = \mu K_t - C_t - \gamma K_t. \tag{10}$$

The law of motion of aggregate capital in (10) is deterministic, because idiosyncratic shocks are washed away.

Aggregating the consumption and investment choices in Proposition (1) and substituting in (10), we obtain the growth rate of aggregate capital:

$$g = \mu - \frac{\rho}{x} - \gamma. \tag{11}$$

For a given capital stock, the higher the fraction of wealth agents invest in the risky production technology, the lower the consumption and the higher the growth rate. The government's budget balance condition is that public spending is financed by taxes and seigneurage:

$$\gamma x = \tau + g_m(1-x). \tag{12}$$

Thus, for given money supply growth g_m and public spending γ, (12) determines τ.

2.5.2 Money:

The third step of our equilibrium analysis is to equalize money supply and demand. We have seen that agents find it optimal to keep a constant fraction $(1-x)$ of their wealth in money and the rest in capital. Therefore, aggregate money demand is $\frac{(1-x)}{x} p_t K_t$. Thus equality between money supply and money demand requires

$$M_t = \frac{1-x}{x} p_t K_t.$$

Since the money supply grows at a rate g_m and prices grow at rate π, this is equivalent to:

$$M_0 e^{g_m t} = \frac{(1-x)}{x} p_0 e^{\pi t} K_0 e^{gt}. \tag{13}$$

Equality between money supply and money demand at time 0 implies

$$\frac{M_0}{p_0} = \frac{(1-x)}{x} K_0, \tag{14}$$

which implies there always exists a rational expectation equilibrium in which the belief that money has no value is self-fulfilling, so that $p_0 = \infty$ and agents invest only in capital ($x = 1$).

There may also exist a monetary equilibrium, that is, an equilibrium in which money has a strictly positive value (p_t is finite for all t) and the demand for money is strictly positive ($x < 1$). Substituting (14) into (13), yields

$$g_m = g + \pi, \tag{15}$$

expressing that, in a monetary equilibrium, nominal growth equals real growth plus the inflation rate.

2.5.3 The nature of equilibrium: monetary vs non-monetary

Substituting in (15) the agents' portfolio optimality condition in a monetary equilibrium (9) and the expression for real growth (11), we obtain an implicit equation for the equilibrium fraction of agents' wealth in the risky asset (x) as a function of government policy (g_m, γ):

$$\sigma^2 x - \frac{\rho}{x} = g_m + \gamma. \tag{16}$$

If the solution x to that equation is strictly lower than 1 then it corresponds to a monetary equilibrium. This is the case if

$$g_m + \gamma < \sigma^2 - \rho. \tag{17}$$

Defining the index of inflationary pressure as

$$I \equiv g_m + \gamma, \tag{18}$$

we obtain our next proposition:

Proposition 2 ***Equilibrium*** *When the index I of inflationary pressure is not too high, as (17) holds, there is a unique monetary equilibrium, characterized by an inflation rate $\pi = \sigma^2 x - \mu$ and a real growth rate $g = \mu - \frac{\rho}{x} - \gamma$, where $x < 1$ is the unique solution of*

$$\sigma^2 x - \frac{\rho}{x} = I. \tag{19}$$

Otherwise, if (17) does not hold, the only equilibrium is hyperinflation: the value of money is zero and agents can only invest in productive capital.

The proposition shows that the nature of the equilibrium (monetary or not) and the level of inflation are determined by the index of inflationary pressure I, which aggregates the growth rate of the money supply and the government expenditure rate.

- If I is too large, the equilibrium is non monetary: $x = 1$ and $p_t \equiv \infty$, i.e., there is hyperinflation. In that case agents cannot use money to smooth their idiosyncratic shocks.

- Otherwise there is a monetary equilibrium ($x < 1$). In that case, the inflation rate π is increasing in I.[16]

Moreover, the initial price level is determined by equation (14): p_0 is proportional to M_0. This is a weak form of neutrality in the sense that initial money supply does not have any real effect, as prices fully adjust to it. The growth of money supply in contrast has real effects, as it impacts investment in productive capital.

In summary, this section has shown that when the government has a monopoly on money issuance, it has complete control of the inflation rate π through the index $I = \gamma + g_m$, which determines x through the equation (19) and thus the inflation rate

$$\pi = \sigma^2 x - \mu. \tag{20}$$

Hereafter, we use x and γ as the policy variables of the government, bearing in mind that the inflation rate is increasing in x and the tax rate is pinned down by the government budget balance.

2.6 Citizens' welfare

In this subsection, we characterize the equilibrium welfare of citizens for a given government policy (x, γ). For simplicity, we assume that the initial capital, K_0, is equally distributed among agents at time $t = 0$.

As can be seen in (8), taking the other aspects of government policy as given, an individual agent's utility is decreasing in inflation. We refer to this as the individual cost of inflation. One could be tempted to conclude from this observation that an agent's utility should be decreasing in inflation, or equivalently in the equilibrium value of x. But this individual cost of inflation does not take into account that inflation and x are related to the other aspects of government policy.

To clarify this countervailing effect, consider again equation (8): it shows that an individual agent's utility is decreasing in the tax rate τ. Now the government budget equation (12) implies that

$$\tau = \gamma x - g_m(1 - x).$$

By (11), (15), and (20) this rewrites as

$$\tau = \gamma - (\sigma^2 x - \frac{\rho}{x})(1 - x).$$

When the right-hand side is decreasing in x, which is the case for example when x is very low, an increase in the equilibrium value of x, corresponding to an increase in inflation, leads to a reduction in taxes, which benefits the agent. We refer to this effect as the aggregate benefit of inflation.

[16] To see this note that in a monetary equilibrium $\pi = \sigma^2 x - \mu$, which is increasing in x and x is such that $\sigma^2 x - \frac{\rho}{x} = I$, so that x is increasing in I.

Because of the coexistence of the individual cost of inflation and the aggregate benefit of inflation, an agent's utility is non-monotonic in inflation, and correspondingly in the equilibrium value of x, as stated in the following proposition.

Lemma 1: *In equilibrium, aggregate welfare $U(x)$ is an inverse U-shaped function of x given by*

$$\rho U(x) = \log(\frac{\rho K_0}{x}) + \frac{1}{\rho}(\mu - \frac{\rho}{x} - \gamma - \frac{\sigma^2 x^2}{2}). \tag{21}$$

The value of x that maximizes citizens' welfare is the unique real solution x^* of the equation

$$\frac{\sigma^2}{\rho}x^3 + x = 1. \tag{22}$$

For this optimal value there is no hyperinflation, i.e., $x^* < 1$. Correspondingly, the inflation rate that maximizes the equilibrium expected utility of citizens is

$$\pi^* = \sigma^2 x^* - \mu. \tag{23}$$

The first term in (21) reflects that, when inflation increases, the agent consumes less. The second and third terms in (21) reflect the above discussed aggregate benefits and individual cost of inflation, respectively. The optimal inflation given in (23) trades off these costs and benefits.

This analysis shows that neither hyperinflation nor severe deflation are optimal for citizens. Hyperinflation is suboptimal since $x^* < 1$. To see that severe deflation is suboptimal note that when x is very small, the utility of citizens is smaller than under hyperinflation.

2.7 Government policy

Given the above analysis, the value function of the government, denoted by U_G, is such that

$$\rho U_G = \log \frac{\rho K_0}{x} + \beta \log \gamma K_0 + \frac{1}{\rho}[(1+\beta)(\mu - \frac{\rho}{x} - \gamma) - \frac{\sigma^2 x^2}{2}]. \tag{24}$$

U_G is the sum of the utility of the agents given in Lemma 1 and of the utility derived by the government from public spending, weighted by β. The government maximizes U_G by choosing (x, γ) under the constraint $x \leq 1$. Taking the first order conditions of U_G with respect to γ and x, the next proposition gives the government's optimal policy, which we denote by (γ_M, x_M).[17]

[17]The subscript M refers to the monopoly of money issuance. In the next section, we consider the case in which there is a private currency, competing with official money, and then we use the subscript C, the first letter of "competition."

Proposition 3 ***Government policy:*** *The policy chosen by the government depends on the magnitude of the conflict of interest between the government and the agents, parametrized by β. When $\beta > 0$, public expenditures equal a multiple of aggregate capital*

$$\gamma_M(\beta) = \frac{\rho\beta}{1+\beta}, \tag{25}$$

which is increasing in β. When β is not too high, as

$$0 < \beta < \frac{\sigma^2}{\rho}, \tag{26}$$

the government implements a monetary equilibrium in which $x_M < 1$ is the unique solution of the cubic equation:

$$\frac{\sigma^2}{\rho}x^3 + x = 1 + \beta, \tag{27}$$

which is increasing in β. In contrast, when (26) does not hold, the government implements the hyperinflation equilibrium in which $x_M = 1$.

The policy that maximizes the welfare of citizens is obtained for $\beta = 0$. Since x_M is increasing in β, from the point of view of citizens, for any $\beta > 0$ the level of x chosen by the government is too high: $x_M > x^*$. Correspondingly, even when there is no hyperinflation, i.e., when $x_M(\beta) < 1$, the inflation rate $\pi_M = \sigma^2 x_M(\beta) - \mu$ is higher than what citizens would like. The interpretation of these results is natural: a self-interested government uses its monopoly power to issue too much money in order to finance large expenditures that are not valuable for citizens.

When β is large, i.e., when the conflict of interest between the agents and the government is severe, inflation is so large that agents invest all their wealth in the real asset. In this hyperinflation environment, the welfare of agents is low as their risk exposure is large since they cannot hold money to buffer productivity shocks. As we will see in the next section, in this context, the ability to hold a private currency immune from hyperinflation is valuable for agents.

When there is no hyperinflation, the agents' value function is

$$\frac{1}{\rho}\log\frac{\rho K_0}{x_M} + \frac{1}{\rho^2}[\mu - \frac{\rho}{x_M} - \gamma_M - \frac{\sigma^2 x_M^2}{2}], \tag{28}$$

where γ_M and x_M are the optimal government policy choices given in Proposition 3. In the next section, we will compute the agents' value function when there is a private currency and compare it to (28) to study the impact of the private currency on the welfare of the agents.

3 Competition between public and private currencies

3.1 Private currency

To capture the risky nature of the private currency, we assume it can crash. We assume there is an exogenous observable Poisson process N_t with intensity λ. At the first jump of this process, which happens at time T, the private currency become worthless.

At any time $t < T$ an agent has capital k_t, m_t units of public currency, and $\hat{m}_t$ units of private currency. The price of the good in private currency is denoted $\hat{p}_t$. Consequently, the composition of the real wealth w_t of an agent is as follows:

$$w_t = k_t + \frac{m_t}{p_t} + \frac{\hat{m}_t}{\hat{p}_t}.$$

As in the previous section, because utility is logarithmic and the environment is stationary (until the first jump of the Poisson process), portfolio shares are constant. We denote by x the fraction of real wealth w_t invested in capital and by b the fraction invested in public currency. The share invested in private currency is $(1 - b - x)$. So we have

$$k_t = xw_t, \frac{m_t}{p_t} = bw_t, \frac{\hat{m}_t}{\hat{p}_t} = (1 - b - x)w_t.$$

Agents rationally anticipate that, at the time of the private currency crash, the equilibrium price of the good in the public currency jumps from p_T to p_{T^+} (we use the superscript $+$ to denote what happens just after the crash). Also, at the time of the crash, an agent's wealth jumps to

$$w_{T^+} = k_T + \frac{m_T}{p_{T^+}} = w_T(x + b\frac{p_T}{p_{T^+}}). \tag{29}$$

As in the case without private currency, our analysis proceeds in three steps: First, we characterize agents' optimal decisions for a given government policy. Second, we characterize market equilibrium for a given government policy. Third, we study how government sets optimal policies, anticipating how agents will react to them.

We consider a stationary rational expectations equilibrium, with constant policy rates γ, g_m and τ until time T. Inflation rates (π for the public currency and $\hat{\pi}$ for the private currency) are also constant. τ is applied to the part $(b + x)w$ of the wealth of the citizens that is observable by the government.[18]

For simplicity we consider the case in which $x > 0$ and $b > 0$. As shown below, this is the case in equilibrium. Also, we hereafter focus on the case $\beta < \frac{\sigma^2}{\rho}$, in which (as stated in Proposition 3) there is no hyperinflation without private currency, i.e., $x_M < 1$, so that, a fortiori, with a private currency we have $x < 1$.

[18] For simplicity we assume private currency holdings are not observable by the government.

3.2 Agents' optimal decisions

The dynamics of an agent's wealth is given by:

$$\frac{dw_t}{w_t} = x(\mu dt + \sigma dZ_t) - \left(\tau(b+x) + \frac{c_t}{w_t} + \pi b + \hat{\pi}(1-b-x)\right) dt \\ -(1-x-b\frac{p_t}{p_t^+})dN_t.$$

The last term reflects the possibility of a private currency crash. When the Poisson process jumps, by (29) agents lose a fraction $(1-x-b\frac{p_t}{p_t^+})$ of their wealth.

As in the previous section, because the utility function is logarithmic, the value function also is logarithmic. Before the private currency crash, there is a constant $\hat{u}(1)$ such that the value function of an agent is

$$\hat{u}(w) = \frac{\log w}{\rho} + \hat{u}(1),$$

while after the crash, the value function is

$$u(w) = \frac{\log w}{\rho} + u(1).$$

Before the crash, the Bellman equation for an agent is thus

$$\begin{aligned} \rho\hat{u}(w) = \quad & \max_{c,x,b} \log c + \tfrac{1}{\rho}(\mu x - \tau(b+x) - (\pi b + \hat{\pi}(1-b-x)) - \tfrac{c}{w}) \\ & -\tfrac{\sigma^2 x^2}{2\rho} - \tfrac{\lambda}{\rho} log \tfrac{w}{w(x+b\frac{p}{p^+})} + \lambda(u(1) - \hat{u}(1)), \end{aligned} \tag{30}$$

under the constraint $x + b \leq 1$. The multiplier associated with this constraint is denoted by ν. The complementary slackness condition is

$$\nu(1-x-b) = 0. \tag{31}$$

Taking first order conditions in (30), optimal consumption is $c = \rho w$ as in the previous section. Moreover, the optimal portfolio structure is characterized in the following lemma:

Lemma 2: *The optimality condition with respect to x is:*

$$\mu + \tau + \hat{\pi} - \sigma^2 x + \frac{\lambda}{x + b\frac{p}{p^+}} = \nu. \tag{32}$$

The optimality condition with respect to b is:

$$-\tau - \pi + \hat{\pi} + \frac{\lambda\frac{p}{p^+}}{x + b\frac{p}{p^+}} = \nu. \tag{33}$$

The optimality condition with respect to x is that the marginal benefit of investing in productive capital is equal to the marginal benefit of investing in the private currency. The marginal benefit of investing in productive capital is equal to expected productivity μ, plus transfer τ, minus risk premium $\sigma^2 x$, plus capital hedging value against a private currency crash $\frac{\lambda}{x+b\frac{p}{p^+}}$, minus the shadow cost ν of the constraint $x+b\leq 1$. The marginal benefit of investing in private currency is the expected increase in the real value of this private currency: $-\hat{\pi}$.

The optimality condition with respect to b is that the marginal benefit of holding the official currency is equal to the marginal benefit of holding the private currency. The benefit of holding official money is the transfer rate τ minus inflation π plus the hedging value of official money against a private currency crash $\frac{\lambda\frac{p}{p^+}}{x+b\frac{p}{p^+}}$, and minus the shadow cost ν of the constraint $x+b\leq 1$.

Relying on Lemma 3, we obtain agents' optimal policy when it involves strictly positive private currency holdings:

Proposition 4 ***Individual choice:*** *Agents find it optimal to invest a strictly positive fraction of their wealth in private currency only if*

$$\pi+\tau>\hat{\pi} \tag{34}$$

Under that condition, the share of an agent's wealth invested in productive capital is

$$\sigma^2 x=\mu+\pi-(1-\frac{p^+}{p})(\pi+\tau-\hat{\pi}). \tag{35}$$

Condition (34) means that the total cost of holding public currency is higher than the cost of holding the private currency when $\lambda=0$. This is a necessary condition for the agents to be willing to hold the private currency.

Equation (35) is similar to its counterpart without private currency, (9). The difference between the two comes from the last term on the right-hand side of (35), which implies that, ceteris paribus, x is lower when there is a private currency than when there is no private currency. This suggests a form of crowding out of real investment by investment in private currency. As clarified below, however, such crowding out can be beneficial for agents by enabling them to reduce their risk exposure.

3.3 Equilibrium

3.3.1 Jumps in aggregate quantities at the time of the crash

After the crash, the private currency is worthless. So agents can only invest their wealth in productive capital and public currency. Denote by W_{T^+} the aggregate wealth of the agents just after the crash. Government policy and equilibrium after the crash are those we analyzed in the previous section for the case without a private currency.

Government policy after the crash is such that agents invest fraction x_M of their wealth in capital and the rest in money. Thus, the aggregate demand for capital after the crash is

$$K_T = x_M W_{T^+}.$$

Before the private currency crash, agents have a fraction x of their wealth invested in productive capital. Noting that the aggregate stock of productive capital is unchanged after the crash, we obtain a condition imposed by the conservation of capital:

$$K_T = xW_T = x_M W_{T^+}. \tag{36}$$

(36) states that K_T, which is the supply of capital at the time of the crash, has to be equal to the demand for capital just before the crash xW_T, as well as the demand for capital just after the crash $x_M W_{T^+}$.

By (29), we have

$$\frac{W_T^+}{W_T} = x + b\frac{p_T}{p_T^+}. \tag{37}$$

Combining (37) with the conservation of capital condition (36) we have

$$x + b\frac{p_T}{p_T^+} = \frac{x}{x_M}. \tag{38}$$

The interpretation of (38) is that the private currency crash reduces the wealth of agents by a factor $\frac{x}{x_M}$. Productive capital and official money offer different hedges against the risk of a private currency crash, because the real value of money is affected by the price change $\frac{p_T}{p_T^+}$. By (38), this price change is

$$\frac{p_T}{p_T^+} = \frac{x(1-x_M)}{bx_M}. \tag{39}$$

3.3.2 Equilibrium outcome

The equilibrium conditions are similar to their counterparts without private currency. The condition for market clearing in the market for goods remains (11) and market clearing in the market for public currency still implies (15). The government budget constraint is slightly modified relative to (12) and becomes

$$\gamma x = \tau(x + b) + g_m b \tag{40}$$

There is also an additional condition: Since the supply of private currency is constant, the inflation rate of the private currency must be the opposite of the growth rate, i.e.,

$$\hat{\pi} = -g. \tag{41}$$

Moreover, there still exists a non-monetary self-fulfilling equilibrium ($x = 1$) in which neither of the two currencies has any value, because both currencies are expected not be accepted as means of payment. We, however, focus on

monetary equilibria ($x < 1$), in which one or both currencies are valued and held. In such equilibria, x is given by the following lemma:

Lemma 3: *With a private currency, in a monetary equilibrium the fraction of their wealth which agents invest in the risky productive technology, x, is such that*

$$I = \sigma^2 x - \frac{\rho}{x} - \lambda[\frac{x_M}{x} - \frac{1 - x_M}{b}], \tag{42}$$

Moreover:

$$\sigma^2 x^2 = \rho + \lambda - \nu. \tag{43}$$

Recall that $I = g_m + \gamma$ is the index of inflationary pressure defined in (18). Since the right-hand side of (42) is increasing in x, the larger the index of inflationary pressure, the larger the fraction of agents' wealth that is invested in the risky asset in a monetary equilibrium.

Equation (43) implies that if the condition $x + b \leq 1$ is slack and $\nu = 0$, meaning that agents invest a strictly positive fraction of their wealth in the private currency, then we have

$$x = x_C \equiv \frac{\sqrt{\rho + \lambda}}{\sigma}. \tag{44}$$

Otherwise, when the condition $x + b \leq 1$ binds, agents don't hold private currency and, by (43), $x < x_C$.

Defining

$$I_C \equiv \frac{\lambda(1 - x_M)}{x_C(1 - x_C)} \tag{45}$$

the above analysis yields our next proposition:

Proposition 5 ***Equilibrium:*** *There are two possible equilibrium regimes, depending on the strength of inflationary pressure:*

- *If $I > I_C$, then agents invest a strictly positive fraction of their wealth in the private currency, $x = x_C$ in the risky technology, and*

$$b = \frac{1}{\frac{I}{\lambda(1-x_M)} - \frac{1}{x_C}} \tag{46}$$

  in the public currency.

- *If $I \leq I_C$, then agents don't invest in the private currency and their investment in the risky technology is $x \leq x_C$ such that*

$$I = \sigma^2 - \frac{\rho - \lambda x_M}{x} + \lambda \frac{1 - x_M}{1 - x}. \tag{47}$$

Proposition 5 implies that the presence of the private currency imposes an upper bound x_C on the fraction of agents' wealth that is invested in the risky technology in equilibrium. With a private currency, x cannot be too high, because inflation cannot be too high, because if it were agents would sop holding the public currency and use only th private currency.

3.4 Government policy

Before the private currency crashes, equilibrium is as given in Proposition 5, and x cannot exceed x_C. After the crash, the equilibrium is as given in Proposition 3, and the constraint $x \leq x_C$ disappears. The fraction of agents' wealth invested in the risky asset is therefore time dependent. We denote it by x_t. Yet before and after the private currency crash the environment is stationary, so we consider the following dynamic for x_t:

$$x_t = x\mathbb{1}_{t<T} + x_M\mathbb{1}_{t\geq T},$$

where T denotes the random time at which the private currency crashes. Similarly, the government spending rate is:[19]

$$\gamma_t = \gamma\mathbb{1}_{t<T} + \gamma_M\mathbb{1}_t \geq T.$$

Thus, since $c_t = \rho w_t$, by (1) and (2) the value function of the government is U_G such that:

$$\rho U_G = E\int_0^\infty \rho e^{-\rho t}\left[\log \rho w_t + \beta \log \gamma_t K_t\right] dt.$$

In this context, the next lemma gives the agent's wealth dynamics and the government's value function before the crash:

Lemma 4: *Before the crash, the dynamics of agent's wealth are*

$$\frac{dw_t}{w_t} = (\mu - \gamma_t - \frac{\rho}{x_t})dt + \sigma x_t dZ_t - (1 - \frac{x_t}{x_M})dN_t, \tag{48}$$

and the government's value function is such that (up to a constant)

$$\rho U_G = \beta \log \gamma + (1+\beta)(\frac{\mu-\gamma}{\rho} - \frac{\rho}{x(\rho+\lambda)}) - \frac{1}{\rho+\lambda}\big(\rho \log x + \frac{\sigma^2 x^2}{2}\big). \tag{49}$$

Before the crash, the government chooses (x, γ) to maximize (49) under the constraint $x \leq x_C$. The optimal government policy is described in our next proposition.

Proposition 6 *When there is a private currency the government spending rate is the same as without a private currency, i.e. $\gamma = \gamma_M$. Monetary policy is such that*

- *When $x_C < x_M$, before the crash agents invest a fraction x_C of their wealth in the productive asset and the rest in the two currencies.*
- *When $x_C \geq x_M$, $x = x_M$, just like without the private currency.*

[19] As shown below, however, the spending rate will be constant.

The intuition of the proposition is the following: When there is no private currency, the government finds it optimal to opt for a monetary policy such that agents invest a fraction x_M of their wealth in the risky productive technology. The presence of a private currency, however, implies the government cannot follow a policy leading to $x > x_C$. So there are two cases. Either $x_C < x_M$ so that the presence of the private currency actually constrains the government, which can only go as far as $x = x_C$. Or $x_C > x_M$ and the constraint imposed by the presence of the private currency is not binding. In that case the government can implement x_M just like without a private currency.

3.5 Are citizens better off when there is a private currency?

We now compare the welfare of agents with and without a private currency. Proposition (6) implies that, when $x_C \geq x_M$ the existence of the private currency does not change the welfare of citizens. But what if $x_C < x_M$?

By equation (28) when there is no private currency the welfare of citizens is

$$\frac{1}{\rho}\left[\log(\rho K_0) + V(x_M)\right] \tag{50}$$

where

$$V(x) \equiv -\log x + \frac{1}{\rho}[(\mu - \frac{\rho}{x} - \gamma) - \frac{\sigma^2 x^2}{2}].$$

As illustrated in the figure, V reaches its maximum at x^*. Moreover, as illustrated in the figure, V is quasi-concave.

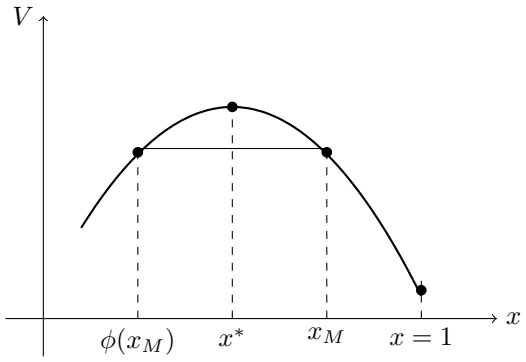

Figure 1: How function ϕ is determined

When there is a private currency and $x_C < x_M$ the welfare of citizens is

$$\frac{1}{\rho}\log(\rho K_0) + \frac{1}{\rho}\left[\frac{\rho}{\rho+\lambda}V(x_C) + \frac{\lambda}{\rho+\lambda}V(x_M)\right] \tag{51}$$

The intuition for (51) is that the weight placed on the value function after the private currency crash ($V(x_M)$) is the expected discount factor until the time of the crash

$$\frac{\lambda}{\rho+\lambda} = E\left[e^{-\rho\tau}\right] = \int \lambda e^{-\lambda T} e^{-\rho T} dT.$$

Comparing (50) and (51), when $x_C < x_M$ agents' utility is larger when there is a private currency than when there is no private currency iff

$$V(x_C) \geq V(x_M).$$

Since V is quasi-concave and unbounded below, there exists a unique $\phi(x_M) < x_M$ such that $V(\phi(x_M)) = V(x_M)$.[20] As can be seen in the figure, if $x_C < \phi(x_M)$, then $V(x_C) < V(x_M)$, i.e., the existence of the private currency makes agents worse off. On the other hand, the existence of the private currency benefits the citizens when $\phi(x_M) < x_C < x_M$. Finally, as directly implied by Proposition 6, the existence of the private currency has no impact on the welfare of citizens when $x_C \geq x_M$. These results are summarized in the following proposition.

Proposition 7 *The impact of the private currency on citizens' welfare depends on the magnitude of x_C relative to x_M:*

1. *When x_C is large relative to x_M, as $x_C \geq x_M$, the private currency has no impact on citizens' welfare.*

2. *When x_C is intermediate, as $\phi(x_M) \leq x_C < x_M$, the private currency makes citizens better off.*

3. *When x_C is low relative to x_M, as $x_C < \phi(x_M)$, the private currency makes citizens worse off.*

Case 1 in Proposition 7 arises when x_C is so large that the private currency does not constrain the goverment and therefore does not change the welfare of citizens. The two other cases arise when $x_C < x_M$ so that the presence of the private currency constrains the government and curbs inflation and x. As can be seen in the figure, in case 2, x_C is not far from x^*. So the impact of the private currency is to bring inflation and investment closer to the level that is optimal for citizens, thus making citizens better off. In case 3, in contrast, x_C is very low, and severely constrains the government. This leads to such a low level of inflation, and correspondingly investment, that citizens are worse off than without the private currency.

We now investigate for which parameter values we are in case 1, case 2, or case 3. As can be seen in equation (27), x_M is increasing in β but independent from λ. This is intuitive: x_M is the investment rate chosen by the government when there is no private currency, in which case λ is irrelevant. Moreover, as β

[20] Intuitively, $\phi(x_M)$ is the unique value of x in the domain $x < x_M$ that gives the agent the same utility as x_M.

increases the government cares more about the level of public spending, which is increasing in x. In addition, as can be seen in (44), x_C is increasing in λ and independent from β. Again, this is intuitive. As λ increases, the private currency becomes more risky, and therefore less attractive. So the constraint imposed by the private currency on government policy is relaxed. This discussion along with Proposition 7 are illustrated in Figure 2.

The lower curve in Figure 2 depicts $\phi(x_M(\beta))$, which, as can be seen in Figure 1, is decreasing in $x_M(\beta)$. The upper curve depicts $x_M(\beta)$. The intercept corresponds to $\phi(x) = x$, i.e., $x = x^*$.

- When x_C is above the upper curve, we are in case 1 of Proposition 7, in which the private currency has no impact. This case arises when the risk of the private currency, is so large that agents don't want to hold it.
- When x_C is between the upper and lower curves, we are in case 2 of Proposition 7. This case arises when λ is intermediate so that the private currency is attractive but not too attractive, and therefore can discipline the government and improve citizens' welfare.
- When x_C is below the lower curve, we are in case 3 of Proposition 7. This case arises when λ is very low, tightening the constraint imposed by the private currency on government policy, which, when the government is rather benevolent, reduces citizens' welfare.

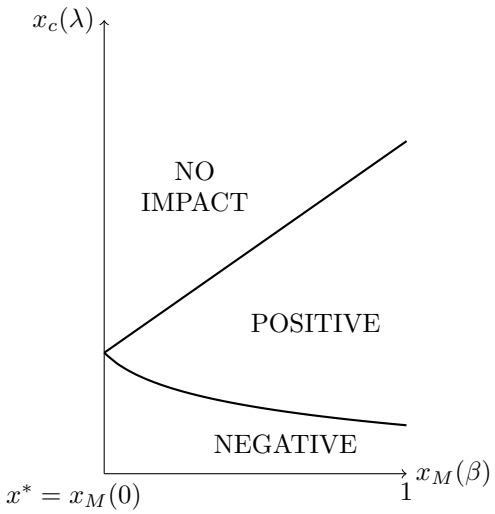

Figure 2: Impact of the private currency on citizens' welfare

4 Conclusion

In our model, fiat money is valuable because it allows agents to buffer uninsurable productivity shocks. When a predatory government has a monopoly over money issuance, it runs an expansionary monetary policy, giving rise to high inflation, compelling agents to save by investing in risky productive assets, which the government taxes to fund large public spending. In the limit, when the government is highly predatory, this leads to hyperinflation, in which case money is valueless and agents only invest in risky assets. The correspondingly large risk exposure reduces agents' welfare.

In this context, a private currency competing with the public currency can, if not too risky, offer an attractive store of value to citizens. We show that competition from the cryptocurrency limits the government's ability to run an inflationary policy. When the government is highly predatory, this increases the welfare of citizens, which echoes Hayek's (1976) advocacy for the denationalization of money. In contrast, when the government is rather benevolent, if competition from the private currency severely constrains the government policy, this reduces citizens' welfare.

A first look at empirical patterns suggests this mechanism may be at play in practice. For example, using cryptocurrency adoption rates from Statista and inflation rates from the World Bank and the International Monetary Fund, we found, for a cross section of 56 countries, an inflation rate of 4 % for the lowest cryptocurrency adoption quartile and an inflation rate of 13.9 % for the highest cryptocurrency adoption quartile. An interesting avenue of further research would be to conduct a more advanced confrontation of our theoretical results with data.

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Appendix A: Proofs

Proof of Proposition 1 :

Equation (5) implies that

$$wu'(w) = -w^2u''(w) = \frac{1}{\rho}.$$

The agent's decision problem (4) therefore simplifies to

$$\max_{c,k\leq w} \log c + \frac{1}{\rho}\left(\frac{(\mu+\pi)k}{w} + \tau - \frac{c}{w} - \pi\right) - \frac{\sigma^2k^2}{2\rho w^2}.$$

Taking the first order conditions with respect to c and k, we obtain the consumption and investment choices stated in Proposition 1. Substituting the optimal choices into the objective, we obtain the value function stated in Proposition 1. Taking the derivative of U with respect to π and using the envelope theorem, we obtain that the agent utility decreases with π.

QED

Proof of Lemma 1:

Substituting into the wealth dynamics equation (3) the consumption and investment choices spelled out in Proposition 1, the dynamics of the agent's wealth rewrites

$$\frac{dw^i}{w^i} = (x^i\mu + \tau - \pi(1-x^i) - \rho)dt + \sigma x^i dZ^i.$$

The budget balance condition (12) and the money growth equation (15) imply that the transfer rate τ is such that

$$\tau = (g+\pi)(1-x) - \gamma x. \tag{52}$$

So the dynamics of the agent's wealth rewrites

$$\frac{dw^i}{w^i} = (x^i\mu + (g+\pi)(1-x) - \gamma x - \pi(1-x^i) - \rho)dt + \sigma x^i dZ^i.$$

Using the real growth rate equation (11) and imposing that in equilibrium, by symmetry, the investment policy is the same for all agents, i.e., $x^i = x$, we obtain the following wealth dynamics

$$\frac{dw}{w} = gdt + \sigma x dZ. \tag{53}$$

Since $c_t = \rho w_t$ and $k_t = xw_t$, by (53) we have

$$logc_t = log\frac{\rho k_t}{x} + (g - \frac{\sigma^2}{2}x^2)t + \sigma x Z_t.$$

This yields (21).

Taking the first-order condition with respect to x in (21), the optimal x is such that

$$\frac{\sigma^2}{\rho}x^3 + x = 1. \tag{54}$$

Now, the unique real solution to (54) is less than one, which completes the proof.

QED

Proof of Lemma 2:

The first-order condition with respect to consumption implies $c = \rho w$. The portfolio problem of the agent is to maximize

$$\mu x + \tau(b + x) - \pi b - \hat{\pi}(1 - b - x) - \frac{\sigma^2 x^2}{2} + \lambda \log(x + b\frac{p}{p^+}), \tag{55}$$

under the constraint $x + b \leq 1$.

The Lagrangian of the agent's portfolio problem is

$$\mathbb{L} = \mu x - \tau(b + x) - \pi b - \hat{\pi}(1 - b - x) - \frac{\sigma^2 x^2}{2} + \lambda \log(x + b\frac{p}{p^+}) + \nu(1 - b - x).$$

The first-order condition with respect to x yields:

$$\mu - \tau + \hat{\pi} - \sigma^2 x + \frac{\lambda}{x + b\frac{p}{p^+}} = \nu. \tag{56}$$

The first-order condition with respect to b yields:

$$-\tau - \pi + \hat{\pi} + \frac{\lambda\frac{p}{p^+}}{x + b\frac{p}{p^+}} = \nu. \tag{57}$$

QED

Proof of Proposition 4:

When the private currency is held, then (33) implies (34).

Substracting the first order condition with respect to b from the first order condition with respect to x, we obtain

$$\sigma^2 x = \mu + \pi + \frac{\lambda}{x + b\frac{p}{p^+}}(1 - \frac{p}{p^+}) \tag{58}$$

That is

$$\frac{\lambda}{x + b\frac{p}{p^+}} = \frac{\sigma^2 x - (\mu + \pi)}{1 - \frac{p}{p^+}}$$

Substituting in the first order condition with respect to b, we obtain (35).

QED

Proof of Lemma 3:

Substituting (37) and (38) into (58), we obtain

$$\sigma^2 x = \mu + \pi + \lambda(\frac{x_M}{x} - \frac{1 - x_M}{b}). \tag{59}$$

Now, in equilibrium we have $g = \mu - \rho/x - \gamma$ and $g_m = g + \pi$, which implies

$$\pi = g_m - \mu + \rho/x + \gamma.$$

Substituting in (59) yields (42).

Substituting (38) into (56), we have

$$\mu - \tau + \hat{\pi} - \sigma^2 x + \lambda \frac{x_M}{x} = \nu. \tag{60}$$

Moreover (37) and (38) imply

$$\frac{\frac{p}{p^+}}{x + b\frac{p}{p^+}} = \frac{x_M}{x} \frac{x(1 - x_M)}{b x_M}.$$

Substituting into (57), we have

$$\tau - \pi + \hat{\pi} + \lambda \frac{x_M}{x} \frac{x(1 - x_M)}{b x_M} = \nu. \tag{61}$$

Multiplying (60) by x and (61) by b and adding the two, we get

$$\sigma^2 x^2 + \lambda + (\mu - \tau + \hat{\pi})x + (-\tau - \pi + \hat{\pi})b = \nu.$$

Using (11), (12) and (41), this simplifies to

$$\sigma^2 x^2 + \lambda + \rho = \nu,$$

which completes the proof.

QED

Proof of Proposition 5:

Consider the case in which $\nu = 0$. In that case, by (43), we have $x = x_C$ and $\sigma^2 x^2 = \rho + \lambda$. Substituting the values of x and ν in (42), we have

$$I = \lambda(1 - x_M)(\frac{1}{x_C} + \frac{1}{b}). \tag{62}$$

Since $\nu = 0$, we have $b < 1 - x_C$. Thus, (62) implies

$$I > \lambda(1 - x_M)(\frac{1}{x_C} + \frac{1}{1 - x_C}) = \lambda(1 - x_M)\frac{1}{1 - x_C}\frac{1}{x_C} = I_C. \tag{63}$$

If (63) holds, there exists a unique $b < 1 - x_C$ such that (62) holds:

$$b = \frac{1}{\frac{I}{\lambda(1 - x_M)} - \frac{1}{x_C}}.$$

Consider the case in which $\nu > 0$. In that case (42) writes

$$I = \sigma^2 x - \frac{\rho}{x} - \lambda[\frac{x_M}{x} - \frac{1 - x_M}{1 - x}]. \tag{64}$$

The right-hand side of (64) is increasing in x, so its root is lower than or equal to x_C if

$$I \leq \sigma^2 x_C - \frac{\rho}{x_C} - \lambda[\frac{x_M}{x_C} - \frac{1 - x_M}{1 - x_C}] = I_C. \tag{65}$$

QED

Proof of Lemma 4:

Equations (30) and (38) along with $c_t = \rho w_t$ imply

$$\frac{dw}{w} = x\mu - (\tau(b + x) + \rho + \pi b + \hat{\pi}(1 - b - x)) - (1 - \frac{x}{x_M})dN + \sigma x dZ \tag{66}$$

By (40)

$$\tau(b + x) = \gamma x - (g + \pi)b.$$

Substituting in (66) we obtain that the deterministic part in (66) is $\mu - \rho/x - \gamma$. Standard computations then yield (49). QED

Proof of Proposition 6:

By equation (49), U_G is quasi-concave in x and has an unconstrained maximum for $x = x_M(\beta)$. Hence, when $x_C > x_M$ the government finds it optimal to set $x = x_M$, while when $x_C \leq x_M$ the government finds it optimal to set $x = x_C$. Equation (49), also implies that the public expenditures ratio is the same as without the private currency, i.e., ¡$\gamma = \gamma_M$ defined in equation (**??**). Since the public expenditures ratio is the same with and without the private currency, official money supply growth is larger without the private currency if and only if the inflationary pressure index is larger with the private currency. By equations (19) and (42) this is equivalent to

$$\sigma^2 x_M - \frac{\rho}{x_M} > \sigma^2 x_C - \frac{\rho + \lambda x_M}{x_C} + \lambda \frac{1 - x_M}{1 - x_C}. \tag{67}$$

That is

$$\sigma^2(x_M - x_C) > \rho(\frac{1}{x_M} - \frac{1}{x_C}) - \lambda(\frac{x_M}{x_C} - \frac{1 - x_M}{1 - x_C}). \tag{68}$$

That is

$$\sigma^2(x_M - x_C) > \rho \frac{x_C - x_M}{x_M x_C} + \lambda \frac{x_C - x_M}{x_C(1 - x_C)}. \tag{69}$$

This holds when $x_M > x_C$.

QED

Appendix B: Microfoundation

This appendix is adapted from Biais, Gersbach, Rochet, von Thadden, and Villeneuve (2025). First we briefly recall the dynamic optimal mechanism analyzed in Biais, Gersbach, Rochet, von Thadden, and Villeneuve (2025), Second we show that, for $\beta = 0$, the equilibrium analyzed in the present paper when there is no private currency implements the dynamic optimal mechanism characterized by Biais, Gersbach, Rochet, von Thadden, and Villeneuve (2025).

There is a principal and a continuum of agents. There is a single good in the economy, which can be consumed or invested in a risky technology, like in the present paper. The principal does not have the skills necessary to operate the technology. So any investment in the productive technology must be delegated to the agents. The latter privately observe their output. The objective of the principal is to maximize the integral of the discounted expected utilities of all the agent. Biais, Gersbach, Rochet, von Thadden, and Villeneuve (2025) characterize the dynamic optimal mechanism, which allocates capital and consumption to the agents, subject to budget and incentive constraints.

Without loss of generality one can focus on incentive compatible direct mechanisms in which agents report their output on each date. The mechanism allocates capital k_t^i and private consumption c_t^i to each agent i as a function of agents' reports. The optimal mechanism is characterized using the recursive approach introduced by Sannikov (2009). It depends on aggregate capital K_t and the continuation pay-offs of the agents:

$$\omega_t^i \equiv \mathbb{E}_t \int_t^\infty \rho e^{-\rho(s-t)} [\log c_s^i] ds.$$

By the martingale representation theorem:

$$d\omega_t^i = \rho(\omega_t^i - \log c_t^i) + \sigma x_t^i dZ_t^i. \tag{70}$$

where x_t^i denotes the sensitivity of future pay-off to current performance. In this context, incentive compatibility is equivalent to $x_t^i \geq k_t^i u'(c_t^i)$. Incentive compatibility implies a minimum sensitivity of an agent's pay-off to his performance. This rules out perfect insurance.

As shown in Biais, Gersbach, Rochet, von Thadden, and Villeneuve (2025), the characteristics of the optimal mechanism are the following:

- Consumption is proportional to capital: $c_t^i = \frac{\rho k_t^i}{x^*}$,
- The growth rate is constant: $g = \mu - \frac{\rho}{x^*}$
- Capital is allocated as a function of individual performance:

$$\frac{dk_t^i}{k_t^i} = gdt + \sigma x^* dZ_t^i.$$

- x^* is the unique solution of

$$\frac{\sigma^2}{\rho} x^3 + x = 1.$$

- There is a constant a^* such that $k_t = a^* \exp \omega_t$.

The allocations obtaining in Biais, Gersbach, Rochet, von Thadden, and Villeneuve (2025) and in the present paper are summarized in Table 1. Inspecting the table reveals that the allocations arising in the equilibrium of the present paper (without private currency) and the optimal mechanism in Biais, Gersbach, Rochet, von Thadden, and Villeneuve (2025) are the same.

Table 1: Comparison of the optimal mechanism in Biais et al (2025) and the equilibrium in the present paper

	Optimal mechanism (Biais et al. 2025)	Equilibrium (this paper)
capital allocations	$k_t = a^* \exp \omega_t$	$k_t = xw_t$
consumption	$c_t = \frac{\rho k_t}{x^*}$	$c_t = \rho w_t$
growth rate	$g = \mu - \frac{\rho}{x^*}$	$g = \mu - \frac{\rho}{x}$
capital dynamics	$\frac{dk_t}{k_t} = gdt + \sigma x^* dZ_t$	$\frac{dk_t}{k_t} = gdt + \sigma x dZ_t$

Appendix C: Cryptocurrencies, stablecoins, and dollarization

Is the economic mechanism in our analysis the same as in dollarization? To address this question one should first note that there are several forms of dollarization (see Levy - Yeyati 2006, 2021). First, *official* (de jure) dollarization occurs when the foreign currency is officially chosen by the government to replace or coexist with the local public currency. Obviously this is different from what we study in the present paper. Second, *financial* dollarization occurs when agents hold financial assets and liabilities in a foreign currency. Again, this differs from the phenomenon studied in this paper, in particular because financial dollarization occurs within the traditional financial system, which the government can control. For example, in Argentina in 2002 the government imposed a compulsory conversion of dollar deposits to pesos ("pesification"). This contrasts with the protection from government predation which is one of the advantages of the private currency in our analysis.[21] Third, a more informal type of dollarization occurs when foreign banknotes (e.g., greenbacks) are used in the home country. Unlike the private currency we analyze, banknotes can be robbed and this can be dangerous for the agent holding them.[22] Moreover, digital currencies whose ownership is registered on blockchain can be safely and rapidly transferred electronically. So, in a sense, the private currency in our paper enables agents to take advantage of a more efficient technology: blockchain, to better achieve what agents were trying to achieve with foreign banknotes.[23]

For countries outside the US, with dysfunctional financial and monetary

[21] It is difficult for governments to prevent cryptocurrency ownership and transfers taking place anonymously on blockchain. Even in China there still is some cryptocurrency ownership, in a legal grey area reflecting the technological difficulty of controlling blockchain. A fortiori, in countries where the government is weaker and less tech-savvy than in China, cryptocurrency ownership remains largely beyond the control of the government.

[22] There are wrench attacks, i.e., physical attacks against cryptocurrency owners to steal their cryptocurrencies, but they remain rare and require technological sophistication, e.g., hacking Know Your Customer databases giving information on the identity of cryptocurrency owners.

[23] Hoarding gold is similar to hoarding foreign banknotes, but like banknotes gold exposes its owner to the risk of being robbed.

systems, stablecoins offer a digital vehicle for dollarization. Indeed, the private currency in our model could be interpreted as a cryptocurrency or as a stablecoin. Both are registered on blockchain and offer agents an instrument to store value and make payments that is immune from local currency inflation. Crash risk and volatility are much larger for cryptocurrencies than for stablecoins, which in our model translates into a larger value of λ for the former than for the latter. That being said, stablecoins also are exposed to crash risk. That risk was vividly illustrated at the time of the Silicon Valley Bank (SVB) crash, which would have triggered a severe loss of value for USDC holders if the US government had not bailed out SVB depositors. The literature also emphasizes that stablecoins remain exposed to the risk of runs leading to sharp drops in value (see eg., Ma, Zeng, and Zhang (2025)).