Financing via Partially Liquid Tokens*

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Abstract

We develop a Diamond-Dybvig-style model in which a non-bank firm issues tokens backed by its future services. Consumers face uncertain liquidity demand and costly ex-post borrowing. Tokens are partially liquid–they provide liquidity for the firm's service but not other consumption goods, creating an endogenous liquidity premium. This premium is influenced by the imperfect substitution between tokens and fully liquid claims, leading to a non-monotonic relationship with the cost of illiquidity. The issuing firm has a stronger incentive to issue tokens when it provides high-value services with infrequent demand or when the demand for the service is positively correlated with the demand for other consumption goods. In terms of token design, we characterize conditions under which the firm may offer consumers more flexibility in token issuance and redemption, potentially making tokens tradable. For tradable tokens, the firm may also permit conversion back to cash at a one-to-one rate, resembling stablecoins prevailing in practice.

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1 Introduction

Firms across various industries are increasingly issuing their own tokens. Airlines reward customers with miles that can be applied towards future flights; retailers, such as Amazon, issue rewards that can be used for future purchases; technology platforms such as Expedia issue rebates that can be applied to future purchases. Recently, firms are exploring corporate “stablecoins”—digital tokens pegged to fiat currency, which can be used for payments within a particular ecosystem. These tokens are partially liquid: they are redeemable for a particular product/service but are not universally convertible into cash. Importantly, such tokens can be viewed through a prism of financing: issuing (selling) tokens can be used to raise funds and supplement traditional fundraising mechanisms.

Theoretical foundations of corporate token issuance remain underdeveloped. Why would a firm issue its own partially liquid tokens rather than rely on standard financing? How do tokens interact with the traditional banking system that already supplies liquidity to firms (and to consumers) through demand deposits and credit lines? Under what conditions can firm-issued claims coexist with bank deposits in providing liquidity?

Our paper develops a model that addresses these questions by extending the canonical Diamond-Dybvig setting to two products/services. In Diamond and Dybvig (1983), banks create liquidity by transforming illiquid long-term assets into demand deposits. We adapt this perspective to examine a non-bank firm that issues redeemable claims—tokens—backed by its own future provision of service. The firm sells tokens to consumers who face uncertain demand for the firm’s service as well as for other consumption. The tokens provide targeted liquidity: they can be redeemed for the firm’s service and are valued at a discount if not redeemed.

The firm’s incentive to issue tokens originates in the liquidity premium, which is the excess value consumers assign to tokens because they relax future liquidity constraints—that consumers are willing to pay up front. Intuitively, consumers face uncertainty about whether they would need to make payments to satisfy future demand, and accessing liquidity at the moment of need is costly. When the expected cost of obtaining liquidity ex-post is high, consumers place greater value on holding an asset that guarantees purchasing power inside the firm’s platform. Essentially, by contracting via tokens, the firm and consumers taken together reduce the expected deadweight loss due to expensive ex-post borrowing to satisfy liquidity needs.

Our paper contributes to two strands of literature. First, it expands the liquidity-creation setting of Diamond and Dybvig (1983) by developing a model of partially liquid corporate claims. The model shows that firms issue redeemable tokens that coexist with demand deposits and provide an alternative financing method. The model characterizes conditions under which firms choose to issue tokens and the price at which the tokens are issued. It also examined how market characteristics—such as the the amount of the firm's service demanded and the frequency at which it is demanded, the correlation between the demand for the firm's service and the demand for other consumption, and design features---such as the possibility of partial redemption, ex-post token issuance, tradability, and convertibility---affect equilibrium token issuance. Whereas the Diamond-Dybvig model focuses on banks transforming long-term assets into liquid deposits, our model shows that non-bank firms can perform an analogous transformation by issuing tokens backed by future products/services.

Second, our paper extends recent theoretical work on redeemable tokens (e.g., Rogoff and You (2023) and Luo (2025)), which emphasizes financing-cost differentials or regulatory arbitrage. In contrast, we focus on a pure liquidity channel: tokens arise in equilibrium because they deliver ex-ante liquidity when ex-post credit is expensive. This interpretation fits the observable behavior of firms that issue tokens that are redeemable for future products/services at consumers' discretion. Despite being inferior to fully liquid money or demand deposits, partially liquid tokens provide targeted liquidity to consumers, allowing issuers to earn liquidity premium without paying high cost of liquidity provision. The liquidity premium the firm earns arises endogenously in the model and depends on the imperfect substitution in liquidity between the tokens and other financial claims.

In our model, the economy contains three types of agents: consumers, a firm, and a bank, operating over three dates. Consumers allocate their initial wealth between the following financial instruments. The first is long-term investment, generating a high return but being illiquid at the intermediate date. The second is zero (or low)-interest deposits, which are fully liquid and can be used for any consumption anytime. The third is tokens issued by the firm, which can be redeemed anytime for the firm's service but not for other consumption goods. Tokens are therefore more liquid than long-term investment but less liquid than deposits. Consumers can also borrow to satisfy their liquidity needs at a rate higher than the return on long-term investment (equivalently, liquidate their investment at a cost). This borrowing cost relative to the return on investment embodies the cost of illiquidity, which gives consumers the incentive to hold liquid claims ex ante. Consumers' objective is to minimize the expected cost of satisfying whatever consumption needs arise once uncertainty is realized. The firm raises funding via two channels: borrowing at a rate equal to consumers' investment return and/or issuing tokens backed by provision of future service. The bank collects consumers' deposits, and it allocates the raised deposits into two types of investment while satisfying consumers' withdrawal of deposits. First, it makes a long-term investment, similar to consumers. Second, it lends money to consumers, providing ex-post liquidity after consumers' consumption demand is realized.

We deliberately assume no difference in interest rates between the firm and consumers, so all financing benefits the firm can receive from tokens must derive from liquidity provision. Specifically, the financing benefits are determined by the difference between consumers' willingness-to-pay for tokens and the firm's cost to satisfy their redemption, which we refer to as the liquidity premium of tokens or the LPT for short.

Our first main result concerns the determination of the LPT. In models with only one liquid claim, the liquid claim features a positive liquidity premium, which is determined by its advantage relative to illiquid claims. A unique feature of our model is that fully liquid deposits and partially liquid tokens coexist. Therefore, the LPT is also affected by the imperfect substitution between them. The first way that the substitution works is that deposits may directly serve as consumers' outside option to tokens. For consumers, the outside option to tokens could be illiquid investment or deposits, and the exact outside option is endogenous. As the cost of illiquidity increases, consumers would prefer to hold higher levels of liquid claims—deposits and tokens. When the cost of illiquidity is very high or very low, consumers decide between holding tokens and holding more deposits. The LPT is then determined by the disadvantage of tokens relative to deposits. When the cost of illiquidity is in an intermediate range, consumers decide between holding tokens and holding more investment. The LPT is then determined by the advantage of tokens relative to investment. As a result of the switching outside option, the LPT has a non-monotonic relationship with the cost of illiquidity, which does not appear in models with a single liquid claim.

When the cost of illiquidity is in an intermediate range, although deposits do not serve as consumers' outside option to tokens, the substitution works in more subtle ways. In that case, consumers will hold deposits that are sufficient for satisfying potential demand for either the firm's service or other consumption. Thus, consumers potentially add tokens to the mix with the sole goal of reducing costly borrowing when the two demands appear simultaneously. On the other hand, without these holdings of deposits, tokens help consumers reduce borrowing whenever there is demand for the service. Therefore, deposits decrease the marginal liquidity provided by tokens and thus decrease the LPT. The substitution also works through the order in which consumers use liquid claims. Without tokens, consumers directly use deposits to satisfy demand for all types of consumption including the service. With tokens, consumers prioritize using tokens when they need the service. The change in the order of liquidity usage makes the firm's liquidity provision more costly, as consumers redeem tokens more often.

The imperfect substitution between liquid claims generally dampens the LPT. We find that the effect could be so strong that it renders the LPT negative and substantially discourage the firm from issuing tokens. This takes place when the demand for the two consumption goods is negatively dependent and the relative cost of borrowing is in an intermediate range.

People tend to think that tokens will make banks worse off, as it decreases deposits due to their substitution in terms of liquidity. However, we find that for a range of cost of illiquidity, tokens in fact increases deposits because of the change in the order of liquidity usage. As discussed above, the change in the order leads to more redemption of tokens. The flip side of this effect is less withdrawal of deposits when the demand for the firm's service is satisfied via tokens. The key new insight here is that the addition of a new liquid claim not only decreases consumers' incentive to hold the current liquid claim, but also decreases consumers' incentive to use it. When the second effect dominates, the issuers of the current liquid claim can in fact benefit from the addition of a new liquid claim.

Our second result concerns how the characteristics of consumption demand affects the LPT. First, the LPT is higher for firms providing products/services that are relatively expensive and infrequently demanded. The higher the price of the firm's service is, the higher the amount of deposits required to be held to satisfy demand for the service, regardless of the frequency of demand for it. In addition, less frequent demand for the service lowers the firm's cost to provide liquidity for it. Thus, the firm has more room to issue tokens to substitute for bank-issued deposits in this case. Second, the LPT is increasing in the correlation between the demand for the firm's service and other consumption. The reason is twofold. When the cost of illiquidity is low, the LPT is partially determined by the disadvantage of tokens relative to deposits, which manifests itself when consumers demand only consumption other than the firm's service—an occurrence whose probability is decreasing in the correlation between demands. When the cost of illiquidity is higher, tokens are more useful when there is demand both for the firm's service and for other consumption—an event whose likelihood is increasing in the correlation between demands. Third, the LPT is decreasing in the size of the demand for consumption other than the service. Since only deposits can provide liquidity for other consumption, greater demand for other consumption induces consumers to hold more deposits, reducing the demand for tokens to a greater extent.

We proceed by extending the baseline model in multiple directions. First, we examine the effects of giving consumers more flexibility in two potential ways: partial redemption of tokens and ex-post token issuance. The former allows consumers to use a combination of tokens and cash for buying the firm's service, giving more flexibility in token redemption. The latter allows consumers to buy tokens after their consumption demand is realized, giving more flexibility in token issuance. From the firm's perspective, giving this flexibility to consumers is double-edged. On the one hand, it introduces additional constraint on the token's price. On the other hand, consumers is better able to decide on the amount of tokens to hold and may gain more liquidity benefits, which may in return increase the LPT that the firm can earn. Our main finding is that the flexibility in the two dimensions is essentially equivalent and beneficial for the firm only when the price of the firm's service exceeds the size of the demand for other consumption and consumers are lukewarm about tokens. The latter takes place when the demand for the two consumption goods is negatively dependent and the relative cost of borrowing is in an intermediate range.

Second, we examine the firm's decision of making tokens tradable. Tradability means consumers can trade tokens in a competitive market at an endogenous price. Tradability have potentially two effects on the quantity and the pricing of tokens. First, tradability makes it easier for consumers to get rid of tokens and imposes an upper bound on the token price. To incentivize consumers to hold tokens into the long term, the firm needs to set the token price to be lower than the long-term value of tokens. Second, tradability increases the liquidity of tokens and potentially increase consumers' holding of tokens. If consumers decide to hold nontradable tokens, they would hold enough to redeem for the firm's service. Tradable token holdings are determined as part of overall liquidity that may be required to satisfy both the demand for the firm's service and the demand for other consumption. We find that making tokens tradable is more attractive for the firm if the long-term value of tokens is higher, due to the first effect, and if the firm providing low-value, high-frequency services, due to the second effect.

Further, we find that given that tradability has made tokens sufficiently liquid, giving consumers more flexibility through partial redemption or ex-post token issuance is not beneficial, but making tokens convertible is. Convertibility refers to that the firm promises to convert tokens back to cash at a pre-specified price at consumers' discretion. Convertibility does not increase the liquidity of tokens but may raise the long-term value of tokens, more so the higher the conversion rate. Thus, the firm would like to set the conversion rate as high as possible, i.e. at a one-to-one rate, resembling a stablecoin. This result is important as it provides a novel justification for the use of corporate stablecoins—which are at the center of current regulatory debate.

The remainder of the paper proceeds as follows. In the rest of the introduction, we review the related literature and discuss examples and economic significance of partially liquid claims. Section 2 describes the setup of the model and the problems each of the types of agents—consumers, the firm, and the bank—are facing, as well as introducing various financial instruments. Section 3 solves the baseline model—first with only a single liquid claims, then with multiple present. Section 4 examines various comparative statics of the model related to the characteristics of consumption goods and discuss policies that give consumers more flexibility in obtaining and redeeming tokens. Section 5 extends the model to allow for token tradability and convertibility. Section 6 concludes.

Related literature

Our paper examines a new form of financing from consumers and is thus related to the literature on raising funds from stakeholders. Trade credit, for instance, raises funds from suppliers (Biais and Gollier 1997; Burkart and Ellingsen 2004; Chod, Lyandres, and Yang 2019; Lehar, Song, and Yuan 2020). The literature argues that suppliers are better able to mitigate the financial frictions faced by external investors, such as asymmetric information and lack of commitment. Inside debt or equity raises funds from executives and could be a superior solution to the agency costs of debt than other forms of executive compensation (Sundaram and Yermack 2007; Edmans and Liu 2011). Crowdfunding and token financing can be used to raise funds from consumers or users. Strausz (2017) considers the benefit of crowdfunding as a way of acquiring information about the eventual payoff of the project if demand is uncertain (see also Astebro, Fernandez Sierra, Lovo, and Vulkan 2017; Chemla and Tinn 2020; Ellman and Hurkens 2019; Cong and Xiao 2024). Lee and Parlour (2022) find that raising fund from consumers rather than investors can be more efficient, because consumers are able to commit to paying for the consumption benefit ignored by investors. Howell, Niessner, and Yermack (2020) and Lyandres, Palazzo, and Rabetti (2022) empirically examine the factors that determine successful initial coin offerings (ICOs). The theory literature on ICOs centers the discussion around the network effect and decentralization (Chod and Lyandres 2021; Cong, Li, and Wang 2021; Gryglewicz, Mayer, and Morellec 2021; Cong, Li, and Wang 2022; Sockin and Xiong 2023b,a; Li and Mann 2024; Goldstein, Gupta, and Sverchkov 2024). Recently, Rogoff and You (2023), He, Rogoff, and You (2024), and Luo (2025) study a firm or a platform issues redeemable tokens to raise funds from consumers. In their frameworks, the gains from trade of this token financing derive from exogenous difference in interest rates between the issuer and consumers, so the issuer basically cares about the amount of funds they can effectively raise from consumers. In contrast, we focus on a pure liquidity channel: tokens arise in equilibrium even absent any difference in interest rates, because they deliver ex-ante liquidity when ex-post credit is expensive. Due to this feature, the issuer has the incentive to make tokens more convenient so that it can earn a higher LPT. Therefore, our paper can shed light on when token issuers should provide more liquidity and what type of liquidity they should provide. Prat, Danos, and Marcassa (2025) consider the liquidity value of tokens by introducing a cash-in-advance constraint. In their framework, consumption demand can only be satisfied using tokens, so tokens provide full liquidity instead of partial liquidity. Also, they focus on the price dynamics of tokens on the secondary market and do not study token issuance and its impact.

Our paper expands the liquidity-creation setting of Diamond and Dybvig (1983) by developing a model of partial liquidity. Whereas the Diamond-Dybvig model focuses on banks transforming long-term assets into liquid deposits, our model shows that non-bank firms can perform an analogous transformation by issuing tokens backed by future products/services. Existing banking literature primarily focuses on the risks associated with the collective withdrawal of deposits—bank runs. However, our analysis considers a previously underexplored consequence: liquidity provision renders the raised funds endogenous and dependent on firms' issuance and redemption policies and business characteristics. This feature could potentially affect the strategies firms employ in issuing rewards as well as the types of firms that choose to issue tokens.

Our paper proposes a novel finance motive for partially liquid tokens such as rewards, thereby contributing to the extensive literature that seeks to rationalize rewards. Behavioral theories posit that rewards can yield psychological benefits or create pricing misconceptions. For instance, Lim, Chun, and Satopää (2021) discovers that certain consumers attribute more value to rewards than to actual money. In the literature of industrial organization, a traditional viewpoint is that rewards implement price discrimination based on purchase history or quantity. Cremer (1984) demonstrates that rewards can be used to differentiate between first-time and repeat buyers. Sun and Zhang (2019) postulate that the use of limited reward expiration terms could be driven by the intent to discriminate between frequent and infrequent customers. In a multi-brand environment, LPs can increase consumer switching costs, thereby weakening competition (Klemperer, 1987, 1995). Particularly, Banerjee and Summers (1987) and Caminal and Matutes (1990) are the first to analyze rewards as a form of endogenous switching cost. Kim, Shi, and Srinivasan (2001) shows that by offering the incentives for repeat purchases, rewards increase a firm's cost to attract competing firms' current customers. Ke, Shin, and Zhu (2024) analyzes rewards from the perspective of product search. In their framework, switching costs originate from search costs, and firms use LPs to make consumers prioritize their products when doing sequential search. Basso, Clements, and Ross (2009) argue that rewards, such as airlines' frequent-flier programs, can exploit agency conflicts between employers and employees. Chod and Lyandres (2023) argues that pricing output in tokens provides a firm with a de facto second-mover advantage. Several recent studies explore firms' optimal decisions in the context of rewards (Kim, Shi, and Srinivasan, 2004; Chun and Ovchinnikov, 2019; Chun, Iancu, and Trichakis, 2020). Taken together, the existing literature predominantly interprets rewards as instruments to boost business, while our paper focuses on their financing benefits.

Examples and Economic Significance of Partially Liquid Claims

Partially liquid claims constitute a diverse and economically significant category of financial instruments. Unlike fully liquid claims, which can be readily converted into cash for unrestricted consumption, partially liquid claims grant holders access only to a specific set of goods or services, often within a defined ecosystem. Their liquidity is thus bounded by scope rather than solely by time or cost. This section outlines prominent examples and underscores their growing role in modern economies.

A canonical example is consumer rewards programs, such as frequent flyer miles and credit card points. These claims are redeemable for goods or services—flights, hotel stays, or merchandise—provided by the issuing entity but are typically non-redeemable for cash. As highlighted in industry data, the aggregate liability from such point systems is substantial, amounting to more than one hundred billion dollars across airlines, hotels, banks, and retailers in the U.S. (see the table below). Their economic footprint is now so pronounced that the former U.S. Transportation Secretary Pete

Buttigieg noted, "points systems like frequent flyer miles and credit card rewards have become such a meaningful part of our economy that many Americans view their rewards points balances as part of their savings."

	Airlines	Hotels	Banks	Retailer
Rewards liabilities (billion USD)	35	12	37	18

In certain service-oriented economies, stored-value cards represent another widespread form of partially liquid claim. Prevalent in sectors like dining, personal care, and wellness in markets such as China, these instruments are redeemable exclusively for services from the issuing merchant. They enhance customer retention and provide upfront capital for firms but restrict the holder's consumption choice to a single provider or network.

The digital realm has introduced new variants, most notably utility tokens within blockchain ecosystems. The value of utility tokens comes from the usefulness of the platform they support rather than from speculation alone. Projects issue these tokens to incentivize network participation, bootstrap early usage, or govern how resources within the system are allocated. These tokens lack general convertibility into cash on equivalent terms. Their liquidity is intrinsically linked to the utility and adoption of the underlying platform. Prominent examples of utility tokens include: Basic Attention Token (BAT), used within the Brave browser ecosystem; Chainlink's LINK token, powering a decentralized oracle network; Filecoin's FIL token, used in a decentralized storage marketplace; Axie Infinity's SLP token used within a gaming context; as well as many utility tokens of DeFi projects, such as Uniswap's UNI, Sushiswap's SUSHI, and Curve's CRV (three of the largest decentralized exchanges), Aave's AAVE and MakerDAO's MKR (two of the largest lending marketplaces), and Lido's LDO (largest liquid staking platform), among many others.

A hybrid case emerges with stablecoins, which are typically convertible to fiat currency but are designed to provide transactional convenience within specific domains. While present-day stablecoins like USDT and USDC facilitate on-ramp to and off-ramp from the crypto ecosystem and on-chain transactions, there is growing speculation that major retail platforms may issue their own stablecoins to streamline payments within their commercial ecosystems. Such instruments would function as partially liquid claims if they offered enhanced utility or incentives for platform-specific transactions, even while maintaining external convertibility.

Collectively, these examples illustrate that partially liquid claims are not peripheral anomalies but are embedded in the infrastructure of contemporary consumption and digital interaction. Their economic significance lies not only in their substantial aggregate value but also in their ability to shape consumption-saving patterns, redefine the boundaries of liquidity, and challenge traditional distinctions between money, credit, and specialty currencies.

2 Model Setup

We modify and extend the Diamond-Dybvig setup by allowing for multiple consumption goods—"service" supplied by the firm and "other consumption", summarizing all other goods and services, and introducing tokens that can be redeemed for the firm's service but not for others. Consider an economy with three dates: 0, 1, and 2. There are three types of agents in the economy: a unit measure of consumers, a firm, and a bank. All agents are risk-neutral and do not discount future cash flows.

2.1 Consumers

Consumers are infinitesimal and homogeneous. They are endowed with an initial wealth of w_0 at date 0, and do not receive any other income throughout the game. At date 0, consumers allocate their wealth among several financial instruments, which are introduced below. At date 1, each consumer learns whether she faces demand shocks for the firm's service as well as for the other consumption. If the demand is satisfied immediately on date 1, it generates a utility of $\alpha > 1$ for each dollar spent. At date 2, all financial instruments deliver cash flows. Since date 2 essentially represents all dates after date 1, we assume that the demand for the two consumption goods at date 2 is sufficiently large that the possible cash flows can all be consumed. The consumption at date 2 generates a utility of 1 for each dollar spent. We denote a consumer's expected utility by U_c.

Table 1 summarizes consumption demand shocks a consumer may face at date 1. With probability q_s, she demands the firm's service. The service is indivisible and costs p_s. With probability q_c, she demands other consumption that costs p_c. With probability q_{cs}, the consumer demands both the firm's service and other consumption. The probability that a consumer demands neither the firm's service nor other consumption is then $1+q_{cs}-q_s-q_c$. Intuitively, p_s and p_c represent the sizes of the consumption demand shocks related to the two goods, respectively, while q_s and q_c represent the frequency of the shocks. $\rho \triangleq q_{cs}/q_cq_s$ represents the dependence between the demand for the two consumption goods. We assume $q_sp_s \leq q_cp_c$. Since the firm's service is just one specific consumption category among many, the expected demand for it is not greater than the total expected demand for other consumption.

	Need a service	Don't need a service
Need other consumption	q_{cs}	$q_c - q_{cs}$
Don't need other consumption	$q_s - q_{cs}$	$1+q_{cs}-q_s-q_c$

Table 1: The probability of consumption demand at date 1

2.2 Financial Instruments

Similar to Diamond and Dybvig (1983), consumers earn proceeds from investment in long-term production, while liquidity is needed to satisfy possible consumption demand during the investment. Here we outline the financial instruments that consumers can utilize to achieve these two goals.

The first instrument is "investment" into long-term production. Investment requires input at date 0, and generates proceeds of $R_L > 1$ per dollar of input at date 2, with the constraint that it cannot be liquidated at date 1.

The second instrument is "deposits" provided by the bank. More precisely, we are referring to demand deposits in practice. Consumers can make deposits and use them for both types of consumption anytime. They earn 0 interest from deposits, or effectively, the gross deposit rate is 1.[1] Throughout the paper, the terms—"deposits", "demand deposits", and "cash"are interchangeable.

The third instrument is "borrowing" from the bank. Consumers can borrow at date 1 and repay R_B per dollar of borrowings to the bank at date 2.[2] More broadly, we can interpret borrowing as any way for consumers to obtain liquidity when consumption shocks are realized, including liquidating investments at some cost. To keep the analysis simple, we assume that consumers never default, and have sufficient initial wealth, $w_0 \gg R_B(p_c + p_s)$, so that they can always borrow up to $p_c + p_s$ to fully satisfy their consumption demand at date 1.

The fourth instrument is "tokens" issued by the firm. Tokens are sold at a price of θ_0 per unit at date 0, which is endogenously set by the firm. Unlike deposits, tokens are only partially liquid: p_s units of token can be redeemed for a service at date 1, but not for other consumption. One token is worth v_2 if it is held to date 2. We assume exogenous $v_2 \leq 1$ to capture the potential devaluation and breakage of tokens over time.[3]

In the baseline model, we assume that partial redemption is not allowed. That is, a service can only be purchased by either all tokens or all cash, but not any combination of them. In Section 4, we consider the case in which the firm can choose to allow partial redemption.

With the four financial instruments, a consumer's decision can be characterized as follows. At date 0, a consumer chooses to hold d_0 in demand deposit and m_0 tokens, and allocates $w_0 - d_0 - m_0\theta_0$ to investment. At date 1, she withdraws Δ_d from the deposit, redeems Δ_m tokens, and borrows b_1 from the bank, all of which depend on a consumer's realized consumption demand. Since con-

[1] In practice, the interest rates of demand deposits are indeed close to zero at many banks. The qualitative results are robust to assuming non-zero return on demand deposits.

[2] Consumers only borrow to satisfy their consumption demand, so borrowing at date 0 or date 2 is irrelevant in the model.

[3] Breakage refers to the case that consumers do not redeem tokens. It may be because consumers do not find opportunities to redeem or simply they forget to do so. In practice, issuers often change token redemption conditions, which is another reason for potential breakage.

sumption shocks are idiosyncratic and consumers are infinitesimal, consumers' total withdrawal of deposits ($E[\Delta_d]$), total redemption of tokens ($E[\Delta_m]$), and total borrowing from the bank ($E[b_1]$) can be evaluated ex-ante.

Since investment only delivers cash flows at date 2, consumers need liquidity to satisfy their consumption demand at date 1. Conceptually, deposits and tokens provide ex-ante liquidity, while borrowing provides ex-post liquidity. They are natural but imperfect substitutes. We focus on the case in which the borrowing cost is sufficiently high that consumers find it unattractive to invest all the wealth in the long-term investment and borrow whenever demand shocks are realized. In other words, they face a non-trivial trade-off between ex-ante liquidity and ex-post liquidity. Specifically, we assume $(q_c + q_s - q_{cs}) \cdot (R_B - 1) > R_L - 1$, which, as shown in Section 3.1, implies that consumers hold positive amounts of deposits absent tokens.

2.3 The Bank

The bank makes profits in two ways. First, it can invest part of the raised deposits into long-term production at date 0. For simplicity, we assume that the bank faces the same long-term investment opportunities as consumers, which requires input at date 0, returns proceeds of R_L per dollar of input at date 2, and cannot be liquidated at date 1. Hence, the bank earns the difference between the deposit rate and the return on investment, $R_L - 1$, at date 2. Second, the bank can lend to consumers at date 1. Lending incurs a cost ε per unit, so the bank earns $R_B - \varepsilon - 1$ from this business. Meanwhile, the bank needs to have sufficient liquidity to satisfy consumers' withdrawal of deposits at date 1.

The deposit rate of 1 and the borrowing rate of R_B are taken as exogenously given. This can stem from the fact that these rates are determined by a broader market and are not sensitive to the decisions of the agents within the scope of the model. To ensure that borrowing plays a meaningful role in the model, we assume that $R_B \leq \alpha$ and $R_B - \varepsilon \geq R_L$. $R_B \leq \alpha$ implies that consumers are willing to borrow when they do not have liquidity to satisfy their consumption demand at date 1, while $R_B - \varepsilon \geq R_L$ implies that the bank is willing to leave sufficient deposits for lending at date 1 instead of investment at date 0.

Taken together, the bank receives d_0 deposits and invests $d_0 - E[\Delta_d] - E[b_1]$ in the long-term production at date 0, expecting $E[\Delta_d] + E[b_1]$ for withdrawal and borrowing at date 1. The bank's payoff is

$$\Pi_B = (d_0 - E[\Delta_d])(R_L - 1) + E[b_1](R_B - \varepsilon - R_L).$$

The first term represents the gain from the liability side—deposits. By providing ex-ante liquidity for consumers, the bank can obtain deposits at a rate lower than the return on investment. The second term represents the gain from the asset side—consumer lending. By providing ex-post liquidity for consumers, the bank can lend to them at a rate higher than the return on investment. Throughout the paper, we consider the bank as a passive player, and analyze the impact of token issuance on it through the lens of its two activities—long-term investment (at date 0) and short-term lending (at date 1).

2.4 The Firm

The firm's production requires a total of I_0 capital at date 0 and date 1 combined. To finance the investment, it borrows funds maturing at date 2. Suppose that the debt requires a repayment of R_F per dollar borrowed. Without tokens, the firm receives $q_s p_s$ from the sale of its service at date 1, so it needs to borrow $I_0 - q_s p_s$. By issuing tokens, the firm receives $m_0 \theta_0$ from the sales of tokens at date 0 and $q_s p_s - E[\Delta_m]$ from the sales of services at date 1, so it needs to borrow $I_0 - m_0\theta_0 - (q_s p_s - E[\Delta_m])$ at date 1. Token issuance reduces debt repayment by $(m_0\theta_0 - E[\Delta_m]) R_F$ at date 2. On the other hand, at date 2, consumers redeem the remaining $m_0 - E[\Delta_m]$ tokens, which reduces the cash flows from the sale of the service by $(m_0 - E[\Delta_m]) v_2$ at date 2. Taken together, the firm's net payoff from tokens is

$$\begin{aligned}\Pi_F &= (m_0\theta_0 - E[\Delta_m]) R_F - (m_0 - E[\Delta_m]) v_2 \\ &= m_0 R_F \left\{ \theta_0 - \frac{E[\Delta_m]}{m_0} - \left(1 - \frac{E[\Delta_m]}{m_0}\right) \frac{v_2}{R_F} \right\}.\end{aligned} \tag{1}$$

On average, for each token issued, the firm pays to consumers $\frac{E[\Delta_m]}{m_0}$ at date 1 and $\left(1 - \frac{E[\Delta_m]}{m_0}\right) v_2$ at date 2. Every dollar of payment at date 1 costs 1 dollar at date 0, since the firm needs to carry money from date 0 to date 1 without any return. Every dollar of payment at date 2 costs $1/R_F$ dollar at date 0, as the firm can borrow at the rate of R_F. Therefore, the firm's cost at date 0 to satisfy redemption is $\frac{E[\Delta_m]}{m_0} + \left(1 - \frac{E[\Delta_m]}{m_0}\right) \frac{v_2}{R_F}$, and the firm earns the difference between the token price and this cost.

We can decompose the firm's net payoff from tokens into two parts as follows:

$$\begin{aligned}\Pi_F = & \underbrace{(m_0\theta_0 - E[\Delta_m]) (R_F - R_L)}_{\text{Gain from difference in interest rates between the firm and consumers}} \\ & + \underbrace{m_0 R_L \left\{ \theta_0 - \frac{E[\Delta_m]}{m_0} - \left(1 - \frac{E[\Delta_m]}{m_0}\right) \frac{v_2}{R_L} \right\}}_{\text{Gain from liquidity provision}}\end{aligned} \tag{2}$$

The first part represents the gain from the difference in interest rates between the firm and consumers. In general, a firm's financing cost (or return on production if the firm cannot obtain sufficient financing) can be higher than a consumer's actual return on investment, $R_F \geq R_L$. This difference could be ascribed to financial frictions and market power of financial intermediaries. The second part represents the gain from liquidity provision when there is no difference in interest rates between the firm and consumers. In our setting, ex-ante liquidity is valuable because the cost of obtaining liquidity through borrowing is higher than the return of illiquid investment, $R_B > R_L$. Hence, the second part can be positive for financial claims that provide liquidity at date 1. Throughout the paper, we refer to $\theta_0 - \frac{E[\Delta_m]}{m_0} - \left(1 - \frac{E[\Delta_m]}{m_0}\right) \frac{v_2}{R_L}$ that appears in equilibrium as the liquidity premium of tokens or the LPT for short.

In the literature, many studies consider tokens as a cheaper way to finance, which relies on the difference in interest rates to motivate financing activities.[4] The primary innovation of our paper is to study how firms issue partially liquid financial claims to raise funds at a lower cost. To focus on the liquidity mechanism, we assume $R_F = R_L$, which shuts down the mechanism based on the difference in interest rates between the firm and consumers. This implies that the firm issues tokens if and only if the LPT is positive.

3 Baseline Model Solution

We first consider the benchmark case without tokens, which helps illustrate the value of liquidity in the model. We then introduce tokens. To better illustrate the main economic channels, we focus on a simplified case in which the sizes of the consumption demand shocks related to the two goods are equal, $p_s = p_c$. We derive the firm's optimal token issuance strategy and implications for the LPT and the welfare impact of tokens. Finally, we present the general case in which the sizes of the consumption shocks can be different, and confirm that our main results remain qualitatively unchanged.

3.1 Benchmark: No Tokens

Suppose that tokens cannot be issued. Consumers hold d_0 in deposits and $w_0 - d_0$ in investment at date 0. They face four possible scenarios at date 1, as shown in Table 1. Since deposits can be used to purchase both the firm's service and other consumption, only the total consumption demand, denoted by C_1, matters. If $d_0 \geq C_1$, the consumer withdraw C_1 from her deposit, earning a payoff of

$$\alpha C_1 + R_L (w_0 - d_0) + (d_0 - C_1).$$

If $d_0 < C_1$, the consumer withdraws her entire deposit and borrows $C_1 - d_0$ from the bank, earning a payoff of

$$\alpha C_1 + R_L (w_0 - d_0) + (d_0 - C_1) - (R_B - 1)(C_1 - d_0).$$

[4] In token financing, for example, Rogoff and You (2022) and Luo (2025).

In the four scenarios in Table 1, C_1 equals $p_s + p_c$ with probability q_{cs}, p_s with probability $q_s - q_{cs}$, p_c with probability $q_c - q_{cs}$, and 0 with probability $1 + q_{cs} - q_s - q_c$. Therefore, the consumer's expected payoff of holding d_0 in deposits is

$$\begin{aligned} E\left[U_c\right] = & (\alpha - 1)\left(p_s q_s + p_c q_c\right) + R_L w_0 - \left(R_L - 1\right) d_0 \\ & - q_{cs}\left(R_B - 1\right)\left(p_s + p_c - d_0\right)^+ - \left(q_s - q_{cs}\right)\left(R_B - 1\right)\left(p_s - d_0\right)^+ \\ & - \left(q_c - q_{cs}\right)\left(R_B - 1\right)\left(p_c - d_0\right)^+, \end{aligned} \tag{3}$$

where $x^+ = \max(x, 0)$. Taking first-order derivative with respect to d_0, we obtain

$$\begin{aligned} \frac{\partial E\left[U_c\right]}{\partial d_0} = & \left(R_B - 1\right)\left[q_{cs}\sigma\left(p_s + p_c - d_0\right) + \left(q_s - q_{cs}\right)\sigma\left(p_s - d_0\right) + \left(q_c - q_{cs}\right)\sigma\left(p_c - d_0\right)\right] \\ & - \left(R_L - 1\right), \end{aligned} \tag{4}$$

where $\sigma(\cdot)$ is an indicator function with $\sigma(x) = 1$ if $x > 0$ and $\sigma(x) = 0$ otherwise. The marginal cost of deposits is foregoing the higher return on investment, $R_L - 1$. The marginal benefit is avoiding the cost of borrowing, $R_B - 1$, which materializes only when the total consumption demand exceeds d_0. This becomes less likely as d_0 increases, so the marginal benefit of deposits is weakly decreasing in d_0. The minimum marginal benefit is 0 and attained when $d_0 \geq p_s + p_c$, while the maximum marginal benefit is $\left(q_c + q_s - q_{cs}\right) \cdot \left(R_B - 1\right)$ and attained when $d_0 < \min\{p_s, p_c\}$. Therefore, consumers hold deposits if and only if $\left(q_c + q_s - q_{cs}\right) \cdot \left(R_B - 1\right) > R_L - 1$.

The following lemma characterizes a consumer's optimal amount of deposits.

Lemma 1. *Let* $q_k \triangleq \begin{cases} q_s, & \text{if } p_s > p_c \\ q_c, & \text{if } p_s \leq p_c \end{cases}$. *Without tokens, a consumer's optimal amount of deposits is as follows:*

$$d_0 = \begin{cases} \min\{p_s, p_c\} & \text{if } q_k < \frac{R_L - 1}{R_B - 1} \\ \max\{p_s, p_c\} & \text{if } q_{cs} < \frac{R_L - 1}{R_B - 1} \leq q_k \\ p_s + p_c & \text{if } \frac{R_L - 1}{R_B - 1} \leq q_{cs}. \end{cases} \tag{5}$$

The optimal amount of deposits must be one of the four corners cases: 0, p_c, p_s, and $p_c + p_s$. The cost of borrowing relative to the return of investment, $\frac{R_B - 1}{R_L - 1}$, captures the cost of illiquidity. For a higher cost of illiquidity, liquidity is more valuable, and a consumer would like to hold a higher level of liquid claims.

3.2 Token issuance

Next, we consider token issuance. In equilibrium, if the firm decides to issue tokens, it must set θ_0 such that its net payoff to the firm from issuing tokens is positive. Hence, it suffices to focus on the case in which $\theta_0 > \frac{v_2}{R_L}$. If $\theta_0 \leq \frac{v_2}{R_L}$, the firm's net payoff from tokens cannot be positive:

$$\begin{aligned}\Pi_F &= m_0 R_L \left\{ \theta_0 - \frac{E[\Delta_m]}{m_0} - \left(1 - \frac{E[\Delta_m]}{m_0}\right) \frac{v_2}{R_L} \right\} \\ &\leq m_0 R_L \frac{E[\Delta_m]}{m_0} \left(\frac{v_2}{R_L} - 1\right) \leq 0.\end{aligned} \tag{6}$$

We next move to consumers' decisions regarding token purchases. First, a consumer chooses either $m_0 = 0$ or $m_0 = p_s$. Since partial redemption is not allowed, if m_0 is smaller than p_s, these tokens cannot be redeemed for a service at date 1 and must be held to date 2. Given $\theta_0 > \frac{v_2}{R_L}$, the return to holding tokens is v_2/θ_0, which is smaller than R_L, the return on investment. Hence, holding fewer than p_s tokens is dominated by investment. Similarly, a consumer does not want to hold more than p_s tokens, since she can redeem at most p_s tokens at date 1 and the part more than p_s tokens must be held to date 2.

Second, consumers prioritize token redemption if they demand the firm's service at date 1. If a consumer purchases the service at date 1, her cash flows at date 2 decrease by $p_s v_2$ if she redeems p_s tokens, decrease by p_s if she uses p_s deposits, and decrease by $p_s R_B$ if she borrows p_s. Since tokens depreciate at least weakly over time—$v_2 \leq 1$, consumers always prefer to use tokens first.

Section 3.1 presents a consumer's expected payoff if she holds no tokens. Here, we present the expected payoff if she holds p_s tokens. Similar to the baseline model, a consumer faces the four possible scenarios at date 1, as described in Table 1. However, since tokens can be redeemed only for the service, not only the total consumption demand but also its composition matters. If a consumer demands the service at date 1, both tokens and deposits can provide liquidity, so her payoff is

$$\alpha C_1 + R_L (w_0 - d_0 - p_s \theta_0) + (d_0 + p_s - C_1) - (R_B - 1)(C_1 - d_0 - p_s)^+ .$$

If a consumer demands other consumption, only deposits can provide liquidity, and tokens are held to date 2, so her payoff is

$$\alpha C_1 + R_L (w_0 - d_0 - p_s \theta_0) + (d_0 - C_1) - (R_B - 1)(C_1 - d_0)^+ + p_s v_2.$$

Therefore, a consumer's expected payoff of holding d_0 in deposits and p_s tokens is

$$\begin{aligned}E[U_c] =& R_L w_0 + (\alpha - 1)(p_s q_s + p_c q_c) - (R_L - 1) d_0 \\ & - [R_L \theta_0 - q_s - (1 - q_s) v_2] p_s - q_c (R_B - 1)(p_c - d_0)^+\end{aligned} \tag{7}$$

Taking the first-order derivative with respect to d_0, we obtain

$$\frac{\partial E\left[U_c\right]}{\partial d_0} = \left(R_B - 1\right) q_c \sigma \left(p_c - d_0\right) - \left(R_L - 1\right).$$

It is straightforward to see that a consumer will choose either $d_0 = 0$ or $d_0 = p_c$.

Based on consumers' equilibrium behavior, the equilibrium with token issuance must be as follows. The firm sets θ_0 such that a consumer is willing to hold p_s tokens instead of 0 tokens. A consumer redeems p_s tokens at date 1 whenever she demands the firm's service, which happens with probability q_s. Hence, the total redemption of tokens is $E\left[\Delta_m\right] = q_s p_s$, and the firm's net payoff from issuing tokens is

$$\Pi_F = p_s R_L \left\{ \theta_0 - q_s - \left(1 - q_s\right) \frac{v_2}{R_L} \right\}. \tag{8}$$

Let θ_{max} be the maximum price the firm can set. If the firm issues tokens, it must set $\theta_0 = \theta_{max}$, so the term, $\theta_{max} - q_s - \left(1 - q_s\right) \frac{v_2}{R_L}$, represents the LPT.

Before proceeding to the case with both tokens and deposits, we present the case with only tokens as another benchmark. Suppose that deposits are not available. A consumer's expected payoff is

$$E\left[U_c\right] = \left(\alpha - R_B\right)\left(p_s q_s + p_c q_c\right) + R_L w_0$$

if she holds 0 tokens, and

$$\begin{aligned} E\left[U_c\right] =& R_L w_0 + \left(\alpha - 1\right)\left(p_s q_s + p_c q_c\right) \\ & - \left[R_L \theta_0 - q_s - \left(1 - q_s\right) v_2\right] p_s - q_c p_c \left(R_B - 1\right) \end{aligned}$$

if she holds p_s tokens. Comparing the two payoffs, we obtain the following lemma.

Lemma 2. *Without deposits, $\theta_{max} = q_s + \left(1 - q_s\right) \frac{v_2}{R_L} + q_s \frac{R_B - R_L}{R_L}$. The LPT is increasing in R_B.*

Without deposits, tokens replace investment, and are used to reduce borrowing when there is demand for the firm's service. The LPT is driven by their advantage relative to investment and thus must be positive. As the short-term borrowing cost R_B increases, the cost of illiquidity increases, so the LPT increases.

3.3 Simplified Case: $p_s = p_c$

Next, we consider the case in which both tokens and deposits are available. We first solve a special case where the sizes of the consumption demand for the two goods are equal, $p_s = p_c$, to illustrate the economic channel without tedious treatment due to the difference in the sizes of the consumption shocks.

Based on the previous analysis, a consumer must choose one of the following corner portfolios: $(d_0, m_0) = (p_c, 0)$, $(d_0, m_0) = (p_c + p_s, 0)$, $(d_0, m_0) = (0, p_s)$, $(d_0, m_0) = (p_c, p_s)$. Comparing these portfolios, we obtain the following proposition.

Proposition 1. *Suppose $p_c = p_s$. The maximum price of tokens that induces each consumer to hold p_s tokens, θ_{max}, and the consumer's portfolio at θ_{max}, (d_0, m_0), are as follows:*

- *if $q_c < \frac{R_L-1}{R_B-1}$, $\theta_{max} = q_s + (1-q_s)\frac{v_2}{R_L} + (1-q_s)\frac{R_L-1}{R_L} - (q_c - q_{cs})\frac{R_B-1}{R_L}$, and $(d_0, m_0) = (0, p_s)$;*
- *if $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_c$, $\theta_{max} = q_s + (1-q_s)\frac{v_2}{R_L} + q_{cs}\frac{R_B-R_L}{R_L} - (q_s - q_{cs})\frac{R_L-1}{R_L}$, and $(d_0, m_0) = (p_c, p_s)$;*
- *if $\frac{R_L-1}{R_B-1} \leq q_{cs}$, $\theta_{max} = q_s + (1-q_s)\frac{v_2}{R_L} + (1-q_s)\frac{R_L-1}{R_L}$, and $(d_0, m_0) = (p_c, p_s)$.*

Similar to the benchmark case with only deposits, consumers prefer to hold a higher level of liquidity as the cost of illiquidity, $\frac{R_B-1}{R_L-1}$, increases, which is reflected by more deposits given that consumers hold p_s tokens.

3.3.1 Switching Outside Option to Holding Tokens

An important feature of our setting is that there are two liquid claims that are not perfect substitutes. Tokens are more liquid than investment but less liquid than deposits. In the presence of the three claims, consumers' outside option to holding tokens could be the illiquid claim, investment, or the more liquid claim, deposits. Proposition 1 suggests that the outside option is endogenous and depends on the cost of illiquidity.

Specifically, consider the case in which $\frac{R_L-1}{R_B-1} \leq q_{cs}$. Due to the high borrowing cost, consumers would hold sufficient liquidity such that they do not borrow at date 1. To this end, they can hold either $p_s + p_c$ in deposits, or p_s tokens and p_c in deposits. Essentially, to be able to sell tokens, the firm needs to convince consumers to hold $p_s(= p_c)$ tokens instead of $p_s(= p_c)$ in deposits. Given that consumers must hold at least p_c in deposits, the marginal benefit of liquidity is to reduce borrowing when a consumer needs both consumption goods. For this purpose, additional p_s tokens are not inferior to additional p_s deposits. Therefore, the maximum token price is lower than one,

$$1 - \theta_{max} = \frac{(1-q_s)}{R_L}(1-v_2),$$

simply because a token depreciates at date 2—$v_2 \leq 1$, and delivers lower cash flows than one dollar deposit.

Now, consider the case in which $q_{cs} < \frac{R_L - 1}{R_B - 1} \leq q_c$. As shown in Lemma 1, without tokens, consumers hold $p_s(= p_c)$ in deposits. With $p_s(= p_c)$ tokens, since $\frac{R_L - 1}{R_B - 1} \leq q_c$, the marginal benefit of deposits is greater than the opportunity cost of foregoing investment, so consumers still hold $p_s(= p_c)$ in deposits. The intuition is that when the cost of illiquidity is in a medium range, consumers prefer to hold a medium level of liquidity. Hence, the firm needs to convince consumers to hold $p_s(= p_c)$ tokens in addition to $p_s(= p_c)$ in deposits. Given that consumers must hold at least p_c in deposits, tokens are used to reduce borrowing when a consumer needs both consumption goods. The LPT is affected by tokens' advantage in liquidity relative to investment, which is reflected by the first term of the LPT, $q_{cs}\frac{R_B - R_L}{R_L}$.

Finally, consider the case where $q_c < \frac{R_L - 1}{R_B - 1}$. As shown in Lemma 1, without tokens, consumers hold $p_s(= p_c)$ in deposits. With $p_s(= p_c)$ tokens, the marginal benefit of deposits is to satisfy the demand for consumption, which arrives with probability q_c. Since $q_c < \frac{R_L - 1}{R_B - 1}$, the benefit is smaller than the opportunity cost of foregoing investment, so consumers hold 0 deposits. Intuitively, when the cost of illiquidity is low, consumers prefer to hold a low level of liquidity. Hence, the firm needs to convince consumers to hold $p_s(= p_c)$ tokens instead of $p_s(= p_c)$ in deposits. The LPT is affected by tokens' disadvantage relative to deposits. That is, tokens cannot be used to reduce borrowing like deposits when a consumer needs only the other consumption. This disadvantage is reflected in the second term of the LPT, $-(q_c - q_{cs})\frac{R_B - 1}{R_L}$, while the first term reflects the LPT if tokens provide the same liquidity as deposits whenever a consumer has consumption needs.

As a result of the switching outside option, the LPT could be affected by either the tokens' advantage in liquidity relative to investment or their disadvantage relative to deposits. In the former case, the LPT is increasing in the cost of illiquidity, as the advantage is greater. In the latter case, the LPT is decreasing in the cost of illiquidity, as the disadvantage is greater. Hence, the switching outside option implies a non-monotonic relationship between the LPT and the cost of illiquidity, as stated in Proposition 2 and illustrated in Figure 1. Note, for comparison, that as stated in Lemma 2, when deposits are not available, the LPT is increasing in the cost of illiquidity.

Proposition 2. *The LPT is decreasing in R_B when $q_c < \frac{R_L - 1}{R_B - 1}$, increasing in R_B when $q_{cs} < \frac{R_L - 1}{R_B - 1} \leq q_c$, and independent of R_B when $\frac{R_L - 1}{R_B - 1} \leq q_{cs}$.*

3.3.2 The Substitution between Liquid Claims

In settings with only one liquid claim, the liquid claim features a positive liquidity premium, which is determined by the cost of illiquidity and its advantage relative to illiquid claims. In settings with multiple liquid claims, their liquidity premiums are also affected by the imperfect substitution between them. In general, such substitution reduces the liquidity premium, as consumers' outside

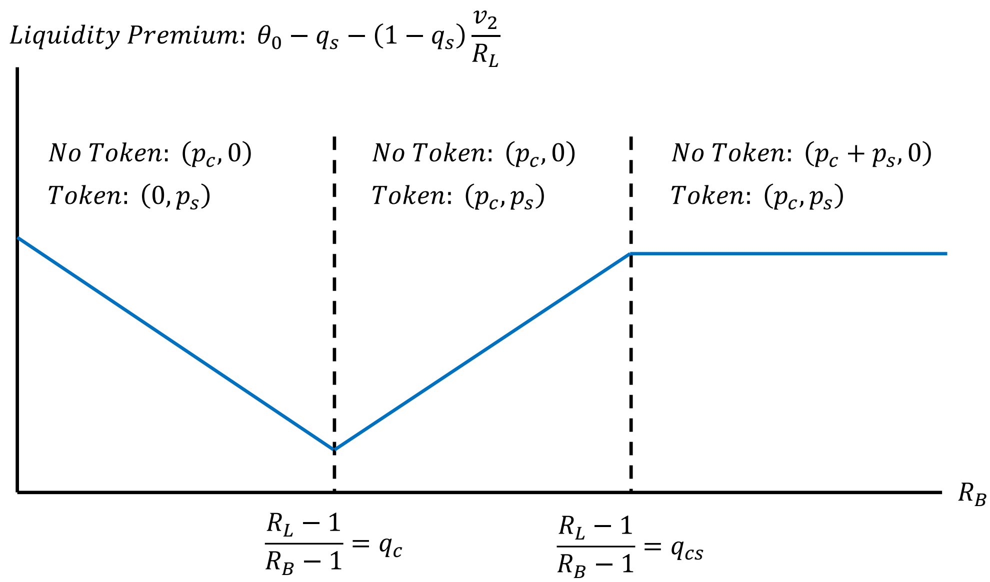

Figure 1: The LPT and the borrowing rate

option could be obtaining liquidity from other liquid claims instead of merely suffering illiquidity. In our model, we identify three ways that this substitution works.

Proposition 3. *The LPT is always lower in the case with deposits than in the case without deposits.*

When $\frac{R_L-1}{R_B-1} > q_c$ or $\frac{R_L-1}{R_B-1} \leq q_{cs}$, the substitution works in a direct way: deposits serve as consumers' outside option to tokens. Deposits improve consumers' outside option to holding tokens and thus decrease the LPT.

When $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_c$, the substitution works in an indirect way. Tokens effectively replace investment but not deposits. Without deposits, tokens reduce borrowing when there is demand for the service, so the LPT is $q_s \frac{R_B-R_L}{R_L}$, as stated in Lemma 2. With deposits, since consumers hold p_c deposits, tokens reduce borrowing only when a consumer needs both consumption goods. Therefore, tokens' advantage in liquidity relative to investment is smaller and contributes only $q_{cs}\frac{R_B-R_L}{R_L}$ instead of $q_s\frac{R_B-R_L}{R_L}$ to the LPT.

However, although tokens provide marginal liquidity only when consumers need both consumption goods, consumers redeem tokens, or put differently, tokens provide actual liquidity whenever they need the firm's service. Since the token value weakly decreases over time, consumers prioritize using tokens. That means, the introduction of tokens also induces a change in the order of liquidity usage. Specifically, when a consumer needs only the service, which happens with probability $q_s - q_{cs}$, she switches from using deposits to using tokens. This change in the order of liquidity usage makes the firm's liquidity provision more costly and further decreases the LPT by $(q_s - q_{cs})\frac{R_L-1}{R_L}$.

3.3.3 Issuance Strategy and Impacts on Bank

Notably, the substitution between tokens and deposits could be so strong that it renders the LPT negative and substantially discourages the firm from issuing tokens. Proposition 4 characterizes the firm's optimal issuance strategy.

Proposition 4. *When $\frac{R_L-1}{R_B-1} \in \left[\frac{q_{cs}}{q_s}, \frac{q_c-q_{cs}}{1-q_s}\right]$, the maximum possible LPT is nonpositive, so the firm does not issue tokens. Otherwise, the firm issues tokens and sets θ_0 to θ_{max} in Proposition 1.*

Propositions 1 and 2 suggest that given R_L, the LPT attains its minimum when $\frac{R_L-1}{R_B-1} = q_c$:

$$\theta_{max} - \left[q_s + (1-q_s)\frac{v_2}{R_L}\right] = \left(\frac{q_{cs}}{q_c} - q_s\right)\frac{R_L-1}{R_L}.$$

When the demand for the two consumption goods is positively correlated (i.e., the firm's service and other consumption are complements), $q_{cs} > q_c q_s$, the LPT is always positive, so the firm always issues tokens. When the demand is negatively correlated (i.e. the firm's service and other consumption are substitutes), the LPT can be negative, and the firm does not issue tokens in that case.

In equilibrium, if tokens are issued, the firm's payoff must increase, and consumers' expected utility does not change. Proposition 5 summarizes the impacts of token issuance on the bank.

Proposition 5. *Suppose tokens are issued in equilibrium.*

- *if $q_c < \frac{R_L-1}{R_B-1}$, the bank's payoff from deposits decreases, and its payoff from borrowing increases;*
- *if $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_c$, the bank's payoff from deposits increases, and its payoff from borrowing decreases;*
- *if $\frac{R_L-1}{R_B-1} \leq q_{cs}$, the bank's payoff from deposits decreases, and its payoff from borrowing does not change.*

It may seem that token issuance would make the bank worse off, as it decreases deposits due to their substitution in terms of liquidity. However, we find that this is not the case when $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_c$. As discussed above. In this case, the introduction of tokens do not decrease the deposits held by consumers at date 0, and induces a change in the order of liquidity usage. This change in the order hurts the firm, as consumers redeem tokens more often. On the other hand, it unexpectedly makes the bank's liquidity provision less costly, as consumers withdraw deposits less often. In general, the addition of a new liquid claim not only decreases consumers' incentive to hold the current liquid claim, but also decreases consumers' incentive to use it. When the second effect dominates, which takes place if $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_c$, the issuers of the current liquid claim (deposits) can in fact benefit from the addition of a new liquid claim (tokens).

Another aspect that the conventional narrative ignores is that banks also make profits by providing ex-post liquidity, which is borrowing in the model. Proposition 5 suggests that tokens have ambiguous impact on borrowing, depending on whether tokens increase or decrease the total ex-ante liquidity. With both deposits and borrowing considered, the bank is certainly worse off only when $\frac{R_L-1}{R_B-1} \leq q_{cs}$

3.4 The general case

Proposition 6. *When $p_s \leq p_c$, Propositions 1, 4, and 5 hold. Suppose $p_s > p_c$.*

1. *The maximum price of tokens that induces each consumer to hold p_s tokens, θ_{max}, and the consumer portfolio under the price, (d_0, m_0), are as follows:*
   - *if $q_c < \frac{R_L-1}{R_B-1}$, $\theta_{max} = q_s + (1-q_s)\frac{v_2}{R_L} + \left(\frac{p_c}{p_s} - q_s\right)\frac{R_L-1}{R_L} + \left[q_s - (q_s + q_c - q_{cs})\frac{p_c}{p_s}\right] \cdot \frac{R_B-1}{R_L}$, and $(d_0, m_0) = (0, p_s)$;*
   - *if $q_s < \frac{R_L-1}{R_B-1} \leq q_c$, $\theta_{max} = q_s + (1-q_s)\frac{v_2}{R_L} - q_s\frac{R_L-1}{R_L} + \left(q_s - q_s\frac{p_c}{p_s} + q_{cs}\frac{p_c}{p_s}\right)\frac{R_B-1}{R_L}$, and $(d_0, m_0) = (p_c, p_s)$;*
   - *if $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_s$, $\theta_{max} = q_s + (1-q_s)\frac{v_2}{R_L} + \frac{R_L-1}{R_L}\left(1 - \frac{p_c}{p_s} - q_s\right) + q_{cs}\frac{R_B-1}{R_L}\frac{p_c}{p_s}$, and $(d_0, m_0) = (p_c, p_s)$;*
   - *if $\frac{R_L-1}{R_B-1} \leq q_{cs}$, $\theta_{max} = q_s + (1-q_s)\frac{v_2}{R_L} + (1-q_s)\frac{R_L-1}{R_L}$, and $(d_0, m_0) = (p_c, p_s)$.*
2. *The LPT is decreasing in R_B when $q_c < \frac{R_L-1}{R_B-1}$, is increasing in R_B when $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_c$, and is equal to that of demand deposits when $\frac{R_L-1}{R_B-1} \leq q_{cs}$.*
3. *When $\frac{R_L-1}{R_B-1} \in \left[1 - \frac{p_c}{p_s} + \frac{q_{cs}}{q_s}\frac{p_c}{p_s}, \frac{(q_s+q_c-q_{cs})\frac{p_c}{p_s} - q_s}{\frac{p_c}{p_s} - q_s}\right]$, the maximum LPT is nonpositive, so the firm does not issue tokens. Otherwise, the firm issues tokens and sets $\theta_0 = \theta_{max}$.*
4. *Proposition 5 holds except that in the case in which $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_s$, the bank's payoff from deposits increases only when $\frac{p_c}{p_s} > \frac{1-q_s}{1-q_{cs}}$.*

For the general case, Proposition 6 presents the LPT and confirms that its properties shown in Section 3.3 qualitatively hold. A notable difference is that if $p_s > p_c$ and $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_s$, the bank's payoff from deposits increases only when $\frac{p_c}{p_s} > \frac{1-q_s}{1-q_{cs}}$. In that case, tokens reduce deposit holdings at date 0 from p_s to p_c. Tokens reduce both consumers' incentive to hold deposits and their incentive to use them. The second effect dominates when $\frac{p_c}{p_s} > \frac{1-q_s}{1-q_{cs}}$.

4 Implications for Token Issuance and Design

The baseline model adopts a simple framework to focus on the liquidity provision role of tokens. In this section, we apply the economic mechanism developed in the baseline model to derive implications for token issuance and design.

4.1 The Characteristics of Consumption

In the model, consumption demand is characterized by the value of the firm's service (p_s), the frequency of the demand for the firm's service (q_s), the value of other consumption (p_c), the frequency of the demand for other consumption (q_c), and the correlation between the demand for the firm's service and other consumption (ρ). We examine how the firm's net payoff from issuing tokens depends on these characteristics. To control for the the overall scale of the firm's business, we hold the total demand for the service, $p_s q_s$, constant. By doing so, we vary p_s and q_s inversely, allowing us to compare firms that offer high-value, low-frequency services with those that provide low-value, high-frequency services.

Proposition 7. *The LPT is weakly higher,*

- *if p_s is higher and q_s is lower, given $p_s q_s$ and ρ;*
- *if ρ is higher;*
- *if p_c is smaller.*

Proposition 7 first states that a firm potentially benefits more from issuing tokens if it supplies high-value, low-frequency service. Intuitively, the cost to the bank and to the firm to provide the liquidity for the service only depends on the expected demand for the service. If consumers decide to obtain liquidity through deposits, they need to hold more deposits for services featuring higher value and lower frequency, even if the expected demand for the service does not change. This implies that using deposits to satisfy such consumption demand is more costly for consumers, as the bank earns more from deposits. This gives the firm more room to earn LPT, especially since it can provide liquidity for such consumption as well as the bank.

Higher correlation between the demand for the two consumption goods makes the LPT weakly higher. This works through the multiple channels discussed in Section 3.3. Take the simplified case with $p_s = p_c$ as an example. If $q_c < \frac{R_L-1}{R_B-1}$, the LPT is determined by the disadvantage of tokens relative to deposits (i.e. that tokens cannot provide liquidity when consumers demand only other consumption, which happens with probability $q_c - q_{cs}$). Conditional on the probability of the demand for the service and other consumption, p_s and p_c, higher correlation implies lower $q_c - q_{cs}$

and thus smaller disadvantage. If $q_{cs} < \frac{R_L - 1}{R_B - 1} \leq q_c$, the LPT is determined by two factors. The first is the advantage of tokens relative to investment (i.e. that tokens can provide liquidity when consumers demand both consumption goods), so higher dependence implies higher advantage of tokens. The second is the change in the order of liquidity usage, which happens when consumers demand only the firm's service. Higher correlation implies this case is less likely, limiting the impact of this change.

Finally, the size of the demand for other consumption negatively affects the LPT. Since only deposits can provide liquidity for other consumption, greater demand for other consumption induces consumers to hold deposits. Deposits can also provide liquidity for the firm's service, thus larger deposits reduce consumers' incentives to hold tokens.

4.2 Partial Redemption of Tokens

In our baseline model, the firm's service can be purchased by either all tokens or all cash. In practice, some firms, such as airlines, may allow consumers to use a combination of tokens and cash to buy a service. We refer to this option as partial redemption. In this section, we examine whether the firm finds it desirable to allow for partial redemption. Consistent with the assumption in the baseline setup that p_s tokens can be redeemed for one unit of service, we assume that tokens are worth one dollar per unit when they are combined with cash.

Partial redemption gives consumers more flexibility in the holdings of liquid claims. Besides holding either 0 or p_s tokens, they can also hold fewer than p_s tokens and use them for the service through partial redemption. From the firm's perspective, giving this flexibility to consumers has two effects. On the one hand, if the firm would like consumers to hold p_s tokens, this flexibility means an additional constraint on the token price, which in general lowers the LPT in this case. On the other hand, consumers may gain more liquidity benefits by holding fewer than p_s tokens, which may in turn increase the LPT that the firm can charge. Therefore, in any equilibrium in which the firm strictly prefers to allow for partial redemption, consumers must hold $m_0 \in (0, p_s)$.

Holding d_0 in deposits and $m_0 \in [0, p_s]$ tokens, a consumer's expected payoff is

$$\begin{aligned} E[U_c] = &R_L w_0 + (\alpha - 1)(p_s q_s + p_c q_c) - (R_L - 1) d_0 - [R_L \theta_0 - q_s - (1 - q_s) v_2] m_0 \\ &- q_{cs}(R_B - 1)(p_c + p_s - d_0 - m_0)^+ - (q_s - q_{cs})(R_B - 1)(p_s - d_0 - m_0)^+ \\ &- (q_c - q_{cs})(R_B - 1)(p_c - d_0)^+ . \end{aligned}$$

The consumer's holdings must satisfy at least one of the following corner conditions: 1) $d_0 + m_0 = p_c + p_s$, 2) $d_0 + m_0 = p_s$, and 3) $d_0 = p_c$. If $d_0 + m_0 = p_c + p_s$, then $d_0 \geq p_c$, and the consumer's expected payoff reduces to

$$E\left[U_c\right] = R_L w_0 + (\alpha - 1)\left(p_s q_s + p_c q_c\right) - (R_L - 1)\, d_0 - \left[R_L \theta_0 - q_s - (1 - q_s)\, v_2\right] m_0.$$

In this equilibrium, the firm must set θ_0 such that $m_0 = p_s$. Hence, we can focus on $d_0 + m_0 < p_c + p_s$. Intuitively, since we have two choice variables here, if the consumer optimally chooses $m_0 \in (0, p_s)$, the other two corner conditions both hold. That means, $p_s > p_c$, and $(d_0, m_0) = (p_c, p_s - p_c)$.

The following proposition characterizes the condition under which the firm prefers to allow for partial redemption and the impact

Proposition 8. *When $p_s \leq p_c$, the firm never strictly prefers to allow for partial redemption. Suppose $p_s > p_c$.*

- *The firm strictly prefers to allow for partial redemption rather than not if and only if $\frac{R_L-1}{R_B-1} \in \left(\frac{q_{cs}}{q_s}, \frac{q_c - q_{cs}}{1-q_s}\right)$.*
- *In that case, consumers' portfolio at date 0 is $(d_0, m_0) = (p_c, p_s - p_c)$.*
- *With the option to allow for partial redemption, the firm always issues tokens.*

When $p_s \leq p_c$, consumers never hold $m_0 \in (0, p_s)$ tokens, so allowing for partial redemption makes no difference from the firm's perspective. Consider the case with $p_s > p_c$. The firm can allow for partial redemption and set a token price θ_0 to induce consumers to hold $(d_0, m_0) = (p_c, p_s - p_c)$. In that case, the firm's net payoff from tokens is

$$(p_s - p_c)\, R_L \left\{\theta_0 - q_s - (1 - q_s)\, \frac{v_2}{R_L}\right\}.$$

The firm strictly prefers to allow for partial redemption if and only if this net payoff exceeds his net payoff without partial redemption,

$$p_s R_L \left\{\theta_{max} - q_s - (1 - q_s)\, \frac{v_2}{R_L}\right\}.$$

Proposition 8 suggests that if the demand for the two consumption goods is independent or positively correlated, the firm always prefers not to allow for partial redemption. If the demand is negatively correlated, the firm strictly prefers to allow for partial redemption when $\frac{R_L-1}{R_B-1}$ is in $\left(\frac{q_{cs}}{q_s}, \frac{q_c - q_{cs}}{1-q_s}\right)$. As suggested by Propositions 2, 4, and 7, without partial redemption, the LPT is low when the demand for the two consumption goods is negatively correlated and $\frac{R_L-1}{R_B-1}$ is in an intermediate region around q_c. The reason is that consumers do not gain much by holding p_s tokens and may benefit from holding fewer tokens. Partial redemption gives consumers this attractive option and allows the firm to earn a higher LPT. Notably, with partial redemption, the firm can always earn a positive LPT, and it always issues tokens.

4.3 Ex-Post Token Issuance

So far, we have assumed that the firm issues tokens only at date 0, before consumption shocks are realized. Potentially, the firm can also issue tokens ex-post after the realization of consumption shocks at date 1 or date 2. We refer to the types of sales as ex-ante and ex-post token issuance.

In practice, firms do not observe the realization of consumers' consumption shocks, and there is no clear timeline of the realization of consumption shocks. Hence, the distinction between the two types of token issuance lies in that consumers can easily time the ex-post one but not the ex-ante one. Therefore, ex-post token issuance usually works through the type of token sales that are always available to consumers. For example, consumers can always purchase miles from airlines' websites and gift cards from Starbucks. Ex-ante token issuance works through the type of token sales that are not always available to consumers or that do not allow consumers to fully control the quantity. For instance, consumers can purchase points at a discount during a special promotion period or earn miles through flying or using credit cards.

Suppose that besides the ability to issue tokens at the price of θ_0 at date 0, the firm always has tokens for sale at the price of $\overline{\theta}$. There are two quick observations about ex-post token issuance. First, any token sold at date 2 must reduce the firm's payoff, since consumers buy tokens at date 2 if and only if $\overline{\theta} < v_2$. Second, tokens bought at the price of $\overline{\theta}$ must be redeemed immediately. Buying tokens in this way and redeeming them later is weakly dominated by buying the tokens right before redemption, given that the price is always $\overline{\theta}$.

In any equilibrium in which the firm strictly prefers ex-post token issuance, consumers must hold $m_0 \in (0, p_s)$ tokens. Consider an equilibrium with $m_0 = 0$. If $\overline{\theta} < 1$, a consumer would buy p_s tokens and redeem immediately at date 1 whenever she needs a service. If $\overline{\theta} \geq 1$, she would buy no tokens at date 1. That means, any token sold at date 1 or date 2 reduces the firm's payoff. Given that no token is sold at date 0, the firm's net payoff from tokens must be nonpositive. Consider an equilibrium with $m_0 \geq p_s$. Given that a consumer already has sufficient tokens for a service, she would not buy any token at date 1. That means, ex-post token issuance would only reduce the firm's cash flows at date 2.

We can focus on the following equilibrium path. Each consumer holds $m_0 \in (0, p_s)$ tokens at date 0. If she needs the service, she spends $(p_s - m_0)\overline{\theta}$ buying $p_s - m_0$ tokens and redeems the total p_s tokens. If she does not need the service, she holds the m_0 tokens to date 2. This equilibrium path is similar to the path under partial redemption. From the firm's perspective, ex-post token issuance does serve the same function as partial redemption, both giving consumers the option to hold fewer than p_s tokens at date 0 and potentially allowing the firm to earn a higher LPT.

Proposition 9. *When $p_s \leq p_c$, the firm prefers not to allow ex-post token issuance. When $p_s > p_c$, the firm prefers to allow ex-post token issuance if and only if $\frac{R_L-1}{R_B-1} \in \left(\frac{q_{cs}}{q_s}, \frac{q_c-q_{cs}}{1-q_s}\right)$, and the price of tokens sold ex-post, $\overline{\theta}$, must be at least 1.*

The model predicts that a firm may issue tokens ex-post at a price higher than 1, the redemption value per token. This result is consistent with the observation that airlines and hotels sell rewards directly to consumers at prices even higher than their redemption value. Certainly, consumers would not buy all the rewards required by a redemption in this way. Instead, it is typically the case that consumers earn some rewards from other sources such as special promotions and credit cards, and they buy additional rewards to reach the minimum amount required by a redemption. Essentially, direct sales are used to unlocking the rewards already held by consumers, thereby increasing the liquidity of rewards. This liquidity benefit allows the issuing firm to charge a price even higher than the rewards' redemption value.

The model also predicts that ex-post token issuance is desirable when the demand for the two consumption goods is negatively correlated. In practice, high-value consumption such as traveling induces large consumption shocks to consumers. Hence, consumers would attempt to adjust the consumption timing to smooth their total consumption over time, which naturally results in a negative correlation between high-value consumption and other consumption. Hence, the condition $\frac{R_L-1}{R_B-1} \in \left(\frac{q_{cs}}{q_s}, \frac{q_c-q_{cs}}{1-q_s}\right)$ is more likely to hold for high-value consumption. This explains why ex-post token issuance at a price even higher than the redemption value is common in the travel and hospitality sector.

5 Tradability and Convertibility

In practice, some rewards and tokens are tradable among consumers. For example, stablecoins, which are gaining popularity, are an example of tradable tokens. When tokens are tradable, although partially liquid claims themselves provide direct liquidity for only certain services and products, their holders may be able to obtain liquidity for other consumption by selling the claims to other consumers. Therefore, tradability effectively increases the liquidity of these claims. For consumers, tradability is definitely a desirable feature, potentially allowing the issuing firm to earn a higher LPT. However, tradability may also reduce the amount of funds that the firm can effectively raise from consumers. In this section, we examine the impact of making tokens tradable, the conditions under which the firm would prefer to allow tradability, and the influence of tradability with other facets of token issuance strategy. Some tokens, such as stablecoins, are redeemable for cash. Thus, we also examine the convertibility feature, combining with tradability.

5.1 Tradable Tokens

Suppose that the firm makes tokens transferable from one consumer's account to another's. Therefore, consumers can trade tokens in a competitive market. We focus on symmetric equilibria in which consumers hold the same portfolio (d_0, m_0) at date 0. This is actually without loss of generality, since in equilibrium, the price of tokens is determined endogenously, and consumers are indifferent to the exact combination of tokens and deposits.

Suppose that the firm sells tokens at $\theta_0 > \frac{v_2}{R_L}$ (If $\theta_0 \leq \frac{v_2}{R_L}$, as shown in Section 3.2, the firm's net payoff from tokens is nonpositive). Consumers only have the incentive to trade tokens at date 1 after the realization of consumption demand. The reason is that they have the same valuation for tokens before the realization of demand or at date 2. Denote the trading price of tokens at date 1 by γ_1. It must be the case that $\theta_0 \leq \gamma_1$. Otherwise, consumers would strictly prefer to buy tokens from the market at date 1 rather than buy tokens at date 0, which means no supply of tokens in the market. Further, the cash that consumers borrow at date 1 must be used immediately. The return of holding money from date 1 to date 2 is 1, and the return of buying tokens at date 1 and holding them to date 2 is $v_2/\gamma_1 < R_L$. Both are smaller than the borrowing cost R_B.

Proposition 10. *In any equilibrium in which the firm decides to issue tradable tokens,*

- $\frac{v_2}{R_L} < \theta_0 = \gamma_1 \leq v_2$, *and*
- *consumers hold no deposits after trade at date 1.*

Proposition 10 provides two key observations. First, $\gamma_1 \leq v_2$. If $\gamma_1 > v_2$, consumers would prefer selling tokens to the market at γ_1 to holding them until date 2, so no one would hold tokens after trade. It implies that all tokens must be redeemed at date 1, so $E[\Delta_m] = m_0$ and the firm's net payoff from tokens is nonpositive:

$$\Pi_F = m_0 R_L \left\{ \theta_0 - 1 - \frac{(1-1)\, v_2}{R_L} \right\} \leq 0.$$

Under $\gamma_1 \leq v_2$, the return of buying tokens on the market and holding them to date 2, v_2/γ_1, is no smaller than 1, so consumers prefer it to holding deposits to date 2. In equilibrium, consumers hold no deposits after the trade at date 1.[5] $\gamma_1 \leq v_2 \leq 1$ implies that to purchase the service, it is cheaper to buy tokens and redeem tokens than use cash.

[5] The firm would like consumers to hold more tokens, so it would set θ_0 such that γ_1 is slightly smaller than v_2 to ensure that consumers hold only tokens after trade at date 1.

Second, $\gamma_1 = \theta_0$. Given the cost of illiquidity induced by $(q_c + q_s - q_{cs}) \cdot (R_B - 1) > R_L - 1$, each consumer would hold at least a certain level of total liquidity at date 0. Because of the liquidity holding, the total borrowing of consumers would not be high, which we find to be surely lower than the cash that consumers need to pay for other consumption. This implies that consumers must hold some deposits at date 0. On the other hand, if $\gamma_1 > \theta_0$, consumers would not hold deposits at date 0 because holding them is dominated by holding tokens at date 0 and selling them in the market. Therefore, $\gamma_1 \leq \theta_0$.

Because of tradability of tokens, tokens and cash are interchangeable at the rate of θ_0 at date 1, so the effective value of a token is θ_0. Also, the effective price of a service at date 1 becomes θ_0, since consumers would always use p_s tokens to buy services instead of p_s dollars. Therefore, a consumer only cares about the total level of effective liquidity in her portfolio, which is $d_0 + m_0\theta_0$, and the total level of effective consumption demand at date 1, $C_{1,c} + C_{1,m}\theta_0$, where $C_{1,c}$ and $C_{1,m}$ are the demand for other consumption and the service respectively. If $d_0 + m_0\theta_0$ exceeds $C_{1,c} + C_{1,m}\theta_0$, the consumer would use all the remaining liquidity to buy tokens and hold them to date 2, receiving a return of v_2/θ_0. Otherwise, she would borrow the shortfall. Taken together, a consumer's expected payoff is

$$\begin{aligned} E[U_c] =& R_L w_0 + \alpha (p_s q_s + p_c q_c) - \left(R_L - \frac{v_2}{\theta_0}\right)(d_0 + m_0\theta_0) - (q_s p_s \theta_0 + q_c p_c)\frac{v_2}{\theta_0} \\ & - q_{cs}\left(R_B - \frac{v_2}{\theta_0}\right)(p_c + p_s\theta_0 - d_0 - m_0\theta_0)^+ - (q_s - q_{cs})\left(R_B - \frac{v_2}{\theta_0}\right)(p_s\theta_0 - d_0 - m_0\theta_0)^+ \\ & - (q_c - q_{cs})\left(R_B - \frac{v_2}{\theta_0}\right)(p_c - d_0 - m_0\theta_0)^+ . \end{aligned}$$

This expression indicates that the marginal cost and the marginal benefit of liquidity decreases in $R_L - \frac{v_2}{\theta_0}$ and $R_B - \frac{v_2}{\theta_0}$. This is because the return of holding liquid claims to date 2 is v_2/θ_0 instead of 1. The following proposition characterizes consumers' portfolios in equilibrium.

Proposition 11. *Given the price of tokens at date 0, θ_0:*

If $p_c \geq p_s\theta_0$, a consumer's portfolio holdings are as follows:

- *If $q_c < \frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}}$, $(d_0, m_0) = ((q_c - q_{cs})\, p_s\theta_0, (1 - q_c + q_{cs})\, p_s)$;*
- *If $q_{cs} < \frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} \leq q_c$, $(d_0, m_0) = \left(q_c p_c - q_{cs} p_s \theta_0, q_{cs} p_s + \frac{(1-q_c)p_c}{\theta_0}\right)$;*
- *If $\frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} \leq q_{cs}$, $(d_0, m_0) = \left(q_c p_c, p_s + \frac{(1-q_c)p_c}{\theta_0}\right)$.*

If $p_c < p_s\theta_0$, a consumer's portfolio holdings are as follows:

- *If* $q_s < \frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}}$, $(d_0, m_0) = \left((q_s + q_c - q_{cs})\, p_c - q_s p_s \theta_0, q_s p_s + \frac{(1 - q_s - q_c + q_{cs}) p_c}{\theta_0} \right)$;
- *If* $q_{cs} < \frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} \le q_s$, $(d_0, m_0) = \left((q_c - q_{cs})\, p_c, p_s - \frac{(q_c - q_{cs}) p_c}{\theta_0} \right)$;
- *If* $\frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} \le q_{cs}$, $(d_0, m_0) = \left(q_c p_c, p_s + \frac{(1 - q_c) p_c}{\theta_0} \right)$.

Consumers' portfolios are shaped by two forces. First, the total level of effective liquidity in the portfolio is determined by the return on investment relative to the cost of borrowing, $\frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}}$. Similarly to the baseline setup with nontradable tokens, consumers prefer to hold more liquidity as this ratio declines. Second, the amount of deposits in the portfolio is determined by the difference between the total demand for other consumption and the total borrowing. Consumers do not hold deposits after trade at date 1, so all cash including deposits and borrowing are used at date 1. Since consumers use only tokens to buy the firm's service, all cash must be used to buy other consumption. On the other hand, other consumption can only be bought using cash. Therefore, the total demand for consumption, $q_c p_c$, equals the sum of the total deposits, d_0, and the total borrowing, $E[b_1]$. The total borrowing depends on the total level of effective liquidity, as a consumer would borrow only when her effective liquidity is not enough to cover her effective demand.

5.2 Issuance Strategy

Next, we analyze the firm's issuance strategy when tokens are tradable. Note that at date 1, all services are bought using tokens, so $q_s p_s$ tokens are redeemed. The firm's net payoff from tokens is

$$\Pi_F = (m_0 \theta_0 - q_s p_s) R_L - (m_0 - q_s p_s) v_2,$$

which is increasing in consumers' holdings of tokens, m_0, and the selling price of tokens, θ_0.

Our main goal is to shed light on the firm's incentive to make tokens tradable, which entails a comparison of the firm's net payoff from tradable and nontradable tokens. When tokens are nontradable, their quantity and pricing are determined in a relatively separate way in equilibrium. Consumers' holdings of tokens are determined or capped by the characteristics of the service: they would never hold more than p_s tokens, and tokens are redeemed with probability q_s. The selling price of tokens is determined by the extent to which they can reduce costly borrowing. When tokens are tradable, their quantity and pricing are closely connected through v_2/θ_0. First, v_2/θ_0 needs to be at least as great as 1 so that consumers would like to hold tokens instead of deposits after trade. This condition imposes a clear upper bound on the selling price of tokens. Second, as indicated by Proposition 11, v_2/θ_0 determines consumers' holdings of tokens. A higher v_2/θ_0

results in a lower $\frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}}$ and thus motivates consumers to hold more tokens. Due to the two forces, the firm's choice of θ_0 is constrained by v_2.

Proposition 12. *The firm's profit from issuing tradable tokens is increasing in v_2. The firm's incentives to make tokens tradable is greater when v_2 is greater.*

According to Proposition 11, it is straightforward to see that given θ_0, as v_2 increases, m_0 weakly increases, and the firm is weakly better off. In fact, the firm can take advantage of the higher v_2 more effectively and be strictly better off. Proposition 12 suggests that the firm's incentive to make tokens tradable is greater when v_2 is greater. This result is not surprising since when tokens are not tradable, the firm's net payoff from tokens does not depend on v_2.

It is straightforward to see that when v_2 is sufficiently small, the firm's net payoff from tradable tokens is negative, no matter what θ_0 is. In that case, the firm is not willing to make tokens tradable. As v_2 increases, the firm's incentive increases. Naturally, we wonder whether the incentive could be great enough so that the firm makes tokens tradable in equilibrium and how this incentive depends on the characteristics of the service. For tradable tokens, since θ_0 affects m_0 through v_2/θ_0, the general characterization of the optimal θ_0 is highly complicated and requires tedious analysis of a number of cases. Here, we focus on the extreme case in which $v_2 = 1$.

Proposition 13. *Suppose $v_2 = 1$. For $\frac{R_L-1}{R_B-1} \leq q_{cs}$ or $q_c < \frac{R_L-1}{R_B-1}$, tradability is preferred; for $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_c$, either tradability is always preferred or there exists κ such that tradability is preferred if and only if $p_s/p_c < \kappa$.*

Proposition 13 conveys two important messages. First, in our setting, the firm prefers to make tokens tradable in some cases. Notably, in Rogoff and You (2023), token issuers never want to make tokens tradable. The reason for the difference is that in our setting, liquidity is valuable, and tradability makes partially liquid tokens effectively more liquid. Second, making tokens tradable is less attractive to firms providing high-value, low-frequency services. For nontradable tokens, consumers' holdings of tokens is determined by the size of the demand for the service, p_s. Hence, firms providing high-value, low-frequency services can benefit more from issuing nontradable tokens than other firms. For tradable tokens, consumers' holdings of tokens hinges on the total level of effective liquidity, which is related to both the size of the demand for the service and that for the other consumption. Hence, the benefit from issuing tradable tokens does not depend on the characteristics of the service as much as the benefit from issuing nontradable tokens.

Finally, we analyze whether the firm benefits from giving consumers more flexibility when issuing tradable tokens.

Proposition 14. *When tokens are tradable, whether to allow for partial redemption or not makes no difference, and the firm prefers not to allow ex-post token issuance.*

Partial redemption becomes irrelevant because consumers always find buying and redeeming tokens cheaper than using cash. To see why the issuing firm does not want to issue tokens ex-post, consider the case in which the firm sets the ex-post price strictly higher than θ_0. In this case, consumers prefer to buy tokens from the market rather than from the firm at date 1. However, this is the only scenario in which the firm may want to issue tokens ex-post. To sum up, tradability makes tokens sufficiently liquid, so giving consumers more flexibility through partial redemption or ex-post token issuance does not help them gain more liquidity benefits.

5.3 Stablecoins with Convertibility

This year, both the United States and Hong Kong have enacted new legislation establishing regulatory guidelines for stablecoins. The reduction of regulatory uncertainty is expected to facilitate the further development and adoption of stablecoins. Major retail platforms—including Amazon, Walmart, Alibaba, and JD.com—are reportedly considering issuing their own stablecoins. A common and important feature of stablecoins is that they can be sold back to the issuers, which we refer to as convertibility. Beyond simple convertibility to fiat currency, stablecoins usually offer consumers certain additional benefits to attract their usage. These additional benefits are likely to be closely integrated with the issuer's core platform business, leveraging their comparative advantages.

In this subsection, we incorporate the convertibility feature into our analysis. Suppose that in addition to redeeming and trading tokens, consumers can also ask the firm to convert tokens back to cash at a pre-specified price, say ϕ dollars per token.[6] ϕ must be no greater than θ_0; otherwise, consumers would buy tokens and convert them immediately at date 0. Convertibility introduces two effects. On the one hand, it increases the liquidity of tokens at date 1, as consumers can convert tokens to cash to satisfy any consumption demand. On the other hand, convertibility may increase tokens' expected value at date 2. A token is worth less than 1 in expectation at date 2, because of devaluation or breakage. In either case, consumers can always choose to receive ϕ dollar per token through conversion. To capture this effect of convertibility, we consider tokens' expected value as a function of ϕ, $v_2(\phi)$, which is potentially increasing in ϕ. Naturally, $v_2(0)$ represents tokens' expected value without convertibility,

Proposition 15. *Suppose that tokens are tradable. If $v_2(\phi)$ is constant in ϕ, convertibility has no impact on the equilibrium, and the firm does not strictly prefer to make tokens convertible. If $v_2(\phi)$ is strictly increasing in ϕ, then the firm strictly prefers to make tokens convertible and sets $\phi = \theta_0$.*

[6] In real-world, stabelcoins are often featured with a fixed $1:1$ exchange ratio. In our model, tokens are valuable in the sense that a token priced as $\theta_0 < 1$ is equivalent to 1 unit of cash when purchasing service goods. It captures trading benefits such as convenience of tokens. One can change the normalization and model that tokens are priced at $\frac{1}{\theta_0}$ to capture the trading benefits tokens may bring.

It turns out that convertibility does not increase the liquidity value of tradable tokens at date 1. If $\phi < \theta_0 = \gamma_1$, consumers do not convert tokens at date 1 because selling tokens to the market dominates. If $\phi = \theta_0 = \gamma_1$, consumers hold the same amount of tokens at the end of date 1, and the difference is that consumers may buy more tokens at date 0 and convert some at date 1. This implies that the firm receives higher cash flows at date 0 and pays out more at date 1. The two amounts are equal, so the firm's net payoff from tokens does not change.

The only impact of convertibility is on tokens' expected value at date 2. As shown in Proposition 12, the firm benefits from a higher v_2. If $v_2(\phi)$ is strictly increasing in ϕ, then the firm strictly prefers to make tokens convertible to increase $v_2(\phi)$ and set ϕ as high as possible, to θ_0. Notably, $\phi = \theta_0$ implies that the optimal design of tokens features a fixed one to one conversion rate. In fact, the token in this case resembles stablecoins observed in practice. To see this, we can re-normalize the prices as follows. Each token is sold at one dollar and can always be converted to one dollar cash. When tokens are used for the service, they are worth $1/\theta_0$ dollars per unit, where $1/\theta_0 > 1$ captures benefits brought by tokens, such as price discounts, premium services,and transaction convenience.

6 Conclusions

This paper provides a framework for understanding when a non-bank firm can profitably issue partially liquid, service-backed tokens and how such tokens shape equilibrium liquidity. The model delivers several contributions.

First, it characterizes the LPT and identifies its non-monotonic relation with the cost of ex-post borrowing. Second, it identifies conditions under which token issuance is optimal for the firm. Third, it evaluates several contract features, such as partial redemption, tradability, and convertibility, and shows how they expand or limit the willingness to issue service-backed tokens.

Taken together, our paper shows when firm-issued tokens have economic value, when they coexist with bank-supplied liquidity, and when design choices meaningfully change their role. The paper contributes a theoretical basis for analyzing digital corporate tokens, which are expected to become an important part of firms' capital structure

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Appendix: Proofs

Proof of Lemma 1

Without tokens, a consumer must choose among the three corner portfolios: $(p_c, 0)$, $(p_s, 0)$, and $(p_s + p_c, 0)$. A consumer's expected payoff is

$$\begin{aligned} E[U_c] = &(\alpha - 1)(p_s q_s + p_c q_c) + R_L w_0 - (R_L - 1) p_c \\ &- q_{cs}(R_B - 1) p_s - (q_s - q_{cs})(R_B - 1)(p_s - p_c)^+ \end{aligned}$$

for $(p_c, 0)$,

$$\begin{aligned} E[U_c] = &(\alpha - 1)(p_s q_s + p_c q_c) + R_L w_0 - (R_L - 1) p_s \\ &- q_{cs}(R_B - 1) p_c - (q_c - q_{cs})(R_B - 1)(p_c - p_s)^+ \end{aligned}$$

for $(p_s, 0)$, and

$$E[U_c] = (\alpha - 1)(p_s q_s + p_c q_c) + R_L w_0 - (R_L - 1)(p_s + p_c)$$

for $(p_s + p_c, 0)$.

Comparing the three portfolios, we obtain

$$(p_c,0) \succ (p_s,0) \Leftrightarrow \frac{R_L-1}{R_B-1}(p_s-p_c) > q_s(p_s-p_c)^+ - q_c(p_c-p_s)^+,$$

$$(p_s+p_c,0) \succ (p_c,0) \Leftrightarrow \frac{R_L-1}{R_B-1} < q_{cs} + (q_s-q_{cs})\frac{(p_s-p_c)^+}{p_s},$$

and

$$(p_s+p_c,0) \succ (p_s,0) \Leftrightarrow \frac{R_L-1}{R_B-1} < q_{cs} + (q_c-q_{cs})\frac{(p_c-p_s)^+}{p_c}.$$

Hence, $(p_s+p_c,0)$ is optimal if and only if $\frac{R_L-1}{R_B-1} < q_{cs}$. When $p_s > p_c$, $(p_c,0)$ is optimal if and only if $\frac{R_L-1}{R_B-1} > q_s$, and $(p_s,0)$ is optimal if and only if $q_{cs} < \frac{R_L-1}{R_B-1} < q_s$. When $p_s < p_c$, $(p_s,0)$ is optimal if and only if $\frac{R_L-1}{R_B-1} > q_c$, and $(p_c,0)$ is optimal if and only if $q_{cs} < \frac{R_L-1}{R_B-1} < q_c$.

Proof of Lemma 2

See the main text.

Proof of Proposition 1

With tokens, a consumer must choose among the following portfolios: $(p_c,0)$, $(p_s,0)$, $(p_s+p_c,0)$, $(0,p_s)$, and (p_c,p_s). A consumer's expected payoff is

$$\begin{aligned}E[U_c] =& R_L w_0 + (\alpha-1)(p_s q_s + p_c q_c) \\ & - [R_L\theta_0 - q_s - (1-q_s)v_2]\, p_s - q_c(R_B-1)\,p_c\end{aligned}$$

for $(0,p_s)$, and

$$\begin{aligned}E[U_c] =& R_L w_0 + (\alpha-1)(p_s q_s + p_c q_c) - (R_L-1)\,p_c \\ & - [R_L\theta_0 - q_s - (1-q_s)v_2]\, p_s\end{aligned}$$

for (p_c,p_s).

Comparing the these portfolios, we obtain

$$(p_c,p_s) \succ (0,p_s) \Leftrightarrow \frac{R_L-1}{R_B-1} < q_c,$$

$$(p_c,p_s) \succ (p_s+p_c,0) \Leftrightarrow \theta_0 < q_s + (1-q_s)\frac{v_2}{R_L} + (1-q_s)\frac{R_L-1}{R_L},$$

$$(p_c, p_s) \succ (p_c, 0) \Leftrightarrow$$
$$\theta_0 < q_s + (1-q_s)\frac{v_2}{R_L} + q_{cs}\frac{R_B - R_L}{R_L} + (q_s - q_{cs})\left[\frac{R_B-1}{R_L}\frac{(p_s-p_c)^+}{p_s} - \frac{R_L-1}{R_L}\right],$$

$$(p_c, p_s) \succ (p_s, 0) \Leftrightarrow$$
$$\theta_0 < q_s + (1-q_s)\frac{v_2}{R_L} + \left(1 - q_s - \frac{p_c}{p_s}\right)\frac{R_L-1}{R_L} + q_{cs}\frac{p_c}{p_s}\frac{R_B-1}{R_L} + (q_c - q_{cs})\frac{R_B-1}{R_L}\frac{(p_c-p_s)^+}{p_s},$$

$$(0, p_s) \succ (p_c, 0) \Leftrightarrow$$
$$\theta_0 < q_s + (1-q_s)\frac{v_2}{R_L} + \left(\frac{p_c}{p_s} - q_s\right)\frac{R_L-1}{R_L} - \left(q_c\frac{p_c}{p_s} - q_{cs}\right)\frac{R_B-1}{R_L} + (q_s - q_{cs})\frac{R_B-1}{R_L}\frac{(p_s-p_c)^+}{p_s},$$

and

$$(0, p_s) \succ (p_s, 0) \Leftrightarrow$$
$$\theta_0 < q_s + (1-q_s)\frac{v_2}{R_L} + (1-q_s)\frac{R_L-1}{R_L} - \left(q_c\frac{p_c}{p_s} - q_{cs}\frac{p_c}{p_s}\right)\frac{R_B-1}{R_L} + (q_c - q_{cs})\frac{R_B-1}{R_L}\frac{(p_c-p_s)^+}{p_s}.$$

Consider the case with $p_s \le p_c$. If $\frac{R_L-1}{R_B-1} > q_c$, $(0, p_s)$ dominates (p_c, p_s), and $(p_s, 0)$ dominates $(p_s + p_c, 0)$ and $(p_c, 0)$. Therefore, the binding constraint for θ_0 is

$$\begin{aligned}\theta_0 &< q_s + (1-q_s)\frac{v_2}{R_L} + (1-q_s)\frac{R_L-1}{R_L} - \left(q_c\frac{p_c}{p_s} - q_{cs}\frac{p_c}{p_s}\right)\frac{R_B-1}{R_L} + (q_c - q_{cs})\frac{R_B-1}{R_L}\frac{(p_c-p_s)^+}{p_s} \\ &= q_s + (1-q_s)\frac{v_2}{R_L} + (1-q_s)\frac{R_L-1}{R_L} - (q_c - q_{cs})\frac{R_B-1}{R_L}.\end{aligned}$$

If $q_{cs} < \frac{R_L-1}{R_B-1} \le q_c$, (p_c, p_s) dominates $(0, p_s)$, and $(p_c, 0)$ dominates $(p_s + p_c, 0)$ and $(p_s, 0)$. Therefore, the binding constraint for θ_0 is

$$\begin{aligned}\theta_0 &< q_s + (1-q_s)\frac{v_2}{R_L} + q_{cs}\frac{R_B - R_L}{R_L} + (q_s - q_{cs})\left[\frac{R_B-1}{R_L}\frac{(p_s-p_c)^+}{p_s} - \frac{R_L-1}{R_L}\right] \\ &= q_s + (1-q_s)\frac{v_2}{R_L} + q_{cs}\frac{R_B - R_L}{R_L} - (q_s - q_{cs})\frac{R_L-1}{R_L}.\end{aligned}$$

If $\frac{R_L-1}{R_B-1} \le q_{cs}$, (p_c, p_s) dominates $(0, p_s)$, and $(p_s + p_c, 0)$ dominates $(p_c, 0)$ and $(p_s, 0)$. Therefore, the binding constraint for θ_0 is

$$\theta_0 < q_s + (1-q_s)\frac{v_2}{R_L} + (1-q_s)\frac{R_L-1}{R_L}.$$

Proof of Proposition 2

Proposition 2 directly follows from the characterization of the LPT in Proposition 1

Proof of Proposition 3

See the main text.

Proof of Proposition 4

Consider the case with $p_s \le p_c$. If $q_c < \frac{R_L-1}{R_B-1}$, the maximum possible LPT is negative is equivalent to

$$\begin{aligned}
&(1-q_s)\frac{R_L-1}{R_L} - (q_c - q_{cs})\frac{R_B-1}{R_L} < 0 \\
\Leftrightarrow &\frac{R_L-1}{R_B-1} < \frac{q_c - q_{cs}}{1-q_s}.
\end{aligned}$$

If $q_{cs} < \frac{R_L-1}{R_B-1} \le q_c$, the maximum possible LPT is negative is equivalent to

$$\begin{aligned}
&q_{cs}\frac{R_B-R_L}{R_L} - (q_s - q_{cs})\frac{R_L-1}{R_L} < 0 \\
\Leftrightarrow &\frac{R_L-1}{R_B-1} > \frac{q_{cs}}{q_s}.
\end{aligned}$$

If $\frac{R_L-1}{R_B-1} \le q_{cs}$, the maximum possible LPT must be positive.

Notice that $q_c < \frac{q_c-q_{cs}}{1-q_s}$ and $q_c > \frac{q_{cs}}{q_s}$ hold if and only if $q_{cs} < q_c q_s$, or the demand for the two consumption goods is negatively dependent. Taken together, when $\frac{R_L-1}{R_B-1} \in \left[\frac{q_{cs}}{q_s}, \frac{q_c-q_{cs}}{1-q_s}\right]$, the maximum possible LPT is nonpositive.

Proof of Proposition 5

Consider the case with $p_s \le p_c$. Without tokens, the following table summarizes the total deposits at date 0, the total deposits after withdrawal at date 1, and the total borrowing at date 1:

	d_0	$d_0 - E[\Delta_d]$	$E[b_1]$
$\frac{R_L-1}{R_B-1} < q_{cs}$	$p_s + p_c$	$p_s + p_c - [q_s p_s + q_c p_c]$	0
$q_{cs} < \frac{R_L-1}{R_B-1} < q_c$	p_c	$p_c - [(q_s - q_{cs})p_s + q_c p_c]$	$q_{cs} p_s$
$q_c < \frac{R_L-1}{R_B-1}$	p_s	$p_s - (q_s + q_c - q_{cs})p_s$	$(q_c - q_{cs})(p_c - p_s) + q_{cs} p_c$

With tokens, the following table summarizes the total deposits at date 0, the total deposits after withdrawal at date 1, and the total borrowing at date 1:

	d_0	$d_0 - E[\Delta_d]$	$E[b_1]$
$\frac{R_L-1}{R_B-1} < q_{cs}$	p_c	$p_c - q_c p_c$	0
$q_{cs} < \frac{R_L-1}{R_B-1} < q_c$	p_c	$p_c - q_c p_c$	0
$q_c < \frac{R_L-1}{R_B-1}$	0	0	$q_c p_c$

Comparing the two tables, we can readily obtain Proposition 5.

Proof of Proposition 6

We first derive θ_{max} for the case with $p_s > p_c$, following the proof of Proposition 1. If $\frac{R_L-1}{R_B-1} > q_c$, $(0, p_s)$ dominates (p_c, p_s), and $(p_c, 0)$ dominates $(p_s + p_c, 0)$ and $(p_s, 0)$. Therefore, the binding constraint for θ_0 is

$$\begin{aligned}\theta_0 &< q_s + (1-q_s)\frac{v_2}{R_L} + \left(\frac{p_c}{p_s} - q_s\right)\frac{R_L-1}{R_L} - \left(q_c\frac{p_c}{p_s} - q_{cs}\right)\frac{R_B-1}{R_L} + (q_s - q_{cs})\frac{R_B-1}{R_L}\frac{(p_s-p_c)^+}{p_s} \\ &= q_s + (1-q_s)\frac{v_2}{R_L} + \left(\frac{p_c}{p_s} - q_s\right)\frac{R_L-1}{R_L}\left[q_s - (q_s + q_c - q_{cs})\frac{p_c}{p_s}\right]\frac{R_B-1}{R_L}.\end{aligned}$$

If $q_s < \frac{R_L-1}{R_B-1} \leq q_c$, (p_c, p_s) dominates $(0, p_s)$, and $(p_c, 0)$ dominates $(p_s + p_c, 0)$ and $(p_s, 0)$. Therefore, the binding constraint for θ_0 is

$$\begin{aligned}\theta_0 &< q_s + (1-q_s)\frac{v_2}{R_L} + q_{cs}\frac{R_B - R_L}{R_L} + (q_s - q_{cs})\left[\frac{R_B-1}{R_L}\frac{(p_s-p_c)^+}{p_s} - \frac{R_L-1}{R_L}\right] \\ &= q_s + (1-q_s)\frac{v_2}{R_L} - q_s\frac{R_L-1}{R_L} + \left(q_s - q_s\frac{p_c}{p_s} + q_{cs}\frac{p_c}{p_s}\right)\frac{R_B-1}{R_L}.\end{aligned}$$

If $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_s$, (p_c, p_s) dominates $(0, p_s)$, and $(p_s, 0)$ dominates $(p_s + p_c, 0)$ and $(p_c, 0)$. Therefore, the binding constraint for θ_0 is

$$\begin{aligned}\theta_0 &< q_s + (1-q_s)\frac{v_2}{R_L} + \left(1 - q_s - \frac{p_c}{p_s}\right)\frac{R_L-1}{R_L} + q_{cs}\frac{p_c}{p_s}\frac{R_B-1}{R_L} + (q_c - q_{cs})\frac{R_B-1}{R_L}\frac{(p_c-p_s)^+}{p_s} \\ &= q_s + (1-q_s)\frac{v_2}{R_L} + \left(1 - q_s - \frac{p_c}{p_s}\right)\frac{R_L-1}{R_L} + q_{cs}\frac{p_c}{p_s}\frac{R_B-1}{R_L}.\end{aligned}$$

If $\frac{R_L-1}{R_B-1} \leq q_{cs}$, (p_c, p_s) dominates $(0, p_s)$, and $(p_s + p_c, 0)$ dominates $(p_s, 0)$ and $(p_c, 0)$. Therefore, the binding constraint for θ_0 is

$$\theta_0 < q_s + (1-q_s)\frac{v_2}{R_L} + (1-q_s)\frac{R_L-1}{R_L}.$$

Second, we show how the LPT varies with R_B. If $q_c < \frac{R_L-1}{R_B-1}$,

$$\frac{\partial\theta_{max}}{\partial R_B} = q_s - (q_s + q_c - q_{cs})\frac{p_c}{p_s} \leq q_s - q_c\frac{p_c}{p_s} < 0;$$

the results are obvious for other regions of $\frac{R_L-1}{R_B-1}$.

Third, we show the firm's issuance strategy. If $q_c < \frac{R_L-1}{R_B-1}$, that the maximum possible LPT is negative is equivalent to

$$\left(\frac{p_c}{p_s}-q_s\right)\frac{R_L-1}{R_L}+\left[q_s-(q_s+q_c-q_{cs})\frac{p_c}{p_s}\right]\cdot\frac{R_B-1}{R_L}<0$$
$$\Leftrightarrow\frac{R_L-1}{R_B-1}<\frac{(q_s+q_c-q_{cs})\frac{p_c}{p_s}-q_s}{\frac{p_c}{p_s}-q_s}.$$

The last inequality follows $\frac{p_c}{p_s}-q_s>\frac{q_c p_c}{p_s}-q_s>0$. If $q_s<\frac{R_L-1}{R_B-1}\leq q_c$, that the maximum possible LPT is negative is equivalent to

$$-q_s\frac{R_L-1}{R_L}+\left(q_s-q_s\frac{p_c}{p_s}+q_{cs}\frac{p_c}{p_s}\right)\frac{R_B-1}{R_L}<0$$
$$\Leftrightarrow\frac{R_L-1}{R_B-1}>1-\frac{p_c}{p_s}+\frac{q_{cs}}{q_s}\frac{p_c}{p_s}.$$

If $q_{cs}<\frac{R_L-1}{R_B-1}\leq q_s$, that the maximum possible LPT is negative is equivalent to

$$\left(1-q_s-\frac{p_c}{p_s}\right)\frac{R_L-1}{R_L}+q_{cs}\frac{p_c}{p_s}\frac{R_B-1}{R_L}<0$$
$$\Leftrightarrow\left(1-q_s-\frac{p_c}{p_s}\right)\frac{R_L-1}{R_B-1}+q_{cs}\frac{p_c}{p_s}<0$$
$$\Rightarrow\left(1-q_s-\frac{p_c}{p_s}\right)q_{cs}+q_{cs}\frac{p_c}{p_s}<0$$
$$\Leftrightarrow(1-q_s)\,q_{cs}<0,$$

which is impossible. If $\frac{R_L-1}{R_B-1}\leq q_{cs}$, the maximum possible LPT must be positive. Notice that $\frac{(q_s+q_c-q_{cs})\frac{p_c}{p_s}-q_s}{\frac{p_c}{p_s}-q_s}>q_c$ is equivalent to$1-\frac{p_c}{p_s}+\frac{q_{cs}}{q_s}\frac{p_c}{p_s}<q_c$. Taken together, when $\frac{R_L-1}{R_B-1}\in\left[1-\frac{p_c}{p_s}+\frac{q_{cs}}{q_s}\frac{p_c}{p_s},\frac{(q_s+q_c-q_{cs})\frac{p_c}{p_s}-q_s}{\frac{p_c}{p_s}-q_s}\right]$, the maximum possible LPT is nonpositive.

Fourth, we show the bank's payoff. Without tokens, the following table summarizes the total deposits at date 0, the total deposits after withdrawal at date 1, and the total borrowing at date 1:

	d_0	$d_0-E[\Delta_d]$	$E[b_1]$
$\frac{R_L-1}{R_B-1}<q_{cs}$	p_s+p_c	$p_s+p_c-[q_s p_s+q_c p_c]$	0
$q_{cs}<\frac{R_L-1}{R_B-1}<q_s$	p_s	$p_s-[(q_c-q_{cs})\,p_c+q_s p_s]$	$q_{cs}p_c$
$q_s<\frac{R_L-1}{R_B-1}$	p_c	$p_c-(q_s+q_c-q_{cs})\,p_c$	$(q_s-q_{cs})(p_s-p_c)+q_{cs}p_s$

With tokens, the following table summarizes the total deposits at date 0, the total deposits after withdrawal at date 1, and the total borrowing at date 1:

	d_0	$d_0-E[\Delta_d]$	$E[b_1]$
$\frac{R_L-1}{R_B-1}<q_{cs}$	p_c	$p_c-q_c p_c$	0
$q_{cs}<\frac{R_L-1}{R_B-1}<q_s$	p_c	$p_c-q_c p_c$	0
$q_s<\frac{R_L-1}{R_B-1}\leq q_c$	p_c	$p_c-q_c p_c$	0
$q_c<\frac{R_L-1}{R_B-1}$	0	0	$q_c p_c$

Proof of Proposition 7

Based on Proposition 6, we denote the firm's net payoff from tokens in different regions of parameters as follows:

$$\Pi_1 \triangleq p_s(1-q_s)(R_L-1)$$

for $\frac{R_L-1}{R_B-1} \leq q_{cs}$,

$$\Pi_2 \triangleq p_s \rho q_s q_c (R_B-1) - p_s q_s (R_L-1)$$

for $q_{cs} \leq \frac{R_L-1}{R_B-1} \leq q_c$ and $p_s \leq p_c$,

$$\Pi_3 \triangleq p_s(1-q_s)(R_L-1) - p_c[(R_L-1) - \rho q_s q_c (R_B-1)]$$

for $q_{cs} \leq \frac{R_L-1}{R_B-1} \leq q_s$ and $p_s \geq p_c$,

$$\Pi_4 \triangleq p_s q_s (R_B-R_L) - p_c(q_s - \rho q_s q_c)(R_B-1)$$

for $q_s \leq \frac{R_L-1}{R_B-1} \leq q_c$ and $p_s \geq p_c$,

$$\Pi_5 \triangleq p_s(1-q_s)(R_L-1) - p_s(q_c - \rho q_s q_c)(R_B-1)$$

for $q_c \leq \frac{R_L-1}{R_B-1}$ and $p_s \leq p_c$,

$$\Pi_6 \triangleq p_s q_s (R_B-R_L) + p_c(R_L-1) - p_c(q_s + q_c - \rho q_s q_c)(R_B-1)$$

for $q_c \leq \frac{R_L-1}{R_B-1}$ and $p_s \geq p_c$.

Part I: Fix $q_s p_s$ and vary (p_s, q_s)

We prove that Π_F is weakly increasing in p_s in two steps. First, Π_F is weakly increasing in p_s in each region as follows:

$$\frac{d\Pi_1}{dp_s} = R_L - 1 > 0,$$

$$\frac{d\Pi_2}{dp_s} = 0,$$

$$\begin{aligned}
\frac{d\Pi_3}{dp_s} &= (R_L-1) + p_c \rho q_c (R_B-1)\frac{dq_s}{dp_s} \\
&= (R_L-1) - p_c \rho q_c (R_B-1)\frac{q_s}{p_s} \\
&\geq \rho q_s q_c (R_B-1) - \rho q_c (R_B-1) q_s = 0,
\end{aligned}$$

$$\frac{d\Pi_4}{dp_s} = -p_c(1-\rho q_c)(R_B-1)\cdot\frac{dq_s}{dp_s} > 0,$$

$$\frac{d\Pi_5}{dp_s} = (R_L - 1) - q_c (R_B - 1) \geq 0,$$

$$\frac{d\Pi_6}{dp_s} = -(1 - \rho q_c) p_c (R_B - 1) \cdot \frac{dq_s}{dp_s} > 0.$$

Second, as p_s increases and q_s decreases, Π_F may move into another region, but there is no jump at that point. Π_F may move from Region 1 to Region 2 at $q_{cs} = \frac{R_L-1}{R_B-1}$, where

$$\Pi_1 = p_s (1 - q_s)(R_L - 1) = p_s q_{cs} (R_B - 1) - p_s q_s (R_L - 1) = \Pi_2.$$

Π_F may move from Region 1 to Region 3 at $q_{cs} = \frac{R_L-1}{R_B-1}$, where

$$\begin{aligned}\Pi_3 &= p_s (1 - q_s)(R_L - 1) - p_c [(R_L - 1) - q_{cs} (R_B - 1)] \\ &= p_s (1 - q_s)(R_L - 1) \\ &= \Pi_1.\end{aligned}$$

Π_F may move from Region 2 to Region 3 at $p_s = p_c$, where

$$\begin{aligned}\Pi_2 &= q_{cs} p_c (R_B - 1) - q_s p_c (R_L - 1) \\ &= p_c (1 - q_s)(R_L - 1) - p_c [(R_L - 1) - q_{cs} (R_B - 1)] \\ &= \Pi_3.\end{aligned}$$

Π_F may move from Region 2 to Region 4 at $p_s = p_c$, where

$$\begin{aligned}\Pi_2 &= q_{cs} p_c (R_B - 1) - q_s p_c (R_L - 1) \\ &= p_c q_s (R_B - R_L) - p_c (q_s - q_{cs})(R_B - 1) \\ &= \Pi_4.\end{aligned}$$

Π_F may move from Region 3 to Region 4 at $q_s = \frac{R_L-1}{R_B-1}$, where

$$\begin{aligned}\Pi_3 &= p_s (1 - q_s)(R_L - 1) - p_c [(R_L - 1) - q_{cs} (R_B - 1)] \\ &= p_s q_s (R_B - R_L) - p_c (q_s - q_{cs})(R_B - 1) \\ &= \Pi_4.\end{aligned}$$

Π_F may move from Region 5 to Region 6 at $p_s = p_c$, where

$$\begin{aligned}\Pi_5 =& p_c (1 - q_s)(R_L - 1) - p_c (q_c - q_{cs})(R_B - 1) \\ =& p_s q_s (R_B - R_L) + p_c (R_L - 1) - p_c (q_s + q_c - \rho q_s q_c)(R_B - 1) \\ =& \Pi_6.\end{aligned}$$

Part II: Vary ρ

We prove that Π_F is weakly increasing in ρ in similar two steps. First, Π_F is weakly increasing in ρ in each region as follows:

$$\frac{d\Pi_1}{d\rho} = 0$$
$$\frac{d\Pi_2}{d\rho} = q_s q_c p_s (R_B - 1) > 0$$
$$\frac{d\Pi_3}{d\rho} = p_c q_s q_c (R_B - 1) > 0$$
$$\frac{d\Pi_4}{d\rho} = p_c q_s q_c (R_B - 1) > 0$$
$$\frac{d\Pi_5}{d\rho} = p_s q_s q_c (R_B - 1) > 0$$
$$\frac{d\Pi_6}{d\rho} = p_c q_s q_c (R_B - 1) > 0$$

Second, as ρ increases, Π_F may move into another region, but there is no jump at that point. Π_F may move from Region 2 to Region 1 at $q_{cs} = \frac{R_L - 1}{R_B - 1}$,

$$\Pi_1 = p_s (1 - q_s)(R_L - 1) = p_s q_{cs} (R_B - 1) - q_s p_s (R_L - 1) = \Pi_2$$

Π_F may move from Region 3 to Region 1 at $q_{cs} = \frac{R_L - 1}{R_B - 1}$,

$$\begin{aligned}\Pi_3 &= p_s (1 - q_s)(R_L - 1) - p_c [(R_L - 1) - q_{cs} (R_B - 1)] \\ &= p_s (1 - q_s)(R_L - 1) \\ &= \Pi_1\end{aligned}$$

Part III: Vary p_c

We prove that Π_F is weakly decreasing in p_c in similar two steps. First, Π_F is weakly decreasing in p_c in each region as follows:

$$\frac{d\Pi_1}{dp_c} = \frac{d\Pi_2}{dp_c} = \frac{d\Pi_5}{dp_c} = 0,$$
$$\frac{d\Pi_3}{dp_c} = -[(R_L - 1) - \rho q_s q_c (R_B - 1)] < 0,$$
$$\frac{d\Pi_4}{dp_c} = -(q_s - \rho q_s q_c)(R_B - 1) < 0,$$
$$\frac{d\Pi_6}{dp_c} = (R_L - 1) - (q_s + q_c - \rho q_s q_c)(R_B - 1) < 0.$$

Second, as p_c increases, Π_F may move into another region, but there is no jump at that point. Π_F may move from Region 3 to Region 2, or from Region 4 to Region 2, or from Region 6 to Region 5, all at $p_s = p_c$. According to the proof in Part I, we know that there is no jump at these points.

Proof of Proposition 8

Still, the firm will set $\theta_0 > \frac{v_2}{R_L}$, and consumers will not hold more than p_s tokens. In equilibrium, if the firm strictly prefers to allow for partial redemption, it cannot be the case that $m_0 = 0$ or $m_0 = p_s$. If $m_0 = 0$, the firm's net payoff from tokens is nonpositive. If $m_0 = p_s$, consumers do not use partial redemption at all.

Consider the equilibrium path with $m_0 \in (0, p_s)$. Holding d_0 in deposits and $m_0 \in [0, p_s]$ tokens, a consumer's expected payoff is

$$\begin{aligned} E[U_c] =& R_L w_0 + (\alpha - 1)(p_s q_s + p_c q_c) - (R_L - 1) d_0 - [R_L \theta_0 - q_s - (1 - q_s) v_2] m_0 \\ & - q_{cs}(R_B - 1)(p_c + p_s - d_0 - m_0)^+ - (q_s - q_{cs})(R_B - 1)(p_s - d_0 - m_0)^+ \\ & - (q_c - q_{cs})(R_B - 1)(p_c - d_0)^+ . \end{aligned}$$

$m_0 > 0$ implies that tokens are dominated by deposits, so

$$\begin{aligned} & R_L \theta_0 - q_s - (1 - q_s) v_2 \le R_L - 1 \\ \Leftrightarrow\ & \theta_0 \le q_s + (1 - q_s)\frac{v_2}{R_L} + \frac{R_L - 1}{R_L}(1 - q_s). \end{aligned}$$

It is not hard to see that the consumer must satisfy at least one of the following corner conditions: 1) $d_0 + m_0 = p_c + p_s$, 2) $d_0 + m_0 = p_s$, 3) $d_0 = p_c$, and 4) $d_0 = 0$. $d_0 = 0$ cannot hold in the equilibrium path with $m_0 \in (0, p_s)$, since the derivative of $E[U_c]$ with respect to d_0 is positive at $d_0 = 0$ for any $m_0 \in (0, p_s)$:

$$\begin{aligned} \frac{\partial E[U_c]}{\partial d_0} =& (R_B - 1)[q_{cs}\sigma(p_s + p_c - m_0) + (q_s - q_{cs})\sigma(p_s - m_0) + (q_c - q_{cs})\sigma(p_c)] \\ & - (R_L - 1) \\ =& (R_B - 1)(q_s + q_c - q_{cs}) - (R_L - 1) > 0. \end{aligned}$$

$d_0 + m_0 = p_c + p_s$ cannot hold in the equilibrium path. Given $d_0 + m_0 = p_c + p_s$, the derivative of $E[U_c]$ with respect to d_0 is

$$\frac{\partial E[U_c]}{\partial d_0} - \frac{\partial E[U_c]}{\partial m_0} = -(R_L - 1) + [R_L \theta_0 - q_s - (1 - q_s) v_2] + (q_c - q_{cs})(R_B - 1)\sigma(p_c - d_0),$$

which is nonpositive when $d_0 \ge p_c$. Hence, she will not hold more than p_c deposits, so $d_0 + m_0 < p_c + p_s$.

Since we have two choice variables here, the other two conditions need to hold simultaneously. That means, $p_s > p_c$, and $(d_0, m_0) = (p_c, p_s - p_c)$. When $p_s \le p_c$, consumers hold either $m_0 = 0$ or $m_0 = p_s$, so the firm never strictly prefers to allow for partial redemption.

Next, we focus on the case with $p_s > p_c$. Since partial redemption essentially allows consumers to hold $(d_0, m_0) = (p_c, p_s - p_c)$, the firm prefers to allow for partial redemption if and only if his net payoff from tokens by inducing consumers to hold $(d_0, m_0) = (p_c, p_s - p_c)$ exceeds that by following the issuance strategy characterized by Propositions 1 and 6.

We first derive the former. Holding $(d_0, m_0) = (p_c, p_s - p_c)$, a consumer's expected payoff is

$$
\begin{aligned}
E[U_c] =& R_L w_0 + (\alpha - 1)(p_s q_s + p_c q_c) - (R_L - 1) p_c \\
& - [R_L \theta_0 - q_s - (1 - q_s) v_2](p_s - p_c) - q_{cs}(R_B - 1) p_c.
\end{aligned}
$$

Comparing $(p_c, p_s - p_c)$ with $(p_c, 0)$, $(p_s, 0)$, $(p_s + p_c, 0)$, $(0, p_s)$, and (p_c, p_s), we obtain

$$
(p_c, p_s - p_c) \succ (p_c, 0) \Leftrightarrow \theta_0 < q_s + (1 - q_s)\frac{v_2}{R_L} + q_s \frac{R_B - 1}{R_L} - q_s \frac{R_L - 1}{R_L}
$$

$$
(p_c, p_s - p_c) \succ (p_s, 0) \Leftrightarrow \theta_0 < q_s + (1 - q_s)\frac{v_2}{R_L} + (1 - q_s)\frac{R_L - 1}{R_L}
$$

$$
(p_c, p_s - p_c) \succ (0, p_s) \Leftrightarrow \theta_0 > q_s + (1 - q_s)\frac{v_2}{R_L} + (1 - q_s)\frac{R_L - 1}{R_L} - (q_c - q_{cs})\frac{R_B - 1}{R_L}
$$

$$
(p_c, p_s - p_c) \succ (p_c, p_s) \Leftrightarrow \theta_0 > q_s + (1 - q_s)\frac{v_2}{R_L} - q_s \frac{R_L - 1}{R_L} + q_{cs} \cdot \frac{R_B - 1}{R_L}
$$

$$
\begin{aligned}
& (p_c, p_s - p_c) \succ (p_s + p_c, 0) \\
\Leftrightarrow & \theta_0 < q_s + (1 - q_s)\frac{v_2}{R_L} - q_s \frac{R_L - 1}{R_L} + \frac{R_L - 1}{R_L}\frac{p_s}{p_s - p_c} - q_{cs}\frac{R_B - 1}{R_L}\frac{p_c}{p_s - p_c}
\end{aligned}
$$

If $\frac{R_L - 1}{R_B - 1} \leq q_{cs}$, $(p_c, p_s - p_c)$ cannot dominate $(p_s, 0)$ and (p_c, p_s) simultaneously, so consumers will not choose it. If $q_{cs} < \frac{R_L - 1}{R_B - 1} \leq q_s$, for $(p_c, p_s - p_c)$ to the optimal for consumers,

$$
\theta_0 \leq q_s + (1 - q_s)\frac{v_2}{R_L} + (1 - q_s)\frac{R_L - 1}{R_L},
$$

so the firm's maximum payoff from tokens is

$$
\Pi_F = (p_s - p_c)(1 - q_s)(R_L - 1).
$$

If $q_s < \frac{R_L - 1}{R_B - 1}$,

$$
\theta_0 \leq q_s + (1 - q_s)\frac{v_2}{R_L} + q_s \frac{R_B - 1}{R_L} - q_s \frac{R_L - 1}{R_L},
$$

so the firm's maximum payoff from tokens is

$$
\Pi_F = (p_s - p_c) q_s (R_B - R_L).
$$

We then derive the condition under which the firm strictly prefers to allow for partial redemption. We refer to the firm's maximum net payoff from tokens without partial redemption as the best alternative. If $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_s$, the best alternative is

$$(p_s - q_s p_s - p_c)(R_L - 1) + q_{cs} p_c (R_B - 1).$$

The firm prefers to allow for partial redemption if and only if

$$\begin{gathered}(p_s - p_c)(1 - q_s)(R_L - 1) > (p_s - q_s p_s - p_c)(R_L - 1) + q_{cs} p_c (R_B - 1) \\ \Leftrightarrow \frac{R_L - 1}{R_B - 1} > \frac{q_{cs}}{q_s}.\end{gathered}$$

If $q_s < \frac{R_L-1}{R_B-1} \leq q_c$, the best alternative is

$$-q_s p_s (R_L - 1) + (q_s p_s - q_s p_c + q_{cs} p_c)(R_B - 1).$$

The firm prefers to allow for partial redemption if and only if

$$\begin{gathered}(p_s - p_c) q_s (R_B - R_L) > -q_s p_s (R_L - 1) + (q_s p_s - q_s p_c + q_{cs} p_c)(R_B - 1) \\ \Leftrightarrow \frac{R_L - 1}{R_B - 1} > \frac{q_{cs}}{q_s}.\end{gathered}$$

If $q_c < \frac{R_L-1}{R_B-1}$, the best alternative is

$$(p_c - q_s p_s)(R_L - 1) + [q_s p_s - (q_s + q_c - q_{cs}) p_c](R_B - 1).$$

The firm prefers to allow for partial redemption if and only if

$$\begin{gathered}(p_s - p_c) q_s (R_B - R_L) > (p_c - q_s p_s)(R_L - 1) + [q_s p_s - (q_s + q_c - q_{cs}) p_c](R_B - 1) \\ \Leftrightarrow \frac{R_L - 1}{R_B - 1} < \frac{q_c - q_{cs}}{1 - q_s}\end{gathered}$$

Notice that $q_c < \frac{q_c - q_{cs}}{1-q_s}$ and $q_c > \frac{q_{cs}}{q_s}$ hold if and only if $q_{cs} < q_c q_s$. Taken together, the firm prefers to allow for partial redemption if and only if $\frac{R_L-1}{R_B-1} \in \left(\frac{q_{cs}}{q_s}, \frac{q_c-q_{cs}}{1-q_s}\right)$.

For $\frac{R_L-1}{R_B-1} \in \left(\frac{q_{cs}}{q_s}, \frac{q_c-q_{cs}}{1-q_s}\right)$, the firm can always obtain a positive net payoff from tokens by inducing consumers to hold $(p_c, p_s - p_c)$. For $\frac{R_L-1}{R_B-1} \notin \left(\frac{q_{cs}}{q_s}, \frac{q_c-q_{cs}}{1-q_s}\right)$, it must be $\frac{R_L-1}{R_B-1} \notin \left[1 - \frac{p_c}{p_s} + \frac{q_{cs}}{q_s}\frac{p_c}{p_s}, \frac{(q_s+q_c-q_{cs})\frac{p_c}{p_s} - q_s}{\frac{p_c}{p_s} - q_s}\right]$, so the firm will also issue tokens. Therefore, when $p_s > p_c$, with the option to allow for partial redemption, the firm always issues tokens.

Proof of Proposition 9

The first must set $\theta_0 > \frac{v_2}{R_L}$. Otherwise, tokens dominate investment, so consumers will hold many tokens at date 0 and do not buy at date 1. The firm's net payoff from tokens cannot be positive. $\theta_0 > \frac{v_2}{R_L}$ implies that consumers will not hold $m_0 > p_s$. We focus on $m_0 \in [0, p_s]$. We focus on the case that $\overline{\theta}$ is no smaller than v_2 so that consumers will not buy tokens at date 2. Following the proof, it is not hard to see that $\overline{\theta} < v_2$ cannot be optimal for the firm.

Part I: three options to satisfy the demand for the service Consumers have four potential options to satisfy their demand for the service. First, they always use cash to buy services. In this case, since tokens are all held to date 2, consumers will choose $m_0 = 0$, so their expected payoff is

$$\begin{aligned} E[U_c] =& R_L w_0 + (\alpha - 1)(p_s q_s + p_c q_c) - (R_L - 1) d_0 \\ & - q_{cs}(R_B - 1)(p_c + p_s - d_0)^+ - (q_s - q_{cs})(R_B - 1)(p_s - d_0)^+ \\ & - (q_c - q_{cs})(R_B - 1)(p_c - d_0)^+ . \end{aligned}$$

If consumers take this first option in equilibrium, their portfolio must be among $(p_c, 0)$, $(p_s, 0)$, and $(p_c + p_s, 0)$.

Second, they always buy $p_s - m_0$ tokens and redeem tokens for services. Their expected payoff is

$$\begin{aligned} E[U_c] =& R_L w_0 + (\alpha - 1)(p_s q_s + p_c q_c) + p_s q_s \left(1 - \overline{\theta}\right) - (R_L - 1) d_0 \\ & - \left[R_L \theta_0 - q_s \overline{\theta} - (1 - q_s) v_2\right] m_0 - q_{cs}(R_B - 1)\left(p_c + p_s \overline{\theta} - m_0 \overline{\theta} - d_0\right)^+ \\ & - (q_s - q_{cs})(R_B - 1)\left(p_s \overline{\theta} - m_0 \overline{\theta} - d_0\right)^+ - (q_c - q_{cs})(R_B - 1)(p_c - d_0)^+ . \end{aligned}$$

If consumers take this second option in equilibrium, their portfolio must be among $(p_c, 0)$, $\left(p_s \overline{\theta}, 0\right)$, $\left(p_c + p_s \overline{\theta}, 0\right)$, $(0, p_s)$, (p_c, p_s) and $\left(p_c, p_s - p_c/\overline{\theta}\right)$, where $\left(p_c, p_s - p_c/\overline{\theta}\right)$ is potentially optimal only when $p_s \overline{\theta} > p_c$.

Third, they use cash to buy services when they only need the service, and buy $p_s - m_0$ tokens and redeem tokens for services when they need both the service and other consumption. Their expected payoff is

$$\begin{aligned} E[U_c] =& R_L w_0 + (\alpha - 1)(p_s q_s + p_c q_c) + p_s q_{cs} \left(1 - \overline{\theta}\right) - (R_L - 1) d_0 \\ & - \left[R_L \theta_0 - q_{cs} \overline{\theta} - (1 - q_{cs}) v_2\right] m_0 - q_{cs}(R_B - 1)\left(p_c + p_s \overline{\theta} - m_0 \overline{\theta} - d_0\right)^+ \\ & - (q_s - q_{cs})(R_B - 1)(p_s - d_0)^+ - (q_c - q_{cs})(R_B - 1)(p_c - d_0)^+ . \end{aligned}$$

If consumers take this third option in equilibrium, their portfolio must be among $(p_c, 0)$, $(p_s, 0)$, $\left(p_c + p_s \overline{\theta}, 0\right)$, $(0, p_s)$, (p_s, p_s), (p_c, p_s), and $\left(p_s, p_s - (p_s - p_c)/\overline{\theta}\right)$, where $\left(p_s, p_s - (p_s - p_c)/\overline{\theta}\right)$ is potentially optimal only when $p_s > p_c$.

Fourth, they use cash to buy services when they need both the service and other consumption, and buy

$p_s - m_0$ tokens and redeem tokens for services when they only need the service. Their expected payoff is

$$\begin{aligned} E[U_c] =& R_L w_0 + (\alpha - 1)(p_s q_s + p_c q_c) + (q_s - q_{cs}) p_s \left(1 - \overline{\theta}\right) - (R_L - 1) d_0 \\ & - \left[R_L \theta_0 - (q_s - q_{cs}) \overline{\theta} - (1 - q_s + q_{cs}) v_2\right] m_0 - q_{cs} (R_B - 1)(p_c + p_s - d_0)^+ \\ & - (q_s - q_{cs})(R_B - 1)\left(p_s \overline{\theta} - m_0 \overline{\theta} - d_0\right)^+ - (q_c - q_{cs})(R_B - 1)(p_c - d_0)^+ . \end{aligned}$$

The fourth option cannot be optimal for consumers. For any (d_0, m_0), we need

$$\frac{p_s\left(1-\overline{\theta}\right) + \left(\overline{\theta} - v_2\right) m_0}{R_B - 1} > \left(p_s \overline{\theta} - m_0 \overline{\theta} - d_0\right)^+ - (p_s - d_0)^+$$

for the fourth to dominate the first, and

$$\left(p_c + p_s \overline{\theta} - m_0 \overline{\theta} - d_0\right)^+ - (p_c + p_s - d_0)^+ > \frac{p_s\left(1-\overline{\theta}\right) + \left(\overline{\theta} - v_2\right) m_0}{R_B - 1}$$

for the fourth to dominate the second. Combining them, we obtain

$$\left(p_c + p_s \overline{\theta} - m_0 \overline{\theta} - d_0\right)^+ - \left(p_s \overline{\theta} - m_0 \overline{\theta} - d_0\right)^+ > (p_c + p_s - d_0)^+ - (p_s - d_0)^+ ,$$

which implies $p_s \overline{\theta} - m_0 \overline{\theta} > p_s$. However, if $p_s \overline{\theta} - m_0 \overline{\theta} > p_s$,

$$0 > \frac{-m_0 v_2}{R_B - 1} > \frac{p_s\left(1-\overline{\theta}\right) + \left(\overline{\theta} - v_2\right) m_0}{R_B - 1} > \left(p_s \overline{\theta} - m_0 \overline{\theta} - d_0\right)^+ - (p_s - d_0)^+ \geq 0.$$

Contradiction! Therefore, the fourth must be dominated by at least one of the first and the second.

Part II: $\overline{\theta}$ must be at least 1 Consider the case with $\overline{\theta} < 1$. Then for any (d_0, m_0), the second option dominates the first and the third options. We focus on that consumers take the second option. As discussed in the main text, in any equilibrium where the firm strictly prefers ex-post token issuance, consumers must hold $m_0 \in (0, p_s)$ tokens. Hence, it must be that $p_s \overline{\theta} > p_c$ and the firm induces consumers to hold $\left(p_c, p_s - p_c/\overline{\theta}\right)$. Then consumers' expected payoff is

$$\begin{aligned} E[U_c] =& R_L w_0 + (\alpha - 1)(p_s q_s + p_c q_c) + q_s (p_s - p_c) - (R_L - 1) p_c \\ & - [R_L \theta_0 - (1 - q_s) v_2]\left(p_s - p_c/\overline{\theta}\right) - q_{cs} (R_B - 1) p_c , \end{aligned}$$

and the firm's net payoff from tokens is

$$\begin{aligned} \Pi_F =& \left\{\left(p_s - p_c/\overline{\theta}\right)\theta_0 + q_s p_c - q_s\left[\left(p_s - p_c/\overline{\theta}\right) + p_c/\overline{\theta}\right]\right\} R_L - \left(p_s - p_c/\overline{\theta}\right)(1 - q_s) v_2 \\ =& [R_L \theta_0 - (1 - q_s) v_2]\left(p_s - p_c/\overline{\theta}\right) - q_s (p_s - p_c) R_L . \end{aligned}$$

Notice that the sum of $E[U_c]$ and Π_F is constant, so the firm would like minimize $E[U_c]$. On the other hand, the firm needs to ensure that $E[U_c]$ is at least as great as consumers' expected payoff from other portfolios. It turns out the binding constraint stems from consumers' expected payoff from the portfolios with $m_0 = 0$. Notice that consumers' expected payoff is strictly decreasing in $\overline{\theta}$ for the portfolios with $m_0 = 0$ and independent of $\overline{\theta}$ for the portfolios with $m_0 = p_s$. $\overline{\theta} = 1$ dominates $\overline{\theta} < 1$ for the firm.

Part III: The third option cannot be optimal Consider the case with $\overline{\theta} \geq 1$. For any portfolio with $m_0 = 0$, it is optimal for consumers to take the first option. For any portfolio with $m_0 = p_s$, the second option dominates the third option. We can then focus on the following combinations of portfolios and options:

- $(p_c, 0)$, $(p_s, 0)$, and $(p_c + p_s, 0)$ with the first option,
- $(0, p_s)$ and (p_c, p_s) with the second option,
- $\left(p_c, p_s - p_c/\overline{\theta}\right)$ with the second option,
- $\left(p_s, p_s - (p_s - p_c)/\overline{\theta}\right)$ with the third option.

For the combinations in the first group, consumers' expected payoff does not depend on $\left(\theta_0, \overline{\theta}\right)$, and has been given by the proof of Lemma 1. For the combinations in the second group, consumers' expected payoff does not depend on $\overline{\theta}$, and has been given by the proof of Proposition 1.

Next, we show that $\left(p_s, p_s - (p_s - p_c)/\overline{\theta}\right)$ with the third option cannot be optimal for consumers. Suppose $p_s > p_c$. Holding $\left(p_s, p_s - (p_s - p_c)/\overline{\theta}\right)$ and taking the third option, consumers' expected payoff is

$$\begin{aligned} E\left[U_c\right] =& R_L w_0 + (\alpha - 1)\left(p_s q_s + p_c q_c\right) - \left(R_L - 1\right) p_s + p_c q_{cs} \\ & - \left[R_L \theta_0 - \left(1 - q_{cs}\right) v_2\right]\left(p_s - (p_s - p_c)/\overline{\theta}\right). \end{aligned}$$

Note that

$$\begin{gathered} \left(p_s, p_s - (p_s - p_c)/\overline{\theta}\right) \succ (p_c, p_s) \Leftrightarrow \\ \left[R_L \theta_0 - (1 - q_{cs}) v_2\right](p_s - p_c)/\overline{\theta} > \left[p_s q_s - p_c q_{cs} + (R_L - 1)(p_s - p_c) - (q_s - q_{cs}) p_s v_2\right] \end{gathered}$$

and

$$\begin{gathered} \left(p_s, p_s - (p_s - p_c)/\overline{\theta}\right) \succ (p_s + p_c, 0) \Leftrightarrow \\ (R_L - 1) p_c + p_c q_{cs} > \left[R_L \theta_0 - (1 - q_{cs}) v_2\right]\left(p_s - (p_s - p_c)/\overline{\theta}\right). \end{gathered}$$

Combining them, we obtain

$$\begin{aligned} & \left[(R_L - 1) p_c + p_c q_{cs}\right](p_s - p_c)/\overline{\theta} > \\ & \left[p_s q_s - p_c q_{cs} + (R_L - 1)(p_s - p_c) - (q_s - q_{cs}) p_s v_2\right]\left(p_s - (p_s - p_c)/\overline{\theta}\right) \\ \Leftrightarrow & \left[(R_L - 1) p_s + p_s q_s - (q_s - q_{cs}) p_s v_2\right](p_s - p_c)/\overline{\theta} > \\ & \left[p_s q_s - p_c q_{cs} + (R_L - 1)(p_s - p_c) - (q_s - q_{cs}) p_s v_2\right] p_s. \end{aligned}$$

Since

$$(R_L-1)\,p_s+p_sq_s-(q_s-q_{cs})\,p_sv_2 \geq (R_L-1)\,p_s+p_sq_s-(q_s-q_{cs})\,p_s>0$$

and $\overline{\theta} \geq 1$, we further obtain

$$\begin{aligned}
&[(R_L-1)\,p_s+p_sq_s-(q_s-q_{cs})\,p_sv_2]\,(p_s-p_c)\\
&>[p_sq_s-p_cq_{cs}+(R_L-1)\,(p_s-p_c)-(q_s-q_{cs})\,p_sv_2]\,p_s\\
\Leftrightarrow& v_2-1>0.
\end{aligned}$$

It contradicts with $v_2 \leq 1$. That means, $\left(p_s, p_s-(p_s-p_c)/\overline{\theta}\right)$ with the third option must be dominated by at least one of $(p_c+p_s, 0)$ with the first option and (p_c, p_s) with the second option.

Part IV: The condition for $\left(p_c, p_s-p_c/\overline{\theta}\right)$ to be optimal Finally, we derive the condition for $\left(p_c, p_s-p_c/\overline{\theta}\right)$ to be optimal for the firm. Suppose $p_s\overline{\theta}>p_c$. Holding $\left(p_c, p_s-p_c/\overline{\theta}\right)$ and taking the second option, consumers' expected payoff is

$$\begin{aligned}
E\left[U_c\right]=&R_Lw_0+(\alpha-1)\,(p_sq_s+p_cq_c)-(R_L-1)\,p_c+q_s\,(p_s-p_c)\\
&-[R_L\theta_0-(1-q_s)\,v_2]\left(p_s-p_c/\overline{\theta}\right)-q_{cs}\,(R_B-1)\,p_c.
\end{aligned}$$

Comparing (p_c, p_s-p_c) with the the combinations in the first and the second groups, we obtain

$$\left(p_c, p_s-p_c/\overline{\theta}\right)\succ(p_c,0)\Leftrightarrow$$
$$[R_L\theta_0-(1-q_s)\,v_2]\left(p_s-p_c/\overline{\theta}\right)<q_s\,(p_s-p_c)+q_{cs}\,(R_B-1)\,(p_s-p_c)+(q_s-q_{cs})\,(R_B-1)\,(p_s-p_c)^+,$$

$$\left(p_c, p_s-p_c/\overline{\theta}\right)\succ(p_s,0)\Leftrightarrow$$
$$[R_L\theta_0-(1-q_s)\,v_2]\left(p_s-p_c/\overline{\theta}\right)<(R_L-1+q_s)\,(p_s-p_c)+(q_c-q_{cs})\,(R_B-1)\,(p_c-p_s)^+,$$

$$\left(p_c, p_s-p_c/\overline{\theta}\right)\succ(p_s+p_c,0)\Leftrightarrow$$
$$[R_L\theta_0-(1-q_s)\,v_2]\left(p_s-p_c/\overline{\theta}\right)<(R_L-1)\,p_s+q_s\,(p_s-p_c)-q_{cs}\,(R_B-1)\,p_c,$$

$$\left(p_c, p_s-p_c/\overline{\theta}\right)\succ(0,p_s)\Leftrightarrow[R_L\theta_0-(1-q_s)\,v_2]\,p_c/\overline{\theta}>(R_L-1)\,p_c+q_sp_c-(q_c-q_{cs})\,(R_B-1)\,p_c,$$

$$\left(p_c, p_s-p_c/\overline{\theta}\right)\succ(p_c,p_s)\Leftrightarrow[R_L\theta_0-(1-q_s)\,v_2]\,p_c/\overline{\theta}>q_sp_c+q_{cs}\,(R_B-1)\,p_c.$$

When $p_s \leq p_c$,

$$\left(p_c, p_s - p_c/\overline{\theta}\right) \succ (p_c, 0) \Leftrightarrow$$
$$\left[R_L\theta_0 - (1-q_s)v_2\right]\left(p_s - p_c/\overline{\theta}\right) < q_s(p_s - p_c) + q_{cs}(R_B - 1)(p_s - p_c),$$

which is impossible. In this case, consumers never choose $\left(p_c, p_s - p_c/\overline{\theta}\right)$.

When $p_s > p_c$, consumers choose $\left(p_c, p_s - p_c/\overline{\theta}\right)$ if and only if

$$\left[R_L\theta_0 - (1-q_s)v_2\right]\frac{p_s - p_c/\overline{\theta}}{p_s - p_c} < \overline{X} \triangleq$$
$$\begin{cases} q_s R_B & \text{if } q_s < \frac{R_L-1}{R_B-1} \\ R_L - 1 + q_s & \text{if } q_{cs} < \frac{R_L-1}{R_B-1} \leq q_s \\ \frac{(R_L-1)p_s - q_{cs}(R_B-1)p_c}{p_s - p_c} + q_s, & \text{if } \frac{R_L-1}{R_B-1} \leq q_{cs} \end{cases}$$

and

$$\left[R_L\theta_0 - (1-q_s)v_2\right]/\overline{\theta} > \underline{X} \triangleq \begin{cases} (R_L - 1) + q_s - (q_c - q_{cs})(R_B - 1) & \text{if } q_c < \frac{R_L-1}{R_B-1} \\ q_s + q_{cs}(R_B - 1) & \text{if } \frac{R_L-1}{R_B-1} \leq q_c \end{cases}$$

Note that the firm wants to maximize $\left[R_L\theta_0 - (1-q_s)v_2\right]\left(p_s - p_c/\overline{\theta}\right)$, and given $\overline{\theta}$, a higher θ_0 will relax the second inequality. Therefore, we can focus on that $\left[R_L\theta_0 - (1-q_s)v_2\right]\frac{p_s - p_c/\overline{\theta}}{p_s - p_c} = \overline{X}$ and derive the set of $\left(\theta_0, \overline{\theta}\right)$ that makes the second inequality hold. It is not hard to see that for smaller $\overline{\theta}$, the second inequality holds more likely.

When $\frac{R_L-1}{R_B-1} \leq q_{cs}$, $\underline{X} < \overline{X}$, so the second inequality cannot hold for any $\overline{\theta} \geq 1$. That means, consumers will not choose $\left(p_c, p_s - p_c/\overline{\theta}\right)$.

When $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_s$, $\left(\theta_0, \overline{\theta}\right)$ must satisfy

$$\left[R_L\theta_0 - (1-q_s)v_2\right]\frac{p_s - p_c/\overline{\theta}}{p_s - p_c} = R_L - 1 + q_s$$

and

$$\left[R_L\theta_0 - (1-q_s)v_2\right]/\overline{\theta} > q_s + q_{cs}(R_B - 1)$$
$$\Leftrightarrow \overline{\theta} < \frac{R_L - 1 + q_s}{q_s + q_{cs}(R_B - 1)}\left(1 - \frac{p_c}{p_s}\right) + \frac{p_c}{p_s}.$$

The firm's net payoff from tokens is

$$\Pi_F = \left[R_L\theta_0 - (1-q_s)v_2\right]\left(p_s - p_c/\overline{\theta}\right) - q_s(p_s - p_c)R_L.$$
$$= (R_L - 1)(1 - q_s)(p_s - p_c).$$

According to Proposition 6, the firm prefers to do ex-post token issuance if and only if

$$\begin{aligned}(p_s - p_c)(1 - q_s)(R_L - 1) > (p_s - q_s p_s - p_c)(R_L - 1) + q_{cs} p_c (R_B - 1) \\ \Leftrightarrow \frac{R_L - 1}{R_B - 1} > \frac{q_{cs}}{q_s}.\end{aligned}$$

When $q_s < \frac{R_L-1}{R_B-1} \leq q_c$, $\left(\theta_0, \overline{\theta}\right)$ must satisfy

$$[R_L \theta_0 - (1 - q_s) v_2] \frac{p_s - p_c/\overline{\theta}}{p_s - p_c} = q_s R_B$$

and

$$\begin{aligned}[R_L \theta_0 - (1 - q_s) v_2] / \overline{\theta} > q_s + q_{cs} (R_B - 1) \\ \Leftrightarrow \overline{\theta} < \frac{q_s R_B}{q_s + q_{cs} (R_B - 1)} \left(1 - \frac{p_c}{p_s}\right) + \frac{p_c}{p_s}.\end{aligned}$$

The firm's net payoff from tokens is

$$\begin{aligned}\Pi_F &= [R_L \theta_0 - (1 - q_s) v_2] \left(p_s - p_c/\overline{\theta}\right) - q_s (p_s - p_c) R_L \\ &= q_s (p_s - p_c)(R_B - R_L).\end{aligned}$$

The firm prefers to do ex-post token issuance if and only if

$$\begin{aligned}q_s (p_s - p_c)(R_B - R_L) > -q_s p_s (R_L - 1) + (q_s p_s - q_s p_c + q_{cs} p_c)(R_B - 1) \\ \Leftrightarrow \frac{R_L - 1}{R_B - 1} > \frac{q_{cs}}{q_s}.\end{aligned}$$

When $q_c < \frac{R_L-1}{R_B-1}$, $\left(\theta_0, \overline{\theta}\right)$ must satisfy

$$[R_L \theta_0 - (1 - q_s) v_2] \frac{p_s - p_c/\overline{\theta}}{p_s - p_c} = q_s R_B$$

and

$$\begin{aligned}[R_L \theta_0 - (1 - q_s) v_2] / \overline{\theta} > (R_L - 1) + q_s - (q_c - q_{cs})(R_B - 1) \\ \Leftrightarrow \overline{\theta} < \frac{q_s R_B}{(R_L - 1) + q_s - (q_c - q_{cs})(R_B - 1)} \left(1 - \frac{p_c}{p_s}\right) + \frac{p_c}{p_s}.\end{aligned}$$

The firm's net payoff from tokens is

$$\begin{aligned}\Pi_F &= [R_L \theta_0 - (1 - q_s) v_2] \left(p_s - p_c/\overline{\theta}\right) - q_s (p_s - p_c) R_L \\ &= q_s (p_s - p_c)(R_B - R_L).\end{aligned}$$

The firm prefers to do ex-post token issuance if and only if

$$(p_s - p_c)\, q_s\, (R_B - R_L) > (p_c - q_s p_s)\, (R_L - 1) + [q_s p_s - (q_s + q_c - q_{cs})\, p_c]\, (R_B - 1)$$
$$\Leftrightarrow \frac{R_L - 1}{R_B - 1} < \frac{q_c - q_{cs}}{1 - q_s}$$

Notice that $q_c < \frac{q_c - q_{cs}}{1 - q_s}$ and $q_c > \frac{q_{cs}}{q_s}$ hold if and only if $q_{cs} < q_c q_s$. Taken together, the firm prefers to do ex-post token issuance if and only if $\frac{R_L - 1}{R_B - 1} \in \left(\frac{q_{cs}}{q_s}, \frac{q_c - q_{cs}}{1 - q_s}\right)$.

Proof of Proposition 10

We prove that $\gamma_1 = \theta_0$. Suppose $\gamma_1 > \theta_0$. Then consumers will not hold deposits in period 0 because it is dominated by holding tokens and sell in the period-1 market. Since consumers can only obtain cash through borrowing in period 1 and they need cash for other consumption, consumers borrow at least $q_c p_c$ in total.

$\gamma_1 \leq 1$. Otherwise, consumers will neither redeem tokens for the service or hold them until date 2, so there is no demand of tokens in the market. Under $\gamma_1 \leq 1$, consumers will always redeem tokens for the service. If $\gamma_1 > v_2$, consumers prefer selling tokens to the market and hold only cash until date 2; otherwise they will buy tokens from the market and hold only tokens until date 2. Their expected payoff is then

$$\begin{aligned}
E[U_c] =& R_L (w_0 - m_0 \theta_0) + \alpha (p_s q_s + p_c q_c) \\
&+ q_{cs} \left[(m_0 \gamma_1 - p_s \gamma_1 - p_c) \max\left\{\frac{v_2}{\gamma_1}, 1\right\} - \left(R_B - \max\left\{\frac{v_2}{\gamma_1}, 1\right\}\right) (p_c + p_s \gamma_1 - m_0 \gamma_1)^+ \right] \\
&+ (q_s - q_{cs}) \left[(m_0 \gamma_1 - p_s \gamma_1) \max\left\{\frac{v_2}{\gamma_1}, 1\right\} - \left(R_B - \max\left\{\frac{v_2}{\gamma_1}, 1\right\}\right) (p_s \gamma_1 - m_0 \gamma_1)^+ \right] \\
&+ (q_c - q_{cs}) \left[(m_0 \gamma_1 - p_c) \max\left\{\frac{v_2}{\gamma_1}, 1\right\} - \left(R_B - \max\left\{\frac{v_2}{\gamma_1}, 1\right\}\right) (p_c - m_0 \gamma_1)^+ \right] \\
&+ (1 - q_c - q_s + q_{cs}) \left[m_0 \gamma_1 \max\left\{\frac{v_2}{\gamma_1}, 1\right\} \right]
\end{aligned}$$

When $m_0 \gamma_1 < \min\{p_c, p_s \gamma_1\}$, taking derivative with respect to m_0, we obtain

$$\begin{aligned}
\frac{\partial E[U_c]}{\partial m_0} &= \gamma_1 \left[-R_0 \frac{\theta_0}{\gamma_1} + \max\left\{\frac{v_2}{\gamma_1}, 1\right\} + \left(R_B - \max\left\{\frac{v_2}{\gamma_1}, 1\right\}\right) (q_s + q_c - q_{cs}) \right] \\
&> \gamma_1 [-R_0 + 1 + (R_B - 1)(q_s + q_c - q_{cs})] > 0.
\end{aligned}$$

Hence, $m_0\gamma_1 \geq \min\{p_c, p_s\gamma_1\}$. Then the total borrowing of consumers is

$$\begin{aligned}
&q_{cs}(p_c + p_s\gamma_1 - m_0\gamma_1)^+ + (q_s - q_{cs})(p_s\gamma_1 - m_0\gamma_1)^+ + (q_c - q_{cs})(p_c - m_0\gamma_1)^+ \\
\leq& q_{cs}(p_c + p_s\gamma_1 - \min\{p_c, p_s\gamma_1\}) + (q_s - q_{cs})(p_s\gamma_1 - \min\{p_c, p_s\gamma_1\}) + (q_c - q_{cs})(p_c - \min\{p_c, p_s\gamma_1\}) \\
=& q_c p_c + q_s p_s \gamma_1 - (q_c + q_s - q_{cs})\min\{p_c, p_s\gamma_1\} \\
=& \max\{q_c p_c + q_s p_s\gamma_1 - (q_c + q_s - q_{cs}) p_c, q_c p_c + q_s p_s \gamma_1 - (q_c + q_s - q_{cs}) p_s\gamma_1\} \\
<& \max\{q_s p_s \gamma_1, q_c p_c\} \\
\leq& q_c p_c.
\end{aligned}$$

The last inequality follows $\gamma_1 \leq 1$ and $q_s p_s \leq q_c p_c$. The total borrowing of consumers is strictly less than $q_c p_c$. Contradiction! Therefore, it must be that $\gamma_1 \leq \theta_0$.

Given $\gamma_1 = \theta_0$, following the argument in the main text, the firm must set $\theta_0 \leq v_2$.

Proof of Proposition 11

Consumers must choose among the three total level of liquidity: $d_0 + m_0\theta_0 = p_s\theta_0$, $d_0 + m_0\theta_0 = p_c$, and $d_0 + m_0\theta_0 = p_s\theta_0 + p_c$. A consumer's expected payoff is

$$\begin{aligned}
E[U_c] =& R_L w_0 + \alpha(p_s q_s + p_c q_c) - (q_s p_s \theta_0 + q_c p_c)\frac{v_2}{\theta_0} - \left(R_L - \frac{v_2}{\theta_0}\right) p_s\theta_0 \\
&- q_{cs}\left(R_B - \frac{v_2}{\theta_0}\right) p_c - (q_c - q_{cs})\left(R_B - \frac{v_2}{\theta_0}\right)(p_c - p_s\theta_0)^+
\end{aligned}$$

for $d_0 + m_0\theta_0 = p_s\theta_0$,

$$\begin{aligned}
E[U_c] =& R_L w_0 + \alpha(p_s q_s + p_c q_c) - (q_s p_s \theta_0 + q_c p_c)\frac{v_2}{\theta_0} - \left(R_L - \frac{v_2}{\theta_0}\right) p_c \\
&- q_{cs}\left(R_B - \frac{v_2}{\theta_0}\right) p_s\theta_0 - (q_s - q_{cs})\left(R_B - \frac{v_2}{\theta_0}\right)(p_s\theta_0 - p_c)^+
\end{aligned}$$

for $d_0 + m_0\theta_0 = p_c$, and

$$E[U_c] = R_L w_0 + \alpha(p_s q_s + p_c q_c) - (q_s p_s \theta_0 + q_c p_c)\frac{v_2}{\theta_0} - \left(R_L - \frac{v_2}{\theta_0}\right)(p_c + p_s\theta_0)$$

for $d_0 + m_0\theta_0 = p_c + p_s\theta_0$. Comparing the three portfolios, we obtain

$$p_c \succ p_s\theta_0 \Leftrightarrow (p_s\theta_0 - p_c)\left[\frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} - q_c - (q_s - q_c)\frac{(p_s\theta_0 - p_c)^+}{p_s\theta_0 - p_c}\right] > 0$$

$$p_s\theta_0 + p_c \succ p_c \Leftrightarrow \frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} < q_{cs} + (q_s - q_{cs})\frac{(p_s\theta_0 - p_c)^+}{p_s\theta_0}$$

$$p_s\theta_0 + p_c \succ p_s\theta_0 \Leftrightarrow \frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} < q_{cs} + (q_c - q_{cs})\frac{(p_c - p_s\theta_0)^+}{p_c}$$

Consumers do not hold deposits after trade at date 1, and cash is not used for the service, so all deposits and borrowing are used for other consumption, so

$$d_0 + E[b_1] = q_c p_c.$$

If $\frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} \le q_{cs}$, consumers choose $d_0 + m_0\theta_0 = p_c + p_s\theta_0$. Consumers do not borrow. Then

$$d_0 = q_c p_c,$$
$$m_0 = p_s + \frac{(1-q_c)\,p_c}{\theta_0}.$$

If $p_c \ge p_s\theta_0$ and $q_{cs} < \frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} \le q_c$, consumers choose $d_0 + m_0\theta_0 = p_c$. Consumers borrow $p_s\theta_0$ only when they need both types of consumption, so the total borrowing is $q_{cs}p_s\theta_0$. Then

$$d_0 = q_c p_c - q_{cs}p_s\theta_0,$$
$$m_0 = q_{cs}p_s + \frac{(1-q_c)\,p_c}{\theta_0}.$$

If $p_c \ge p_s\theta_0$ and $q_c < \frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}}$, consumers choose $d_0 + m_0\theta_0 = p_s\theta_0$. Consumers borrow p_c when they need both types of consumption and borrow $p_c - p_s\theta_0$ when they need only other consumption, so the total borrowing is $q_{cs}p_c + (q_c - q_{cs})(p_c - p_s\theta_0)$. Then

$$d_0 = (q_c - q_{cs})\,p_s\theta_0,$$
$$m_0 = (1 - q_c + q_{cs})\,p_s.$$

If $p_c < p_s\theta_0$ and $q_{cs} < \frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} \le q_s$, consumers choose $d_0 + m_0\theta_0 = p_s\theta_0$. Consumers borrow p_c when they need both types of consumption, so the total borrowing is $q_{cs}p_c$. Then

$$d_0 = (q_c - q_{cs})\,p_c,$$
$$m_0 = p_s - \frac{(q_c - q_{cs})\,p_c}{\theta_0}.$$

If $p_c < p_s\theta_0$ and $q_s < \frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}}$, consumers choose $d_0 + m_0\theta_0 = p_c$. Consumers borrow $p_s\theta_0$ when they need both types of consumption and borrow $p_s\theta_0 - p_c$ when they need only the service, so the total borrowing is

$q_{cs}p_s\theta_0 + (q_s - q_{cs})(p_s\theta_0 - p_c) = q_s p_s\theta_0 - (q_s - q_{cs})p_c$. Then

$$d_0 = (q_s + q_c - q_{cs})p_c - q_s p_s \theta_0,$$
$$m_0 = q_s p_s + \frac{(1 - q_s - q_c + q_{cs})p_c}{\theta_0}.$$

Proof of Proposition 12

Consider any (v_2, θ_0) that may emerge in equilibrium. Then $\frac{v_2}{R_L} < \theta_0 \leq v_2$. We show that for $v_2' > v_2$, setting $\theta_0' = \theta_0 \cdot v_2'/v_2$ leads to higher profits of the firm than that under (v_2, θ_0). Denote the firm's profit under (v_2, θ_0) and (v_2', θ_0') as Π_F and Π_F' respectively.

If $\frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} \leq q_{cs}$,

$$\begin{aligned}
\Pi_F' &= \left(p_s + \frac{(1-q_c)p_c}{\theta_0 \cdot v_2'/v_2}\right)\left(\theta_0 R_L \cdot v_2'/v_2 - v_2'\right) - q_s p_s \left(R_L - v_2'\right) \\
&= \left(p_s \cdot v_2'/v_2 + \frac{(1-q_c)p_c}{\theta_0}\right)(\theta_0 R_L - v_2) - q_s p_s \left(R_L - v_2'\right) \\
&> \left(p_s + \frac{(1-q_c)p_c}{\theta_0}\right)(\theta_0 R_L - v_2) - q_s p_s (R_L - v_2) = \Pi_F
\end{aligned}$$

If $p_c \geq p_s\theta_0$ and $q_{cs} < \frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} \leq q_c$, $p_s\theta_0'$ could be smaller or greater than p_c. If $p_c \geq p_s\theta_0'$,

$$\begin{aligned}
\Pi_F' &= \left(q_{cs}p_s + \frac{(1-q_c)p_c}{\theta_0 \cdot v_2'/v_2}\right)\left(\theta_0 R_L \cdot v_2'/v_2 - v_2'\right) - q_s p_s \left(R_L - v_2'\right) \\
&= \left(q_{cs}p_s \cdot v_2'/v_2 + \frac{(1-q_c)p_c}{\theta_0}\right)(\theta_0 R_L - v_2) - q_s p_s \left(R_L - v_2'\right) \\
&> \left(q_{cs}p_s + \frac{(1-q_c)p_c}{\theta_0}\right)(\theta_0 R_L - v_2) - q_s p_s (R_L - v_2) = \Pi_F.
\end{aligned}$$

If $p_c < p_s\theta_0'$ and $\frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} \leq q_s$, since

$$p_s - \frac{(q_c - q_{cs})p_c}{\theta_0'} \geq q_{cs}p_s + \frac{(1-q_c)p_c}{\theta_0'}$$

when $p_s\theta_0' \geq p_c$,

$$\begin{aligned}
\Pi_F' &= \left(p_s - \frac{(q_c - q_{cs})p_c}{\theta_0'}\right)\left(\theta_0' R_L - v_2'\right) - q_s p_s \left(R_L - v_2'\right) \\
&\geq \left(q_{cs}p_s + \frac{(1-q_c)p_c}{\theta_0'}\right)\left(\theta_0' R_L - v_2'\right) - q_s p_s \left(R_L - v_2'\right) \\
&> \Pi_F.
\end{aligned}$$

If $p_c < p_s\theta_0'$ and $\frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} > q_s$, since

$$q_s p_s + \frac{(1-q_s-q_c+q_{cs})\,p_c}{\theta_0'} \geq q_{cs} p_s + \frac{(1-q_c)\,p_c}{\theta_0'}$$

when $p_s\theta_0' \geq p_c$,

$$\begin{aligned}
\Pi_F' &= \left(q_s p_s + \frac{(1-q_s-q_c+q_{cs})\,p_c}{\theta_0'}\right)\left(\theta_0' R_L - v_2'\right) - q_s p_s \left(R_L - v_2'\right) \\
&\geq \left(q_{cs} p_s + \frac{(1-q_c)\,p_c}{\theta_0'}\right)\left(\theta_0' R_L - v_2'\right) - q_s p_s \left(R_L - v_2'\right) \\
&> \Pi_F.
\end{aligned}$$

If $p_c \geq p_s\theta_0$ and $q_c < \frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}}$, $p_s\theta_0'$ could be smaller or greater than p_c. If $p_c \geq p_s\theta_0'$

$$\begin{aligned}
\Pi_F' &= (1-q_c+q_{cs})\,p_s\left(\theta_0' R_L - v_2'\right) - q_s p_s \left(R_L - v_2'\right) \\
&> (1-q_c+q_{cs})\,p_s\left(\theta_0 R_L - v_2\right) - q_s p_s \left(R_L - v_2\right) = \Pi_F
\end{aligned}$$

If $p_c < p_s\theta_0'$ and $\frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} \leq q_s$, since

$$p_s - \frac{(q_c - q_{cs})\,p_c}{\theta_0'} \geq (1-q_c+q_{cs})\,p_s$$

when $p_s\theta_0' \geq p_c$,

$$\begin{aligned}
\Pi_F' &= \left(p_s - \frac{(q_c-q_{cs})\,p_c}{\theta_0'}\right)\left(\theta_0' R_L - v_2'\right) - q_s p_s \left(R_L - v_2'\right) \\
&> (1-q_c+q_{cs})\,p_s\left(\theta_0' R_L - v_2'\right) - q_s p_s \left(R_L - v_2'\right) \\
&> (1-q_c+q_{cs})\,p_s\left(\theta_0 R_L - v_2\right) - q_s p_s \left(R_L - v_2\right) = \Pi_F
\end{aligned}$$

If $p_c < p_s\theta_0'$ and $\frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} > q_s$, since

$$q_s p_s + \frac{(1-q_s-q_c+q_{cs})\,p_c}{\theta_0} \geq (1-q_c+q_{cs})\,p_s$$

when $p_s\theta_0 \leq p_c$,

$$\begin{aligned}\Pi_F' &= \left(q_s p_s + \frac{(1-q_s-q_c+q_{cs})\,p_c}{\theta_0'}\right)\left(\theta_0' R_L - v_2'\right) - q_s p_s\left(R_L - v_2'\right) \\ &> \left(q_s p_s + \frac{(1-q_s-q_c+q_{cs})\,p_c}{\theta_0}\right)\left(\theta_0 R_L - v_2\right) - q_s p_s\left(R_L - v_2\right) \\ &\geq (1-q_c+q_{cs})\,p_s\left(\theta_0 R_L - v_2\right) - q_s p_s\left(R_L - v_2\right) = \Pi_F\end{aligned}$$

If $p_c < p_s\theta_0$ and $q_{cs} < \frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} \leq q_s$,

$$\begin{aligned}\Pi_F' &= \left(p_s - \frac{(q_c-q_{cs})\,p_c}{\theta_0'}\right)\left(\theta_0' R_L - v_2'\right) - q_s p_s\left(R_L - v_2'\right) \\ &> \left(p_s - \frac{(q_c-q_{cs})\,p_c}{\theta_0}\right)\left(\theta_0 R_L - v_2\right) - q_s p_s\left(R_L - v_2\right) = \Pi_F\end{aligned}$$

If $p_c < p_s\theta_0$ and $q_s < \frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}}$,

$$\begin{aligned}\Pi_F' &= \left(q_s p_s + \frac{(1-q_s-q_c+q_{cs})\,p_c}{\theta_0'}\right)\left(\theta_0' R_L - v_2'\right) - q_s p_s\left(R_L - v_2'\right) \\ &> \left(q_s p_s + \frac{(1-q_s-q_c+q_{cs})\,p_c}{\theta_0}\right)\left(\theta_0 R_L - v_2\right) - q_s p_s\left(R_L - v_2\right) = \Pi_F\end{aligned}$$

Since v_2 does not affect the firm's net payoff from tokens when tokens are nontradable, the firm is more willing to make tokens tradable when v_2 is higher.

Proof of Proposition 13

Note that if $\theta_0 = 1$, $\frac{R_L - \frac{v_2}{\theta_0}}{R_B - \frac{v_2}{\theta_0}} = \frac{R_L - 1}{R_B - 1}$.

Part I: $\frac{R_L-1}{R_B-1} \leq q_{cs}$ or $q_c < \frac{R_L-1}{R_B-1}$

Consider $\frac{R_L-1}{R_B-1} \leq q_{cs}$. Then the firm's net payoff from nontradable tokens is

$$p_s\,(1-q_s)\,(R_L-1)\,.$$

The firm's net payoff from tradable tokens at $\theta_0 = 1$ is

$$p_s\,(1-q_s)\,(R_L-1) + p_c\,(1-q_c)\,(R_L-1) > p_s\,(1-q_s)\,(R_L-1)\,.$$

Hence, tradability is preferred.

Consider $q_c < \frac{R_L-1}{R_B-1}$. If $p_s \leq p_c$, the firm's net payoff from nontradable tokens is

$$p_s(1-q_s)(R_L-1) - p_s(q_c-q_{cs})(R_B-1).$$

The firm's net payoff from tradable tokens at $\theta_0 = 1$ is

$$\begin{aligned}&(1-q_c-q_s+q_{cs})\,p_s(R_L-1)\\>&p_s(1-q_s)(R_L-1) - p_s(q_c-q_{cs})(R_B-1).\end{aligned}$$

Hence, tradability is preferred. If $p_s > p_c$, the firm's net payoff from nontradable tokens is

$$(p_c - q_s p_s)(R_L-1) + [q_s p_s - (q_s+q_c-q_{cs})\,p_c](R_B-1).$$

The firm's net payoff from tradable tokens at $\theta_0 = 1$ is

$$\begin{aligned}&(1-q_s-q_c+q_{cs})\,p_c(R_L-1)\\>&(1-q_s-q_c+q_{cs})\,p_c(R_L-1) + p_s q_s(R_B-R_L) - p_c(q_s+q_c-q_{cs})(R_B-R_L)\\=&(p_c-q_s p_s)(R_L-1) + [q_s p_s - (q_s+q_c-q_{cs})\,p_c](R_B-1)\end{aligned}$$

The first inequality follows from $p_s q_s \leq p_c q_c < p_c(q_s+q_c-q_{cs})$. Hence, tradability is preferred.

Part II: $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_s$

Consider $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_s$. The firm's net payoff from nontradable tokens is

$$\begin{cases} p_s[q_{cs}(R_B-1) - q_s(R_L-1)], & \text{if } p_s \leq p_c \\ p_s(1-q_s)(R_L-1) - p_c[(R_L-1) - q_{cs}(R_B-1)] & \text{if } p_s > p_c \end{cases}.$$

The firm's net payoff from tradable tokens at $\theta_0 = 1$ is

$$\begin{cases} -p_s(q_s-q_{cs})(R_L-1) + p_c(1-q_c)(R_L-1), & \text{if } p_s \leq p_c \\ p_s(1-q_s)(R_L-1) - p_c(q_c-q_{cs})(R_L-1) & \text{if } p_s > p_c \end{cases}.$$

The firm can also set a lower θ_0 for tradable tokens so that $\frac{R_L-\frac{v_2}{\theta_0}}{R_B-\frac{v_2}{\theta_0}}$ is lower and m_0 is higher. Since $q_{cs} < \frac{R_L-1}{R_B-1} \leq q_s$, the only choice is set $\theta_0 = \frac{1-q_{cs}}{R_L-q_{cs}R_B} < 1$ such that $\frac{R_L-\frac{1}{\theta_0}}{R_B-\frac{1}{\theta_0}} = q_{cs}$. The firm's net payoff from tradable tokens at that point is

$$p_s\left[\frac{1-q_{cs}}{R_L-q_{cs}R_B}R_L - 1 - q_s(R_L-1)\right] + (1-q_c)\,p_c R_L - \frac{(1-q_c)\,p_c}{\frac{1-q_{cs}}{R_L-q_{cs}R_B}}$$

Given all other parameters, we have two observations regarding p_s/p_c. First, for sufficiently small p_s/p_c, tradability is preferred. Second, the derivative of the firm's net payoff from nontradable tokens with respect to p_s/p_c is never smaller than that of the firm's net payoff from tradable tokens under either strategy. That means, if tradability is not preferred for p_s/p_c, it is not preferred for any p'_s/p'_c greater than p_s/p_c. To sum up, either tradability is always preferred or there exists κ^* such that tradability is preferred if and only if $p_s/p_c < \kappa^*$.

Part III: $q_s < \frac{R_L-1}{R_B-1} \leq q_c$

Consider $q_s < \frac{R_L-1}{R_B-1} \leq q_c$. Either tradability is always preferred or there exists κ^* such that tradability is preferred if and only if $p_s/p_c < \kappa^*$.

The firm's net payoff from nontradable tokens is

$$\begin{cases} p_s\left[q_{cs}\left(R_B-1\right)-q_s\left(R_L-1\right)\right], & \text{if } p_s \leq p_c \\ p_s q_s\left(R_B-R_L\right)-p_c\left(q_s-q_{cs}\right)\left(R_B-1\right) & \text{if } p_s > p_c \end{cases}.$$

The firm's net payoff from tradable tokens at $\theta_0 = 1$ is

$$\begin{cases} -p_s\left(q_s-q_{cs}\right)\left(R_L-1\right)+p_c\left(1-q_c\right)\left(R_L-1\right), & \text{if } p_s \leq p_c \\ p_c\left(1-q_s-q_c+q_{cs}\right)\left(R_L-1\right) & \text{if } p_s > p_c \end{cases}.$$

The firm can also set a lower θ_0 for tradable tokens so that $\frac{R_L-\frac{v_2}{\theta_0}}{R_B-\frac{v_2}{\theta_0}}$ is lower and m_0 is higher. The first choice is to set $\theta_0 = \frac{1-q_{cs}}{R_L-q_{cs}R_B} < 1$ such that $\frac{R_L-\frac{1}{\theta_0}}{R_B-\frac{1}{\theta_0}} = q_{cs}$. The firm's net payoff from tradable tokens at that point is

$$p_s\left[\frac{1-q_{cs}}{R_L-q_{cs}R_B}R_L-1-q_s\left(R_L-1\right)\right]+\left(1-q_c\right)p_c\left(R_L-\frac{R_L-q_sR_B}{1-q_s}\right)$$

If $\frac{R_L-\frac{p_s}{p_c}}{R_B-\frac{p_s}{p_c}} < q_s$, which is possible only when $p_s > p_c$, there is a second choice. The firm can set $\theta_0 = \frac{1-q_s}{R_L-q_sR_B} < 1$ such that $\frac{R_L-\frac{1}{\theta_0}}{R_B-\frac{1}{\theta_0}} = q_s$. The firm's net payoff from tradable tokens at that point is

$$p_s\left[\frac{1-q_s}{R_L-q_sR_B}R_L-1-q_s\left(R_L-1\right)\right]-\left(q_c-q_{cs}\right)p_c\left(R_L-\frac{R_L-q_sR_B}{1-q_s}\right).$$

Given all other parameters, we have the same two observations regarding p_s/p_c as above. Therefore, either tradability is always preferred or there exists κ^* such that tradability is preferred if and only if $p_s/p_c < \kappa^*$.

Proof of Proposition 14

See the main text.

Proof of Proposition 15

Suppose $\phi < \theta_0 \leq v_2(\phi)$. The firm can increase ϕ to $\phi' = \theta_0$, and v_2 increases to $v_2(\phi')$. According to the proof of Proposition 12, if the firm increases θ_0 to $\theta_0' = \theta_0 v_2(\phi') / v_2(\phi)$, its net payoff from tokens is higher. Hence, (ϕ', θ_0') dominates (ϕ, θ_0). The firm can iterate this process. Since $\theta_0 \leq 1$, ϕ and θ_0 must converge to the same number. So, $\phi < \theta_0$ must be dominated by certain $\phi = \theta_0$.