A Unified Theory of Delegated Capital Management*

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Abstract

We develop a unified theory of delegated capital management by extending the paradigm of Berk and Green (2004) from mutual funds to alternative assets. With competitive markets and rational investors, we derive the optimal contract and account for observed regularities — performance fees, persistent alpha, and limits on capital. The key distinction between mutual funds and alternatives is the liquidity of the underlying assets. When assets are illiquid, it is optimal to acquire information about the manager's skill. A positive alpha is therefore necessary to compensate informed investors and a performance based contract is required to induce these investors to allocate their capital optimally. At the same time, a free-rider problem emerges that requires capital constraints on uninformed investors. Thus, liquidity of the underlying assets explains the contrasting contract structures across sectors.

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The capital markets have seen a seismic shift in the last 70 years. In 1950 individuals and institutions invested their capital directly. In that environment, financial economists derived the now widely accepted paradigm that explains the risk-return tradeoff these individuals and institutions face.[1] By making the dual assumptions of competitive capital markets (an infinite supply of investment capital for positive net present value (NPV) investment opportunities) and rational expectations, financial economists derived the important result that, with the exception of the necessarily small set of investors who have a competitive advantage in collecting, processing and trading on information, for the rest of investors the expected return of any investment in the capital markets is determined solely by its riskiness. Because of the information revealed in prices, for these investors, other factors, like the quality of the underlying business, or the skill of the company's managers, are irrelevant.

Today capital markets are organized differently. Rather than investing directly, most market participants invest indirectly though intermediaries known as money managers. Although the shift was largely completed by the turn of the century, financial economists were slow in extending the paradigm to explain the new market structure. Indeed, for many years financial economists theorized that investors who invested through money mangers should face a different risk-return tradeoff to investors who invested directly, and they were perplexed when the empirical evidence suggested otherwise. However, Berk and Green (2004) showed that under the same assumptions of competitive capital markets and rational expectations (hereafter "perfectly competitive capital markets"), investors who invest in one sector of asset management, namely mutual funds, all face the same risk-return tradeoff. Their expected return is only a function of the risk of the fund and does not depend on the skill of the manager. In this case, rather than the price revealing information, information is revealed by the flow of funds.

Our objective in this article is to complete the process of extending the paradigm by deriving the implications of the assumptions of perfectly competitive capital markets to the rest of the money management sector. In broad terms, money managers are divided into two sectors, publicly available investments mainly in the form of mutual funds, and privately available investments known as "alternatives." Although, superficially, managers in both sectors appear to do the same thing — manage money on behalf of investors — as Kaplan and Schoar (2005) first pointed out, important differences between the sectors exist that are puzzling given the apparent similarities. In particular, although the logic in Berk and Green (2004) can explain some of the empirical regularities that are observed in the private

[1]The seminal contributions that coalesced into the paradigm are Sharpe (1964), Lintner (1965), Mossin (1966) and Treynor (1961) (who derived the relation between risk and return), and Muth (1961), Fama (1965), LeRoy (1976), Grossman (1976) and Grossman and Stiglitz (1980) (who derived the role of private information).

sector, for example, over a long enough time span a strong flow of funds-performance relation exists, it does not appear to explain other regularities, such as the apparent existence of positive investment alpha and the persistence in that alpha (see Kaplan and Schoar (2005)), along with limits on the flow of incoming capital. Why do managers appear give up some of their rents to investors by providing a positive alpha, rather than simply raising their fees or increasing their fund size to extract all the rents?

Likely because performance in the mutual fund sector is unpredictable, there is no evidence that some investors in the sector systematically pick outperforming funds. In contrast, there is evidence in the alternative sector that some investors, particularly university endowments, consistently earn excess returns. This observation suggests that these investors have skill selecting managers presumably because they possess an informational advantage.

Another stark difference between the two sectors is the form of the managerial contract. In the alternatives sector, managers are compensated as a function of performance as well as a fraction of assets under management (AUM). In the mutual fund sector managers are compensated only as fraction of AUM. This difference is particularly puzzling in light of the fact that Berk and Green (2004) show, in the context of mutual funds, that a fee proportional to AUM is optimal. Why is the same contract not optimal in the alternatives sector?

An additional puzzling difference is that many funds in the alternatives sector limit their own AUM. In the mutual fund sector, funds sometimes limit new outside investment, but they rarely, if ever, limit additional investment from existing investors. In the alternative space, managers routinely place a limit on the size of the fund, and if their fund reaches the limit, they ration all investors including existing investors. Why do they ration rather than raise their fees and capture additional rents?

Finally, as Begenau and Siriwardane (2024) document, significant fee dispersion exists in alternatives, implying that different investors in the same fund earn different returns. Moreover, Begenau and Siriwardane (2024) show that in the alternative space some investors consistently pay lower fees implying that some investors consistently earn higher returns than other investors. Why do the investors who pay higher fees put up with this equilibrium?

We show that a simple parsimonious explanation for all of these puzzles exists. Just as in the mutual fund sector, in the alternative sector the perfectly competitive markets paradigm can explain the observed behavior. The observed differences between the sectors derives from the nature of the underlying assets under management. Unlike the mutual fund sector, the alternative sector is characterized by illiquid and opaque underlying assets. The opacity limits investor's ability to learn about managerial ability from performance, and the illiquidity of the underlying assets limits the ability investors to react to this information. As a consequence investors have a much greater incentive to invest in acquiring information about managerial ability rather than simply learning from performance. In essence, the industry is bifurcated by the endogenous choices investors make about the need to assess managerial skill. When the underlying assets are transparent and liquid, investors optimally choose not to acquire information and instead continuously evaluate how much capital to commit as they learn about managerial skill from performance. When the underlying assets are illiquid and opaque, investors have limited ability to both observe performance and react by moving capital, so they must commit the capital for an extended period. In this environment it is optimal to acquire information prior to the initial investment.

In perfectly competitive capital markets all investment opportunities earn zero alpha. But because investors must spend resources acquiring information, if these costs are ignored, the alpha of an informed investor is positive. Furthermore, if investor capital allocation decisions are observable, once one investor investigates, it is suboptimal for other investors to acquire information, thereby removing any incentive for the first investor to acquire information. The only way to resolve this free-rider problem is to offer a different contract to informed and uninformed investors. That is, different investors will earn different returns, and if some investors have a competitive advantage acquiring information, the differences will persist across investors, consistent with the empirical evidence in Begenau and Siriwardane (2024). This result mirrors the result in Grossman and Stiglitz (1980) — when the costs of information are ignored, informed investors earn higher risk-adjusted returns.

Perhaps the most surprising result of our model is that the management contract observed in the alternatives industry is optimal. Furthermore, the mutual fund contract, which Berk and Green (2004) prove is optimal in the public sector, is a suboptimal contract in the private sector. The reason for this difference derives from the information asymmetry between the privately informed investor and the manager.

To achieve optimality the contract must deliver the same equilibrium as would arise if the manager managed his own money. That means the contract must deliver two things. First, it must ensure that the manager extracts all the rents. Second it must ensure that once investors acquire information, they choose to invest optimally. That is, the contract must ensure that whenever the informed investor chooses to invest, it is also in the manager's interests to invest, and the amount of capital that the manager is able to raise is optimal.

The dual requirements of the optimal contract requires that the contract have three components. A contract that contains both a management fee, a performance fee and a cap on AUM delivers a first-best allocation because the management fee is used to ensure that incentives are aligned, and the performance fee is used to ensure that the manager extracts all rents. The cap is required to limit free-riding by uninformed investors — without it the informed investor would not invest. This insight explains why the two sectors in the money management space have two different contracts. The mutual fund contract, because it only consists of a management fee, cannot both align incentives and extract all rents. Using it in private markets would lead to a suboptimal capital allocation.

Although some investors acquire information, managers bear the cost because they are forced to compensate investors for the information acquisition costs by providing a positive alpha. Consistent with perfectly competitive markets, managers do in fact extract all rents after information acquisition costs are accounted for. Of course, once the information acquisition costs are sunk, managers have an incentive to extract more rents by enlarging the size of the fund. They can do this by lowering the fee for uninformed investors. Thus, to induce investors to collect information, managers must commit to a maximum fund size. That explains why managers routinely limit the size of their funds. Once they have made this commitment, they will set fees for uninformed investors to extract all rents. Moreover, the cap on fund size is binding even though no investor earns economic rents. The key insight is that without the cap the manager would have an incentive to lower fees, making the cap binding. The model thus resolves the aforementioned puzzle of why managers limit the size of their funds but nevertheless extract all rents.

It is important to note that in this simple framework, the excess returns investors earn (before taking into account information costs) do not arise because of a premium for liquidity. In our setting, if managerial skill was directly observable, all investors would earn zero alpha even though they would need to commit capital for an extended period. The reason some investors earn above market returns is the limited ability of the capital markets to endogenously communicate information and thereby resolve the information asymmetry. That is not to say that a liquidity premium does not exist in the alternatives sectors, just that it is not required to explain the observed facts. Because the underlying assets themselves are illiquid it is quite plausible that the underlying asset returns contain a premium that compensates for the lack of liquidity.

This article is organized as follows. We build the model in three steps. First we abstract away from the principal-agent problem and, in Section 2 consider the case when the manager manages his own money. There we show that the liquidity of the underlying assets is determines the value of prior due diligence on the manager, thus bifurcating the industry into two sectors. Then, in Section 3, we focus on the case where managers choose to manage illiquid assets, and explicitly consider the problem of a manager raising capital from a set of investors who might potentially investigate and become informed regarding the quality of the manager. We first simplify the problem by restricting attention to linear contracts. We then extend the analysis to option-based performance contracts and show the standard contract in the alternatives industry is optimal. Section 5 discusses the implications of our results and

Section 6 concludes the paper. The next section explains how this paper extends the existing literature.

1 Background

The first paper to systematically analyze the compensation contracts in the venture capital industry is Gompers and Lerner (1999), and as the authors note in their paper, the contractual terms are similar in other private equity funds. Managers (limited partners) are paid a management fee that is a fraction of AUM and a performance fee that is a percentage of any positive returns earned by the fund. In that article the authors derive two theoretical models of money management, one based on learning and the other on signaling and conclude that the empirical evidence supports the learning model. The authors do not show that the observed contract is optimal in their setting, and the model applies equally well to the mutual fund sector, leaving open the question of why the two sectors have such different contractual terms. The paper does not consider how the liquidity of the underlying assets interacts with learning.

One of the most influential studies of performance in the alternatives industry is Kaplan and Schoar (2005). That article documents the apparent existence of positive and persistent investment alpha for investors in successful private equity funds. But subsequent researchers have questioned the quality of the underlying dataset pointing out that it relies on voluntary reporting rather than the mandated requirements imposed by the SEC in the mutual fund sector. However, Kaplan and Sensoy (2015) survey the subsequent literature that uses alternative data sources and conclude the original finding of outperformance and persistence is robust.[2]

The evidence of performance in the hedge fund industry has similar reliability issues because the data relies on self-reporting to vendors. In a recent paper, Barth, Joenväärä, Kauppila and Wermers (2023) use confidential regulatory filings to augment the vendor databases and conclude that the selection bias *lowers* reported returns because successful funds choose not to report to vendors. They find that while the funds that choose to report to vendors do not deliver positive outperformance, the funds that choose not to report provide substantial alpha to their investors. Their performance is also persistent, in contrast to the funds that choose to report.

Even if one were to ignore the empirical evidence (presumably because of the issues associated with the reliability of the data), there is indirect evidence that some funds generate

[2]Korteweg (2023) provides an extensive survey of the performance literature, both of the empirical findings as well as the theoretical and empirical issues associated with measuring performance.

predictable positive and persistent alpha. Lerner, Schoar and Wongsunwai (2007) show that some investors who invest in alternatives, particularly endowments, earn significant excess returns, suggesting that these investors are able to select better managers and, more importantly, these managers allow their investors to earn a positive alpha. Subsequent research that focused on the performance of endowments largely validated the original result, leaving as a puzzle why this particular class of investors are able to achieve such high returns.[3] The favored explanation, championed by David Swenson, Yale University's highly successful endowment manager, is that endowments have a competitive advantage picking managers because they employ better human capital. Binfarè, Brown, Harris and Lundblad (2023) and Binfare, Harris, Brown and Lundblad (2017) find evidence consistent with this explanation while Mittal (2024) finds that funds that employ worse human capital perform poorly.

As Kaplan and Sensoy (2015) point out, the existence of persistence is puzzling because it appears to suggest that, unlike mutual fund managers, alternative managers are not able to extract the rents they earn from their superior skills.[4] Furthermore, the evidence appears to show that persistence is not as strong in some sectors in alternatives (like buyout funds) as it is in other sectors (venture capital). Hochberg, Ljungqvist and Vissing-Jørgensen (2014) propose, as an explanation for the persistence, the hold up problem if one assumes existing investors have an informational advantage. But this explanation cannot explain how a new fund comes into existence, that is, how the manager raises initial capital in a perfectly competitive capital market.

Begenau and Siriwardane (2024) show that different investors in the alternative sector earn different returns, which implies that investors face different contractual terms. This contrasts with the mutual fund sector. Although fee dispersion exists in some mutual funds, it is small and a function of the size of the investment. Furthermore, the fee structures are publicly disclosed and available to all investors (based on the size of the investment). In contrast, the contractual differences in the alternative sector were largely unknown until the analysis in Begenau and Siriwardane (2024) and is persistent across investors. That is, some investors are able to consistently get lower fees, and Begenau and Siriwardane (2024) demonstrate that this persistence is not explained by the size of the investment. This evidence also suggests that the positive and persistent alpha documented in the literature was likely only available to a select group of investors.

Lerner and Schoar (2004) study the role of liquidity in private equity. In contrast to us, that article focuses on the liquidity demands of investors, rather than the liquidity of the

[3]See Lerner, Schoar and Wang (2008), Brown, Garlappi and Tiu (2010)

[4]That article comprehensively documents the evidence on persistence and subsequent research continues to find evidence of persistence in venture capital funds, but shows that persistence in buyout funds appears to have disappeared, see Harris, Jenkinson, Kaplan and Stucke (2023)

underlying assets. In our model, we assume that investors do not demand a premium for illiquid investments, or put another way, there is always an excess of capital for positive alpha opportunities regardless of the length of time the capital must be committed. We make this assumption for simplicity. Were investors to demand a premium for illiquidity, our results would care through, with better informed investors earning an additional premium over this liquidity premium.

2 Optimal Information Acquisition and Capital Allocation

We begin by abstracting from the delegation aspect of asset management and consider the optimal information acquisition decision and capital allocation to a potentially skilled manager. In this simple setting, the manager's skill (or the quality of the investment strategy) is uncertain (to everyone, including the manager herself), but can be learned upfront via costly due diligence, or over time from the strategy's realized performance. We use this case to derive the first-best efficient allocation and the optimal information acquisition decision, which will serve as the benchmark for the delegated setting analyzed later.

The value of conducting initial due diligence depends critically on the ability to reallocate capital ex post, and therefore on the liquidity and the opacity of the investment strategy. When the manager can rebalance frequently, as in liquid public market securities such as mutual funds, the opportunity to update the portfolio using realized returns substitutes for upfront due diligence. By contrast, when rebalancing opportunities are infrequent due to illiquidity and the value of the underlying assets are not easily observable, as in private equity, the benefits of acquiring information in advance are much greater. Having established this relationship, in the next section we focus on the delegation problem in which the manager and investor are distinct parties, and derive an optimal contract that elicits first-best information acquisition and capital allocation.

We model investment skill as in Berk and Green (2004). Managers, or investment strategies, differ in their ability to generate expected returns in excess of those provided by alternative investment opportunities available to all investors with the same risk. We denote by α the initial level of this excess return for a given manager/strategy.

We also assume there are diminishing returns associated with the scale of investment, as individual investment opportunities may be limited in scale or subject to price impact. As a result, the effective return of the strategy falls by $\beta > 0$ for each additional dollar invested.

Therefore, given total invested capital q, the realized return of the investment in period t in excess of the expected return on a market investment of equivalent risk (i.e., the appropriate passive benchmark) is given by

$$R_t(q) = \alpha - \beta q + \epsilon_t. \tag{1}$$

We assume that the strategy's exposure to any systematic sources of risk is known and can be hedged via benchmarks, so that the residual risk, ϵ, is idiosyncratic and mean zero conditional on α and any prior information.

There is also a fixed cost $\delta \geq 0$ of implementing this strategy, which represents the manager's opportunity cost and any necessary resources (data feeds, trading infrastructure, etc.). We assume both β and δ are known and independent of the manager's ability α.

Incorporating these costs, the total value added of the manager at time t, given committed capital q_t, is given by

$$\pi_t = q_t \, R_t(q_t) - \delta. \tag{2}$$

We assume the manager allocates capital in order to maximize the expected value added each period based on his perception of his ability. Specifically, let $\hat{\alpha}_s$ denote the manager's expectation of his ability given the information he has available at time s:

$$\hat{\alpha}_s \equiv E\left[\alpha | I_s\right].$$

Naturally, the optimal allocation will depend on the perceived profitability, $\hat{\alpha}$, of the strategy given the available information, relative to its expected price impact, β.

Suppose the manager must decide on date s the capital allocation for a future date $t \geq s$. Then, maximizing (2) with respect to q_t, we have the following result regarding the optimal capital allocation and the expected value added:

Proposition 1. *The optimal capital allocation on date t, given the information available on date $s \leq t$, is given by*

$$q^*_{t|s} = \frac{\hat{\alpha}_s}{2\beta}.$$

With that allocation, the fund's expected return is $\hat{\alpha}_s/2$, and the (per period) expected value added of the fund is[5]

$$E\left[\pi_t | I_s\right] = \frac{\hat{\alpha}_s^2}{4\beta} - \delta.$$

Proof of Proposition 1. Maximizing the expectation of (2) with respect to q_t, we have the

[5]For simplicity, here we implicitly assume $\hat{\alpha}_s \geq 0$ (or, equivalently, that the manager can short the strategy when $\hat{\alpha}_s < 0$). Later we will introduce constraints on the minimum expected alpha required for investment.

first-order condition $E[\alpha - 2\beta q_{t|s}|I_s] = 0$, which is solved by $q^*_{t|s} = \hat{\alpha}_s/(2\beta)$, and therefore

$$E[\pi_t|I_s] = E[\alpha|I_s]q^*_{t|s} - \beta(q^*_{t|s})^2 - \delta = \hat{\alpha}^2/(4\beta) - \delta.$$

□

The optimal allocation increases with the expected alpha of the strategy. Consequently, to optimally manage money, the manager must adjust the amount of invested capital in response to new information about alpha. An important difference between public- and private-market investments is the frequency with which the manager can respond to new information by adjusting the capital allocation.

Suppose that the capital can only be adjusted every T periods. That is, the capital allocation decision q_t on date t is fixed until period $t + T$. The optimal allocation will then be determined based on the information available on each "reallocation date" according to Proposition 1. To keep the model simple, we will make two further assumptions. First, we assume that the excess return that is earned at any point in this interval only depends on the initial capital investment q_t.[6] Second, we assume that the manager does not have the option to shut down the fund.[7] Under these assumptions the (per period) expected value added is constant between adjustment periods, so the expected value added of the strategy will then be a sequence of T-period annuities, which can be evaluated as follows:

Proposition 2. *Suppose the amount of capital invested can be adjusted every T periods. Given risk-free rate r, the present value of investing is given by*

$$E\left[\int_0^\infty e^{-rt}\pi_t dt\right] = \left(\frac{1 - e^{-rT}}{r}\right)\sum_{n=0}^{\infty} e^{-rnT}\left(\frac{E\left[\hat{\alpha}^2_{nT}\right]}{4\beta} - \delta\right). \tag{3}$$

Proof of Proposition 2. Recall that $q_t = q^*_{t|s}$ for $t \in (s, s + T]$. The expected value added is therefore constant over this period, and equal to $\hat{\alpha}^2_s/(4\beta) - \delta$, which we can evaluate as annuity starting on date s:

$$e^{-rs}\left(\frac{1 - e^{-rT}}{r}\right)\left(\frac{E\left[\hat{\alpha}^2_s\right]}{4\beta} - \delta\right).$$

The result then follows by summing this result over each adjustment date nT for all n. □

Now suppose the manager can pay a cost $C \geq 0$ to learn his true α. To compute the gain associated with paying this cost, we can compare equation (3) with and without the

[6] The economic justification for this assumption is that between adjustment periods the scale is fixed, that is, and any intermediate excess returns earn a passive, zero alpha return (or are consumed).

[7] We relax this assumption shortly.

ex-ante information. When the manager pays the cost and acquires the information, he will learn his true alpha upfront and allocate capital optimally at all times. In that case, the term $E[\hat{\alpha}_{nT}^2]$ in (3) becomes $E[\alpha^2]$ when calculating the expected value added of the fund. Thus, the value of information depends on the difference between $E[\alpha^2]$ and $E[\hat{\alpha}_{nT}^2]$.

By the law of total variance, for any s, we have

$$E\left[\alpha^2\right] - E\left[\hat{\alpha}_s^2\right] = Var(\alpha) - Var(\hat{\alpha}_s) = E\left[Var(\alpha|I_s)\right].$$

In words, the gain from knowing the true α at any date is proportional to the expected residual uncertainty that would remain without that information. This gain declines with s since the residual uncertainty falls when additional returns are observed.

So, with full information, capital can be efficiently deployed at the optimal level. Without full information, capital is inefficiently deployed, but this inefficiency will be reduced on every adjustment date by using information that is learned in the interim to refine the estimate of α. The longer the interval of illiquidity, the more costly the inefficiency. The cost also increases when returns are noisier, slowing the rate of learning. The inefficiency is maximal when it is impossible to ever adjust capital, so if the costs of acquiring information are strictly less than the size of this inefficiency, there will be a sufficiently long illiquidity period such that it makes sense to pay for information. The following proposition formalizes these insights:

Proposition 3. *Given illiquidity period T, the gain from learning the true alpha of the strategy upfront is given by*

$$G(T) = \left(\frac{1-e^{-rT}}{4\beta r}\right)\left(\sum_{n=0}^{\infty} e^{-rnT} E\left[Var(\alpha|I_{nT})\right]\right). \tag{4}$$

The value of information, G, is higher when the illiquidity period T is longer, the manager's skill is more uncertain, or when returns are noisier. In particular, let $\sigma_\alpha^2 \equiv E[Var(\alpha|I_0)]$, then note that

$$G(T) \to \frac{\sigma_\alpha^2}{4\beta r} \quad as \quad T \to \infty.$$

Proof of Proposition 3. The expression for G follows immediately from (3) and the law of total variance. Because the residual variance of alpha, $Var(\alpha|I_t)$, declines with t (as more information is learned), we can think of (4) as the left Riemann sum approximation of the integral of a decreasing function, which declines with a shorter step size (T). Thus, G is monotone in T. G also increases if the initial uncertainty σ_α^2 is higher, or returns are noiser (in the usual sense), since that will raise the residual uncertainty that remains at any point. Finally, when T approaches infinity, there is no opportunity to update, so the residual uncertainty is constant in perpetuity. □

The limiting value of G provides a necessary condition to pay for information. So long as

$$C < \frac{\sigma_\alpha^2}{4\beta r}, \tag{5}$$

it will be profitable to learn α when the rebalancing horizon T is sufficiently long.

Thus far, we have assumed the manager invests every period and ignored the option to shut down the fund. In Appendix A we generalize the above results to include this option and show that again, the value of information increases with the illiquidity period.

From Proposition 1, when the manager has the option to shut down the fund (earning an outside option of zero), it is profitable to operate the fund given the information available on date s if

$$\frac{\hat{\alpha}_s^2}{4\beta} - \delta \geq 0. \tag{6}$$

We can use (6) to define the threshold belief, α_M, that determines when the manager will operate the fund — that is, when the value added from investing exceeds the manager's opportunity cost δ.[8] Solving (6) gives an explicit expression for the threshold α_M:

$$\alpha_M \equiv 2\sqrt{\beta\delta}. \tag{7}$$

Indeed, consider the case with $T = \infty$, so that there is no opportunity to adjust capital. Then from Proposition 1, without acquiring information, the time 0 expected value added of the fund is given by

$$E\left[\int_0^\infty e^{-rt}\pi_0 dt\right] = \frac{1}{r}E\left[\pi_0\right] = \frac{1}{r}\left(\frac{\hat{\alpha}_0^2}{4\beta} - \delta\right). \tag{8}$$

Depending on the prior belief about alpha, this expression need not be positive. Specifically, if $\hat{\alpha}_0 < \alpha_M$, (8) is negative and the fund will fail to launch.

Now consider the decision to acquire information. Again, once alpha is learned, the manager will only invest if $\alpha > \alpha_M$. Therefore, investigating is optimal only if the expected gain from running fund when it is worthwhile to do so exceeds the cost of acquiring information:

$$\frac{1}{r}E\left[\left(\frac{\alpha^2}{4\beta} - \delta\right)\mathbf{1}_{\{\alpha > \alpha_\mathbf{M}\}}\right] - C > 0. \tag{9}$$

[8]Note that this condition is sufficient but not necessary. Because the manager has the option to adust the size of the fund on a future adjustment date, it might be optimal to operate the fund when $\hat{\alpha}_s < \alpha_M$ because of operating affords the option to learn about ability. Obviously when $T = \infty$ the option does not exist and so then the condition is also necessary.

Here, the indicator function captures the fact that the fund will only launch when the manager learns that $\alpha > \alpha_M$. That is, once the manager learns his skill, he will choose not to manage money if $\alpha < \alpha_M$.

Going forward, we assume $\hat{\alpha}_0 < \alpha_M$ so that (8) is negative — without the opportunity to acquire information or rebalance, the fund would not open. We also assume that (9) holds — for large enough T, it is optimal to investigate and learn the quality of the investment opportunity.

3 Delegated Capital Management

We now explicitly introduce the delegation problem between managers and investors. We consider a potentially skilled manager with no capital, who must raise capital from investors. As in the previous section, initially, neither the manager nor the investors know the skill of the manager. But some investors have the ability to do costly due diligence and learn the manager's type.

The case when the assets are liquid, and returns are sufficiently informative so that it is suboptimal to acquire information upfront, is the focus of Berk and Green (2004). In that article, the authors demonstrate the optimality of the standard mutual fund contract, in which managers receive a fee proportional to their assets under management and investors can redeem or invest at any time. Competition between investors ensures that the manager fully captures the value added, or put differently, all investors earn an alpha of zero. Because fund flows are unrestricted, market forces guarantee that each period the market provides the manager with an optimal amount of capital given all available information. Importantly, the contract delivers a first best allocation — the manager is indifferent between managing his own capital or taking outside capital.

The focus of this article is to analyze the equilibrium when the assets are sufficiently illiquid so that it is optimal to pay for information. Consequently, we henceforth assume that the rebalancing horizon T is sufficiently long for this to be the case.

One possibility is that the fund manager could investigate himself — that is, pay the cost to discover and reveal alpha. For now, we assume it is not possible for the manager to do so credibly. As we will see, this assumption is without loss of generality. In the equilibrium we derive, there is no incentive for the manager to investigate himself. Rather, the manager will offer a contract to an outside investor that will induce them to investigate and reveal their information.

Why would an investor be willing to pay to investigate the manager? She must expect that if she does investigate, she will be compensated ex post. That is, the decision to collect information must have at least zero NPV. Hence, the expected return to the informed investor from investing after investigating must have a positive net alpha, and the expectation of earning this positive net alpha ex post must offset the cost of investigating ex ante.

But if an informed investor must pay to acquire information, a free-rider problem emerges. If all investors receive the same terms, then an uninformed investor could invest and earn the same positive net alpha without bearing the investigation cost. This opportunity to free-ride removes the incentive to collect information in the first place. Therefore, to sustain an equilibrium in which some investors choose to incur the information collection cost, different investors must receive different terms. Managers must commit to these terms ex ante and, as we will see, must put limits on their ability to raise additional capital.[9]

3.1 An Optimal Fully-Revealing Contract

In this section, we derive an optimal contract between the manager and investors. This contract will turn out to be linear and achieve the first-best allocation given the information constraints, and so no alternative contract exists that can make any party better off without making another party worse off. Importantly, the contract delivers the same payoff to the manager as would be obtained if manager had sufficient capital to invest on his own. We will also show that the standard mutual fund contract (that pays a fee proportional to AUM) will not deliver this result. Instead, the optimal contract will feature management fees, carried interest, and constraints on total fund raising.

As before, we assume that α is the manager's skill, with decreasing return to scale determined by β, and subject to a fixed cost δ, so that the total value added is given by (2). The manager, who does not know α, proposes the following contract to a informed investor:

- A management fee of f_i per dollar invested by the informed investor;
- A fractional share ("carry") s of the net return that the manager will retain;
- A cap $\bar{q}$ on overall fundraising that depends on the amount of capital q_i that the informed investor commits.

Within this framework, we show that

- There exist terms $(f_i, s, \bar{q})$ that implement the first-best allocation and represent an optimal contract for the manager;

[9] Note that another way to avoid the free-rider problem is for the informed investor to hire the manager directly and close the fund to outside investors. We assume that the informed investor has limited capital that she is willing to invest in a single fund, so that this solution is not optimal. We will discuss this tradeoff further once we have the solved for the optimal contract with delegation.

- Given the capital commitment q_i of the informed investor, the manager's contract with uninformed investors has the same carry, s, but has a higher management fee f_u, and the capital constraint $\bar{q}$ is binding.

The above contract features a linear sharing rule s, so that the manager must pay the investor if the net return is negative. This linear sharing rule greatly simplifies the analysis, but is unrealistic, and so after establishing the main intuition here we will generalize our results in the next section to the case when carried interest is paid only on returns in excess of a target hurdle rate.

With the linear sharing rule, we will show that first-best allocation can be obtained with a cap on overall fundraising that is linear in the amount of informed capital, q_i:

$$\bar{q}(q_i) = q_0 + k \cdot q_i \tag{10}$$

We will show later that the manager will raise funds up to this cap, so that q_0 represents the minimum fund size, and $k > 0$ limits how much the fund can expand relative to the contribution of the informed investor. Importantly, the choice of k, hereafter called the *participation factor*, limits the amount of uninformed capital the manager can raise along side the informed investor. Specifically, if we subtract the level of informed capital from (10), the cap on the uninformed capital that the manager can raise is:

$$q_u = \bar{q}(q_i) - q_i = q_0 + (k - 1)q_i \tag{11}$$

For now, we restrict attention to the case with $k \geq 1$, so that the amount of uninformed capital is increasing in the amount of informed capital. Later we will discuss the equilibrium determinants of k.[10]

We begin the analysis by analyzing the decision faced by the informed investor under this contract.

3.1.1 Informed Investor's Problem

Consider the informed investor's decision regarding whether to become informed and the amount of capital to commit. In analyzing this decision, we assume for now that the informed investor expects that the manager will always choose to raise the maximum amount of capital $\bar{q}$, and show later that it will indeed be optimal for the manager to do so.

[10]The case $k \geq 1$ is certainly more consistent with practice, and, because we will show later that there is a degree of indeterminacy in k, choosing $k > 1$ may be without loss of generality. We will also show that the linear form of the cap is uniquely optimal in this case — that is, it is the only functional form that supports the efficient outcome.

Suppose the investor chooses to investigate and learn alpha. Once informed, the investor will then choose an amount q_i to invest to maximize her profits from investing (that is, the value added on her invested capital net of fees):

$$q_i(1-s)E\underbrace{\left[R_t - \beta\bar{q}(q_i) - f_i\right]}_{\text{pre-carry net return}} = q_i\left(1-s\right)\left(\alpha - \beta(q_0 + kq_i) - f_i\right). \tag{12}$$

The optimal q_i solves the first order condition:

$$\left(1-s\right)\left(\alpha - \beta(q_0 + kq_i) - f_i\right) - q_i\left(1-s\right)\beta k = 0.$$

Solving for q_i provides the amount of capital the informed investor will choose to invest as a function of the manager's skill:

$$q_i(\alpha) = \frac{\alpha - (\beta q_0 + f_i)}{2\beta k} = \frac{\alpha - \alpha_I}{2\beta k}, \tag{13}$$

where α_I is defined as

$$\alpha_I \equiv \beta q_0 + f_i. \tag{14}$$

Notice that the informed investor will only invest $q_i > 0$ if $\alpha > \alpha_I$; that is, α_I is the minimum level of managerial ability required for the informed investor to commit capital, hereafter the *investment threshold*. At this level of managerial ability, the informed investor's expected net return is zero.

Given this investment rule, the net return of the informed investor when she chooses to invest is equal to

$$\left(1-s\right)\left(\alpha - \beta(q_0 + kq_i) - f_i\right) = \left(1-s\right)\frac{\alpha - \alpha_I}{2}. \tag{15}$$

Multiplying (15) by the optimal investment, (13), the informed investor's expected profit is given by:

$$\pi_i(\alpha) = \begin{cases} \left(\frac{1-s}{k}\right)\frac{(\alpha-\alpha_I)^2}{4\beta} & \text{if } \alpha > \alpha_I \\ 0 & \text{if } \alpha \leq \alpha_I \end{cases} \tag{16}$$

Recall that to become informed, the informed investor incurs a cost C upfront. In order to ensure that she can recoup this cost over the T-period life of the investment, the informed investor's per-period expected profit must be at least c_T, where,[11]

$$c_T \equiv \left(\frac{r}{1-e^{-rT}}\right) C, \tag{17}$$

that is, the equivalent per-period annuitized cost.

Using our earlier notation $\hat{\alpha}_0$ to represent the expected ability of the manager prior to investigating, the informed investor therefore finds it optimal to investigate as long as

$$E\left[\pi_i(\alpha)\right] - c_T \geq E\left[\pi_i(\hat{\alpha}_0)\right], \tag{18}$$

where $E\left[\pi_i(\hat{\alpha}_0)\right]$ is the expected profit from investing without information. Note that if $\hat{\alpha}_0$ is below the investment threshold α_I, then the right-hand side of (18) is zero, and the optimality condition for investigation becomes,

$$E\left[\pi_i(\alpha)\right] \geq c_T. \tag{19}$$

3.1.2 Manager's Problem

Next we consider the manager's optimization problem. We will show that by choosing the contract parameters (s, f_i, q_0, k) appropriately, the manager can achieve the first-best allocation and extract all surplus. The reason the first-best allocation is attainable is that once the manager and uninformed investors observe the informed investor's capital commitment q_i, they can infer α from (13) and information becomes symmetric. Importantly, because the first best is attained, the resulting contract must be optimal for the manager.

To set the terms of the contract to achieve the first best, there are two conditions that must be satisfied. First, the fund should operate whenever the value added from investing exceeds the manager's opportunity cost; that is, if $\alpha > \alpha_M$ as defined by (7). Hence, the manager's and the informed investor's investment thresholds must coincide:

$$\alpha_I = \alpha_M = 2\sqrt{\beta\delta}. \tag{20}$$

Second, from Proposition 1, the total capital $\bar{q}$ that is invested should match the efficient level. Therefore,

$$\bar{q}(q_i) = q_0 + kq_i = \frac{\alpha}{2\beta}. \tag{21}$$

[11]Because the informed investor's information will be fully revealed in equilibrium, we assume he must recoup his investigation cost before the subsequent funding round in period T. In principle, the manager could write a longer term contract and the informed investor could recoup the investment over a longer horizon. In this case the net alpha will remain positive and will persist over funding rounds.

Substituting the optimal investment function $q_i(\alpha)$ of the informed, (13), into the above, we have

$$q_0 + \frac{\alpha - \alpha_I}{2\beta} = \frac{\alpha}{2\beta}. \tag{22}$$

Because (22) must hold for all α, we can solve for q_0 and apply (20) to derive

$$q_0 = \frac{\alpha_I}{2\beta} = \frac{\alpha_M}{2\beta} = \sqrt{\frac{\delta}{\beta}}. \tag{23}$$

Finally, substituting this result for q_0 into the definition of α_I, (14), implies that the optimal management fee for the informed investor is

$$f_i = \frac{\alpha_M}{2} = \sqrt{\beta\delta}. \tag{24}$$

Together, (23) and (24) determine the (f_i, q_0) that ensure the capital allocation is optimal. The remaining contract terms (s, k) and the management fee f_u paid by the uninformed are determined so that all investors break even, thereby ensuring that all rents accrue to the manager.

The uninformed fee is set to ensure that the uninformed investor's earns zero net return:

$$(1 - s)\left(\alpha - \beta\bar{q}(q_i) - f_u\right) = 0, \tag{25}$$

which, using (21), requires setting the fee to be

$$f_u = \alpha - \beta\left(\frac{\alpha}{2\beta}\right) = \frac{\alpha}{2}. \tag{26}$$

That is, the uninformed management fee is set equal to the gross expected excess return of the fund, so that the uninformed earn a zero net alpha.[12] While α is not known to the uninformed investors (or the manager) initially, it is revealed by the size of the informed investor's commitment q_i (or equivalently by the size of the cap, $\bar{q}$, on the total capital raised). Thus, the fee f_u can be implemented conditional upon the informed investor's committing to invest.[13] Note that because the informed investor only invests when $\alpha > \alpha_M$, if the fund is launched, $f_u > f_i$ implying "most favored nation" treatment of the informed investor.

Lastly, the manager sets the contract so that the informed investor earns zero rents net

[12] Note that because the carry is charged symmetrically, uninformed investors do not pay carry in expectation.

[13] It is common in practice for funds to use the commitment of a high reputation investor to raise additional capital.

of her investigation cost. That is, under the assumption that $\hat{\alpha}_0 \leq \alpha_M$ the participation constraint, (19), holds with equality, implying that the final parameters (s,k) satisfy:

$$E\left[\pi_i(\alpha)\right] = \left(\frac{1-s}{k}\right) E\left[\frac{(\alpha-\alpha_M)^2}{4\beta} \cdot \mathbf{1}_{\{\alpha>\alpha_M\}}\right] = c_T. \tag{27}$$

Notice that (s,k) are not uniquely determined. There is a one-dimensional family of solutions that satisfy (27). That is, the manager can either set the ratio of informed to uninformed capital by specifying k and then let the carry be determined by the distribution of skill in the market, or he can set the carry and let the ratio be a function of the distribution of skill in the market. This indeterminacy mirrors a similar result in the mutual fund space. As Berk and Green (2004) show, a mutual fund manager can either choose the management fee and let the market determine the size of the fund he manages or he can choose the size of the fund and let the market decide the management fee. Because the cross-sectional variation in fund size is much larger than management fees, in that space it appears that managers opt to set the fee. It appears that a similar result holds in the alternative space. The cross-sectional variation of the carry charge is low, likely implying that the participation factor k, which controls the ratio of informed to uninformed capital, adjusts based on variation in market perception of managerial ability.

3.1.3 Binding Capital Constraint

In the analysis above, we assumed that the manager will always choose to raise the maximum level of capital, $\bar{q}$. We now show why this constraint on the maximum capital always strictly binds.

Note first that because the manager sets q_u and f_u after α is revealed, f_u can be set so that the uninformed earn zero expected net return, and the uninformed investors will willingly invest the level of capital allowed. As a result, we can assume the manager captures all rents attributed to uninformed capital. Therefore, the manager's payoff for a given choice of q_u is given by

$$\begin{aligned}
\pi_m(q_u) &= E[\text{ informed fees} + \text{uninformed fees} - \delta\ ] \\
&= q_i(f_i + s\left[\alpha - \beta(q_i+q_u) - f_i\right]) + q_u(f_u + s\underbrace{\left[\alpha - \beta(q_i+q_u) - f_u\right]}_{=0}) - \delta \\
&= (sq_i + q_u)\left[\alpha - \beta(q_i+q_u)\right] + (1-s)q_i f_i - \delta. \qquad (28)
\end{aligned}$$

The first-order condition for the manager's choice of q_u, if total capital is below the cap, is then

$$\begin{aligned}\pi'_m(q_u) &= [\alpha - \beta(q_i + q_u)] - \beta(sq_i + q_u) \\ &= \alpha - 2\beta(q_i + q_u) + \beta(1-s)q_i. \end{aligned} \tag{29}$$

The final term in (29), $\beta(1-s)q_i$, is always strictly positive given $s < 1$. This term equals the reduction in the expected profit of the informed agent, whose net return falls by $\beta(1-s)$ for each additional dollar of capital raised. Then, given a total quantity below the efficient level — that is, $q_i + q_u \leq \alpha/(2\beta)$ — the first term in (29) is also non-negative:

$$\begin{aligned}\pi'_m(q_u) &> \alpha - 2\beta(q_i + q_u) \\ &\geq \alpha - 2\beta(\alpha/2\beta) = 0, \end{aligned} \tag{30}$$

Hence, if the cap is set to the efficient level of capital, taking additional uninformed capital both increases the total value added of the fund and lowers the net return to the informed investor, both of which benefit the manager. Therefore, the investment cap (21) strictly binds.

The result that the capital constraint binds explains one of the most perplexing features of private market investing. Unlike open-ended mutual funds, private investment funds often have binding limits on the capital raised. It might therefore appear that the fund manager could increase his rents by raising fees or taking additional capital.

In our model, the cap is binding even while the manager extracts all rents. The reason is that without the cap, since $\pi'_m(\bar{q}) > 0$, the manager has an incentive to take more capital by lowering the fee to uninformed investors. Increasing the amount of capital beyond the first best is a second-order loss, but the manager captures a first-order gain by reducing the return to the informed investor. Knowing this, the informed investor will not invest without an ex ante commitment from the manager to limit the total amount of capital to the first-best level.

The following proposition summarizes the equilibrium:

Proposition 4. *A contract (s, f_i, q_0, k) satisfying (23), (24), and (27) is optimal for the manager and achieves the first-best capital allocation. Under this contract, the manager starts the fund and raises capital $\bar{q}_u$ from uninformed investors, charging $f_u > f_i$ given by (26), if and only if the informed investor commits $q_i > 0$. All investors break even, and the manager earns an expected per period profit of* [14]

$$E\left[\left(\frac{\alpha^2}{4\beta} - \delta\right) \mathbf{1}_{\{\alpha > \alpha_{\mathrm{M}}\}}\right] - c_T, \tag{31}$$

[14] As a reminder, the fact that the fund is only launched when $\alpha > \alpha_M$ is equivalent to $\alpha^2/(4\beta) > \delta$.

consistent with (9).

Proof of Proposition 4. Follows from the analysis in the text. □

3.1.4 The Role of Carry and the Capital Constraint

A distinguishing feature of private equity contracts compared to mutual fund contracts is the existence of a cap on AUM and a performance-based carry fee in addition to a management fee. As we have seen, unlike the mutual fund setting, to achieve the first best two conditions need to be satisfied. First, the set of states in which the fund invests must be efficient. Second, all extracted rents must accrue to the manager. Because the informed investor determines the set of states when the fund invests, his contract must satisfy this dual purpose. As shown in (24), the management fee is set to ensure that the set of states in which investment occurs is optimal by determining the critical alpha threshold. The role of carry and the cap on total capital, on the other hand, is to ensure that the informed investor breaks even.

As we have already noted, there is an indeterminacy between k and s, that is the participation factor and the carry fee. The reason for this indeterminacy is that both variables affect the rents. The manager can directly reduce the rents to informed capital by raising the carry s. But he can also reduce rents by increasing the ratio of uninformed capital to informed capital because uninformed capital does not earn rents. Intuitively, because free riding by uninformed investors comes at the expense of informed investors, the manager can extract rents from informed investors either by increasing the amount of free riding (by increasing k), or limiting the free riding (by keeping k low) and instead increasing the carry, s.

Notice that in this equilibrium, the management fee paid by the informed is not set to extract rents. Instead, it is set to ensure that the manager makes at least his opportunity cost even in the worst-case scenario when the manager turns out to have ability α_M. The form of the optimal contract also makes it clear why the manager has to take uninformed capital at all. The informed investor is indifferent about investing in a manager with abiliy α_M, which necessarily means that the optimal invest at that level is zero. But when a α_M skilled manager invests his own capital, he has strictly positive investment (he breaks even at this level). Therefore, to achieve first best investment, the manager must take uninformed capital. At this minimal capital level, the management fee exactly covers the fixed costs of managing capital.

This role of the management fee accords well with the common justification given in the industry for charging a management fee — to insure that the fixed costs of managing money are covered. If the fee was not set in this way, there would be states in which either the manager would choose to quit when his ability is revealed, or the informed investor would choose not to invest even though starting the fund is optimal.

In contrast, in a public-market mutual fund the assets are liquid and investors have insufficient incentive to investigate ex ante. Because managers and investors are therefore symmetrically informed, they always choose to invest in the same states. As a result, the manager can use the management fee to extract the rents. In private markets, when the manager and investor are asymmetrically informed, an additional instrument is required for rent extraction.

A technical issue we have not yet addressed are the conditions for which (27) admits a solution with both $s \geq 0$ and $k \geq 1$. As long as the investigation costs satisfies

$$c_T \leq E\left[\frac{(\alpha - \alpha_M)^2}{4\beta} \cdot \mathbf{1}_{\{\alpha > \alpha_M\}}\right], \tag{32}$$

then (27) implies $(1-s)/k \leq 1$, and a solution with $s \geq 0$ and $k \geq 1$ is possible. However, for an investigation cost in the range

$$E\left[\frac{(\alpha - \alpha_M)^2}{4\beta} \cdot \mathbf{1}_{\{\alpha > \alpha_M\}}\right] < c_T < E\left[\frac{(\alpha^2 - \alpha_M^2)}{4\beta} \cdot \mathbf{1}_{\{\alpha > \alpha_M\}}\right], \tag{33}$$

then, while it is still efficient to investigate, the informed investor's participation constraint may require $k < 1$ or $s < 0$.

To see why this wedge exists, consider the extreme case when the costs of investigation equals the total available surplus. In that case, all rents must accrue to informed capital to compensate the informed investor for investigating. But recall that the manager captures all the rents associated with uninformed capital via the uninformed management fee. Hence, in states in which management talent is high, these fees must be rebated to informed capital (so that, in expectation, the manager earns nothing after paying his fixed costs), which necessarily requires a negative s. Indeed, in this scenario, we might expect delegation to fail, with the informed investor hiring the manager directly instead. In summary, the costs of investigation need to be low enough to ensure that any rents the manager earns from uninformed fees do not exceed the total expected rents from managing money.

3.1.5 Implementation

The manager implements the optimal contract as follows. First he calculates the minimum skill level at which he would still choose to manage money, α_M, from his opportunity cost, δ, and the technology parameter β using (20). Next, he uses this parameter to calculate f_i using (24) and q_0 using (23), and picks any reasonable s. He then calculates k using his choice of s

based on the investigation cost and (27). Using these parameters, he presents potentially informed investors with the contract (f_i, s, q_0, k). On seeing this contract, an investor chooses to incur the cost C to acquire information, and if the resulting α exceeds α_M she chooses to invest q_i as given by (13). On seeing this investment level, uninformed investors (and the manager) infer α by inverting (13) as follows:

$$\alpha = \alpha_M + 2\beta k q_i.$$

The manager then offers the remaining (uniformed) investors the contact $\{f_u, s\}$, where f_u is given by (26). Note that since $\bar{q}(q_i) = \alpha/2\beta$, we can equivalently set the uninformed management fee proportional to the maximum fund size:

$$f_u = \beta\bar{q}. \tag{34}$$

The manager then solicits the maximum allowable capital from uninformed investors, which will equal $\bar{q}(q_i) - q_i$ as specified in (11).

This equilibrium implementation closely models reality. New managers almost always seek a cornerstone investor with a successful reputation. In return for investing, this investor is offered a larger capital allocation and a break in fees. The manager then uses the information that this investor chose to invest in his fund to solicit capital from other investors. Moreover, managers routinely cap the size of their funds, that is, they agree up front to limit the amount of capital they will accept.

The main difference between this contract and those observed in practice is the linear carry charge. Because it would require the manager to pay the investors in the event of a sufficiently negative idiosyncratic shock to the return, it requires unlimited liability for the manager. In reality the manager's liability is limited by both his personal wealth and bankruptcy law. Consequently, the linear contract is not readily implementable.

To remedy this issue and restore limited liability, in the next section we modify the carry charge as is done in practice: carry is only charged when returns exceed a hurdle. We will show that imposing limited liability in this way does not materially affect the outcome or the interpretation of the contract, and, most importantly, that the first-best allocation is still attained.

3.2 The Limited Liability Contract

Although the linear contract in the previous subsection delivers a first-best outcome, it requires unlimited liability for the manager: When the fund has a negative return, the manager must reimburse the investors for a fraction s of the loss. Such a contract is likely infeasible in practice because the manager's ability to make such a guarantee is uncertain and bankruptcy law limits the ability of investors to enforce the terms of the contract.

In practice, private contracts limit the manager's liability by making the carry fee asymmetric: The manager only receives a share s of the amount of the return in excess of a target "hurdle rate". As long as the hurdle rate is non-negative, so is the minimum level of carry is non-negative, the liability of the manager is limited. Typical hurdles rates for private equity range from zero to ten percent, depending on the specific asset class, with zero being common for U.S. venture funds and 8% being the most common for private equity.[15] In light of this evidence, we extend the contract to preserve limited liability for the manager by including a hurdle rate, and show that the first-best outcome can still be obtained.

Let $h \geq 0$ be the hurdle rate. Investors are charged the carry s on the portion of their return net of management fees that exceeds the hurdle h. When the investor's net return is below h, carry is zero (and the manager does not rebate the investors). That is, the carry owed to the manager by an investor with management fee f, given (gross) return R, can be calculated as[16]

$$\text{Carry}(R) = s(R - f - h)^+$$

We refer to this carry formula as "option-based" carry, as opposed to the "linear" carry of the prior section.

As we show below, all of the main insights of the prior section continue to hold. The important contract features, such as the cap on uninformed capital, and a lower management fee for the informed investor, continue to be optimal. The main qualitative change is that the amount of capital invested by the informed, as well the cap on uninformed capital, are no longer linear in the manager's perceived alpha.

3.2.1 Gross versus Net Returns

Recall that a manager whose trading strategy has quality alpha, and who invests total capital q, generates a gross return equal to

$$R(q) = \alpha - \beta q + \epsilon. \tag{35}$$

[15] See Goodwin Insight, November 2023. Note that hurdles may be soft or hard; hard hurdles compensate the manager only on the portion of the return that exceeds the hurdle (as modeled here), whereas soft hurdles compensate the manager based on the entire return once the hurdle is met.

[16] As a simplification, we compute carry each period. In private equity, carry is computed based on the current cumulative return, and may include clawback provisions if the fund subsequently underperforms. Hedge funds pay carry periodically but often contain a high water mark, effectively paying carry on cumulative returns only.

For an investor with management fee f, the expected *pre*-carry return is therefore

$$\bar{R}^{pre} = E\left[R(q) - f\right] = \alpha - \beta q - f. \tag{36}$$

Given carry s with hurdle h, this investor has an expected *net* return, post carry, given by

$$\text{Expected Net Return} = \bar{R}^{net} = F(\bar{R}^{pre}) \equiv \bar{R}^{pre} - sE\left[\left(\bar{R}^{pre} + \epsilon - h\right)^{+}\right]. \tag{37}$$

For example, with this definition of F, the expected net return for the informed investor given a manager of quality α managing a fund at the efficient investment level, $\frac{\alpha}{2\beta}$, is equal to $F(\alpha/2 - f_i)$. In the linear carry model, $F(\alpha/2 - f_i) = (1 - s)(\alpha/2 - f_i)$, and therefore the expected net return is linear in alpha. With option-based carry, the net return is increasing and concave in alpha. The concavity arises because better performance raises the probability that carry will be charged.

The concavity of F has two important implications. First, because expected net returns are no longer linear in alpha, the informed investor's optimal capital commitment will no longer increase linearly with alpha. Second, the concavity of F raises the break-even level of the pre-carry return. To see why, note that $F(0) < 0$, since even with a zero expected pre-carry return, the expected carry is positive — the manager will earn carry when the idiosyncratic shock is above the hurdle (that is, the option component of managerial compensation is always positive regardless of managerial ability).

The importance of the second implication is that the break-even pre-carry expected return is not zero as was the case in the linear model. Before, investors invested whenever the pre-carry return was positive. With option-based carry, the break-even pre-carry expected return, r_0, is strictly positive, and is defined as the unique solution to

$$F(r_0) \equiv 0. \tag{38}$$

Note that, solving this equation using (37) implies that r_0 is equal to the expected carry charge at break even,

$$r_0 = sE\left[(r_0 + \epsilon - h)^{+}\right] > 0.$$

Intuitively, $r_0 > 0$ is the pre-carry return that just offsets the value of the manager's carry option. The informed investor will invest whenever his expected pre-carry return exceeds r_0 (rather than zero, as in the linear case). Managers set fees to ensure that uninformed investors earn exactly r_0 (rather than zero, as in the linear case). Notice that although r_0 is an increasing function of the carry charge, s, that is, it does not depend on the manager's ability, α. Figure 1 illustrates the difference the option contract makes.

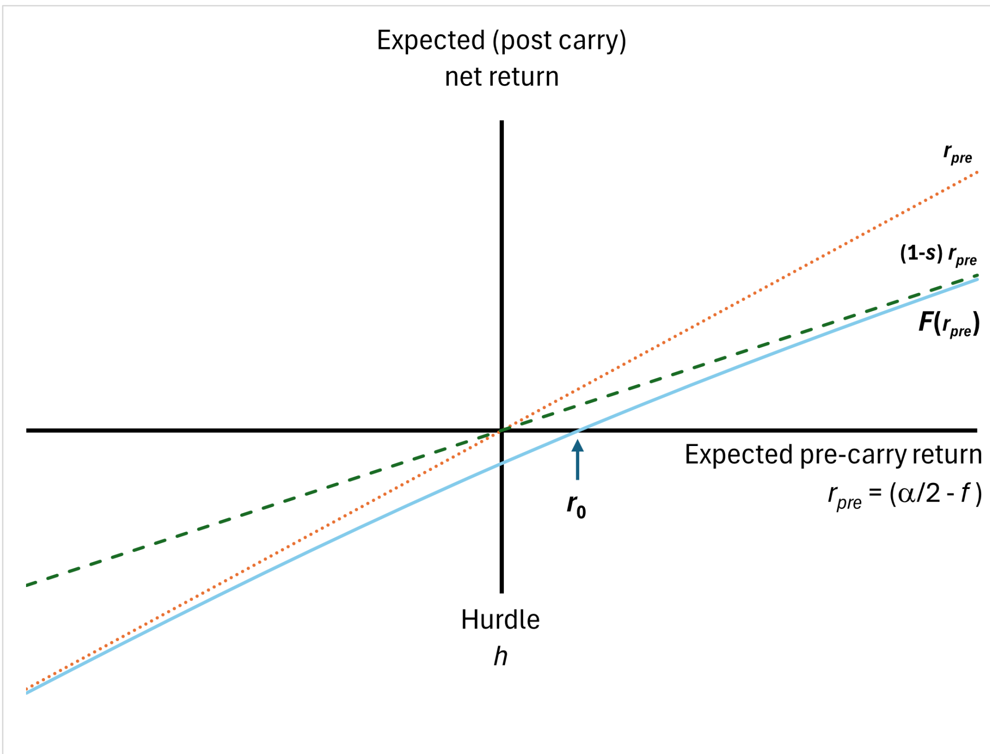


Figure 1: Break-even Expected Pre-Carry Return r_0. The green dashed line is the net return earned in the linear case. The solid blue line is the net return earned in the option based carry case. The optimal investment policy in the linear case is to invest when ever the net return exceeds the hurdle, while in the option case, it is optimal to invest only when the net return exceeds r_0.

3.2.2 Informed Investment

The crucial feature of an optimal contract is that informed investor's capital commitment q_i reveals alpha. The remaining contract terms then allow the manager to set the total capital optimally and extract the rents.

With linear carry, the informed investor's expected net return, and therefore the choice of q_i, varied linearly with alpha. With option-based carry, however, the expected net return is concave in alpha, leading to a non-linear relationship between alpha and q_i. Because of this non-linearity, we do not know the functional form of how the inferred alpha and investment cap vary with the informed investor's quantity choice. Therefore, we cannot write down the informed investor's maximization problem directly.

We therefore adopt a different solution method to the one we used in the linear case. We begin by assuming an investment schedule $q_i(\alpha)$ that the informed investor will follow, with $q_i(0) = 0$ and $q_i' > 0$ when $q_i > 0$. Given such a schedule, if the informed investor invests, the manager will infer α by inverting q_i and will set the cap $\bar{q}(q_i)$ equal to the optimal fund size. Anticipating this response, we begin by deriving the schedule $q_i(\cdot)$ that is incentive compatible for the informed investor (that is, the schedule such that the informed investor, taking into account the reaction of the manager and uninformed investors, finds it optimal to use).[17]

Consider an equilibrium investment schedule $q_i(\alpha)$. Suppose that after learning the true α, the informed investor deviates and instead communicates $\hat{\alpha}$ by choosing to invest $q_i(\hat{\alpha}) > 0$. In that case, from Proposition 1, the manager will believe that the efficient size of the fund is $\hat{\alpha}/(2\beta)$. Assuming the manager will run the fund at the inferred efficient size (which will be true in equilibrium), the expected pre-carry return for the informed investor is

$$\alpha - \beta\left(\frac{\hat{\alpha}}{2\beta}\right) - f_i = \alpha - \frac{\hat{\alpha}}{2} - f_i.$$

The informed investor will choose $\hat{\alpha}$ to maximize her net investment payoff:

$$\max_{\hat{\alpha}} \quad q_i(\hat{\alpha})F(\alpha - \hat{\alpha}/2 - f_i). \tag{39}$$

The first order condition is therefore

$$q_i' F - \tfrac{1}{2} q_i F' = 0. \tag{40}$$

[17]Technically, the model is similar to a standard signaling model in which the informed investor is the agent and the manager is the principal. The key difference is that the manager commits upfront to the total capital raised as a function of q_i, and this commitment allows the manager to select the equilibrium that is most favorable (eliminating the usual need for equilibrium refinements).

Incentive compatibility requires that (39) be maximized when $\hat{\alpha} = \alpha$.[18] When $q_i > 0$, this requirement implies that (40) can be written as follows:

$$\frac{q_i'(\alpha)}{q_i} = \frac{\frac{1}{2}F'(\alpha/2 - f_i)}{F}$$

Integrating both sides in the range when $q_i > 0$, we get $\ln q_i(\alpha) = \ln F(\alpha/2 - f_i) + \ln \theta$, where $\ln \theta$ is an arbitrary constant of integration. Exponentiating both sides gives

$$q_i(\alpha) = \theta \; F(\alpha/2 - f_i). \tag{41}$$

For ease of comparison with the linear case, we introduce a new constant of integration k,

$$k \equiv \frac{1}{\beta\theta(1-s)}. \tag{42}$$

Substituting this expression leads to the following general form for any incentive compatible investment schedule:

$$q_i(\alpha) = \begin{cases} \frac{F(\alpha/2 - f_i)}{\beta k(1-s)} & \text{if } F(\alpha/2 - f_i) > 0 \\ 0 & \text{if } F(\alpha/2 - f_i) \leq 0 \end{cases} \tag{43}$$

where the scale factor $k > 0$, the arbitrary constant of integration is still to be determined.[19] To see that this result matches that of the linear model, note that in the linear model, $F(\alpha/2 - f_i) = (1-s)(\alpha/2 - f_i)$. Hence in that case, when $\alpha > \alpha_I = 2f_i$, (43) becomes

$$q_i(\alpha) = \frac{(1-s)(\alpha/2 - f_i)}{\beta k(1-s)} = \frac{\alpha - 2f_i}{2\beta k} = \frac{\alpha - \alpha_I}{2\beta k}. \tag{44}$$

matching (13).

Equation (43) states that for any contract in which the informed investor reveals α and the manager implements the efficient investment level $\alpha/(2\beta)$, the informed investor commits capital proportional to her expected net return. Note also that the investment threshold α_I

[18] Given $s < 1$, the objective in (39) satisfies the Spence-Mirrlees single-crossing property $\frac{\partial^2}{\partial q_i \partial \alpha} q_i F = F' > 0$. Thus, we can use the approach of Mailath (1987 CITATION) to characterize the separating equilibrium in our setting with continuous types, which we follow here.

[19] The boundary for positive investment, $\alpha_0 = \inf\{\alpha | q_i(\alpha) > 0\}$, can be derived as follows. Since the informed investor earns a non-negative profit, $F(\alpha/2 - f_i) \geq 0$ for $\alpha > \alpha_0$, and so by continuity, $F(\alpha_0/2 - f_i) \geq 0$. But if $F(\alpha_0/2 - f_i) > 0$, then type $\alpha_0 - \epsilon$ could profit by deviating from choosing zero investment, to choosing $q_i(\alpha_0 + 2\epsilon) > 0$, thereby earning net return $F(\alpha_0 - \epsilon - (\alpha_0 + 2\epsilon)/2 - f_i) = F(\alpha_0/2 - 2\epsilon - f_i) > 0$ for small enough ϵ. Hence $F(\alpha_0/2 - f_i) = 0$.

is defined by

$$F(\alpha_I/2 - f_i) = 0 = F(r_0),$$

and therefore,

$$\frac{\alpha_I}{2} - f_i = r_0. \tag{45}$$

The left side of (45) is the expected return of the fund, $\alpha_I/2$, less the management fee, that is, the informed investor's pre-carry expected return. So the investment threshold is the level of managerial ability at which the investor's pre-carry expected return is equal to r_0, the break-even pre-carry expected return. (Again, this matches the linear case, for which $r_0 = 0$.)

3.2.3 Manager's Problem

Having determined the set of implementable investment schedules for the informed investor, we now consider how the manager will set the terms of the contract to extract the first-best surplus. To do so, the manager again commits to the following upfront:

- A management fee of f_i per dollar invested by the informed investor,
- A fractional share ("carry") $s \in [0, 1)$ of the net return in excess of a hurdle rate $h \geq 0$ that the manager will retain;
- A cap $\bar{q}(\cdot)$ on overall fundraising that depends on the amount of capital q_i that the informed investor commits.

First, we must insure the informed investor's investment threshold α_I matches the efficient level α_M defined in (7). From (45), this requires

$$f_i = \frac{\alpha_M}{2} - r_0. \tag{46}$$

Given any schedule $q_i(\alpha)$ based on (43), the manager and other investors can invert the schedule to assess the informed investor's determination of alpha based on his capital commitment. The cap on total capital is then set to the efficient level:

$$\bar{q}(q_i) = \frac{\alpha}{2\beta} = \frac{q_i^{-1}(q_i)}{2\beta}. \tag{47}$$

Note that because q_i increases with alpha, $\bar{q}$ will increase with q_i. To compare with the linear model, we can compute the slope of $\bar{q}$ as follows:

$$\begin{aligned}\bar{q}'(q_i) &= \frac{1}{2\beta q_i'(\alpha)} = \frac{k(1-s)}{F'(\alpha/2-f_i)} = \frac{k(1-s)}{1-s\Pr(\alpha/2-f_i+\epsilon>h)} \\ &= \frac{k(1-s)}{1-s\Pr(\text{carry}|\alpha)} \end{aligned} \tag{48}$$

Therefore, $\bar{q}$ is convex in q_i, with a slope that increases and approaches k as the likelihood that carry will be charged approaches one (matching the linear model).

Uninformed investors, upon observing the capital cap $\bar{q}$, can also infer $\alpha = 2\beta\bar{q}$ and will therefore correctly anticipate a gross return of the fund equal to $\alpha - \beta(\alpha/(2\beta)) = \alpha/2$. Given that, the manager will set the fee f_u to make the uninformed investors indifferent. That is, the uninformed expected net return will be set to zero, implying that the uninformed investors' pre-carry expected return must be set equal to r_0, the break-even pre-carry expected return:

$$\frac{\alpha}{2} - f_u = r_0,$$

implying

$$f_u = \frac{\alpha}{2} - r_0. \tag{49}$$

Note that, like f_i, the uninformed fee is similarly reduced by r_0, the break-even expected carry. Consequently, the difference in management fees,

$$f_u - f_i = \frac{\alpha - \alpha_M}{2},$$

is the same as in the linear case, and because $\alpha > \alpha_M$ if the fund launches, we have $f_u > f_i$.

Thus, given any parameters (s, h, k), by choosing $(f_i, \bar{q}, f_u)$ according to (46), (47), and (49), the manager can be assured that the schedule q_i given by (43) incentive compatible for the informed investor. The final condition for efficiency is that the informed investor finds it worthwhile to investigate. The informed investor's profit upon investigating is given by

$$\pi_i(\alpha) = \begin{cases} q_i(\alpha)F(\alpha/2-f_i) = \frac{F(\alpha/2-f_i)^2}{\beta k(1-s)} & \text{if } \alpha > \alpha_M \\ 0 & \text{if } \alpha \le \alpha_M \end{cases} \tag{50}$$

Therefore, for a given option-based carry (s, h), we can choose k so that the expected profit of the informed investor from investigating equals the investigation cost c_T:[20]

$$E\left[\pi_i(\alpha)\right] = \frac{1}{\beta k(1-s)} E\left[F(\alpha/2 - f_i)^2\ \mathbf{1}_{\{\alpha>\alpha_M\}}\right] = c_T \tag{51}$$

The final step is to show that, in this equilibrium, the manager will indeed choose to accept capital until the cap, $\bar{q} = \alpha/2\beta$, is reached. As in the linear model, the manager will find it optimal to do so. Because the manager earns all rents attributed to the uninformed investors, the manager's expected profit is equal to the total value added of the fund less the amount captured by the informed investor. If the total capital is below the efficient level — i.e. the cap — then each dollar raised both improves efficiency, and so increases the total value added of the fund, while it reduces the net return earned by the informed investor, both of which improve the manager's payoff. We prove this formally as part of the following proposition:

Proposition 5. *For a given (s, h), if the manager chooses $(k, f_i, \bar{q}, f_u)$ according to (46)-(51), then the informed investor will choose to investigate and invest according to (43), and the manager will raise uninformed capital until the cap $\bar{q}$ binds at the first-best allocation. This contract is optimal for the manager, who will earn the first-best surplus (31).*

Proof of Proposition 5. The essence of the proof is contained within the main text. Here we formalize two details. First, we verify the global optimality of the informed agent's optimal investment choice. Recall that the first-order condition (40) holds by construction of $q_i(\alpha)$. To see that the first-order condition is sufficient, note that F is increasing and concave, and therefore q_i, which is proportional to F, is also increasing and concave. The second derivative with respect to $\hat{\alpha}$ of the objective in (39) is then:

$$q_i''F - q_i'F' + \tfrac{1}{4}q_iF'' < 0.$$

Second, we verify that the manager will choose raise capital until $\bar{q}$ binds. Here we use the fact that, because the uninformed earn zero rents, the manager's expected payoff is equal to the total surplus net of the informed agent's profit:

$$\pi_m = (q_i + q_u)(\alpha - \beta(q_i + q_u)) - \delta - \pi_i.$$

Therefore,

$$\tfrac{\partial}{\partial q_u}\pi_m = (\alpha - 2\beta(q_i + q_u)) - \tfrac{\partial}{\partial q_u}\pi_i.$$

[20]Note that we continue to assume $\hat{\alpha}_0 < \alpha_M$, so that the investor would never invest without investigating first. (Indeed, with option-based carry, an investor who does not investigate earns an even lower expected payoff than $\pi_i(\hat{\alpha}_0)$, since their higher residual uncertainty raises the expected cost of the carry option.)

Because higher q_u reduces the expected return for the informed investor, the second term, $-\frac{\partial}{\partial q_u}\pi_i$, is strictly positive. The first term is weakly positive as long as $q_i + q_u \leq \alpha/(2\beta)$. Therefore, the cap $\bar{q} = \alpha/(2\beta)$ strictly binds. □

4 Implications

In this section, we calibrate the model and identify the main parameters as follows. First, we consider a minimum fund size $\bar{q}(0) = \$50$ million, with manager opportunity cost $\delta = \$1$ million per year. Because the minimum efficient alpha is α_M, the minimum fund size must equal to $\frac{\alpha_M}{2\beta}$, with total value added $\bar{q}(0)\alpha_M/2 = \delta$. Therefore we can calibrate

$$\alpha_M = \frac{2\delta}{\bar{q}(0)} = 4\% \quad \text{and} \quad \beta = \frac{\alpha_M}{2\bar{q}(0)} = 0.04\%.$$

We consider a standard carry rates of 10–20% and use a hurdle rate of 0%. Assume idiosyncratic risk $\epsilon \sim N(0, \sigma_\epsilon^2)$. Then we can calculate r_0 from (38) as follows:

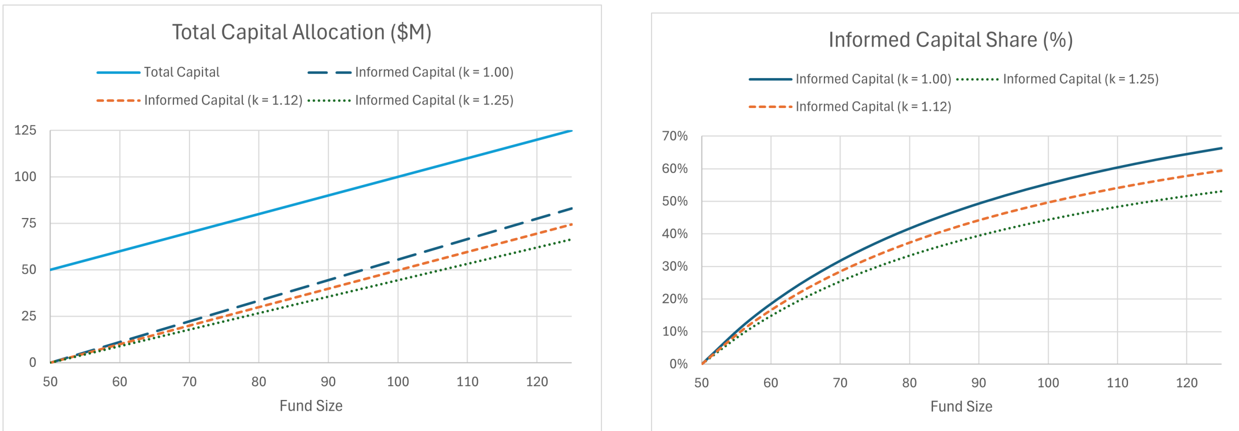
	σ_ϵ		
s	10%	15%	20%
10%	0.420	0.630	0.841
15%	0.648	0.972	1.297
20%	0.890	1.335	1.780

Table 1: Implied r_0 (in percent) given carry s, zero hurdle, and idiosyncratic risk σ_ϵ.

Hence with a 15% volatility and 20% carry, $r_0 = 1.34\%$. Recall that we can interpret r_0 as the minimum expected carry payment. Therefore, the optimal informed management fee with these parameters is $f_i = \alpha_M/2 - r_0 = 0.66\%$.

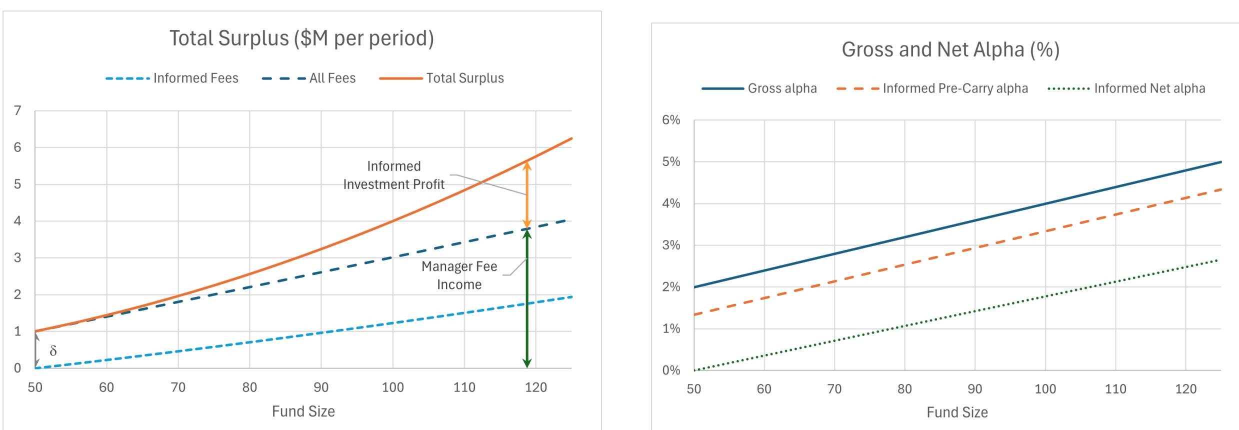
The figures below show the allocation of capital and surplus for different realizations of alpha, which correspond to different fund sizes. We show the capital allocation to the informed investor for different choices of the co-investment factor k.

The next set of figures show the total surplus, total fees, and gross and net alpha as function of realized fund size.



(a) Total Capital Allocation **(b)** Informed Share

Figure 2: Total Capital Allocation and Informed Capital Share versus Fund Size



(a) Total Surplus and Fees **(b)** Gross and Net Alpha

Figure 3: Total Surplus, Fee Income, and Gross and Net Alpha versus Fund Size

5 Discussion

The model we have outlined provides an parsimonious explanation for the optimality of the standard contract observed in private capital management. It is derived from two important assumptions. First that the underlying assets are illiquid for a significant period of time, and second, that investors do not initially know the skill of the manager. The success of the model therefore suggests that these two features of the private management industry are important and likely explain the differences that are observed between public and private capital management.

As in the competitive model of mutual funds, our model presumes that managers with skills in short supply extract the rents. Informed investors earn a predictable positive alpha as compensation for the cost of acquiring information and validating the manager's skill. The model can therefore deliver the empirically observed phenomenon of persistent excess returns.

That said, in our current model, informed investors do not outperform once their costs of diligence are included. However, a simple extension to the model could deliver such a result. For simplicity, we assumed all investors have identical investigation costs. If instead we assume that investors are heterogeneous, facing different costs of investigation, then it is natural that investors with the lowest costs would have a competitive advantage and would earn excess returns. Only the marginal investor would break even.[21] This is precisely the argument David Swensen has put forward for why endowments have outperformed.

The model also delivers the result that the contracts that are observed in the two sectors of the industry are optimal, and more importantly, that the industry is optimally bifurcated based on the liquidity of the underlying investments. That is, the contract in the mutual fund sectors, if used in the alternatives sector, would lead to a suboptimal outcome. Moreover, the model predicts that the terms offered investors will differ in the two sectors. Within the mutual fund sector, the optimal allocation can be achieved with investors all facing the same terms. This is not true within the alternative sector, to achieve an optimal allocation, different investors must face different terms.

In the model, the manager places a limit of the size of the fund, by limiting the amount of uninformed capital he will accept. One might argue that these limits are inconsequential. Since managers earn all the rents, all uninformed investors are indifferent about investing an additional dollar. Informed investors, because they recognize how their investment affects the information inference, are worse off increasing their investment. But what these observations ignore is that without the limit the manager has an incentive to attract additional uninformed

[21] To model this in our context, all one would need to assume is the existence of an unobservable small (finite) set of investors with costs strictly less than C.

capital by lowering his fees. Because of these incentives, the limit on the amount of capital is crucial to implementing the optimal allocation.

In our model there is an indeterminacy in the participation factor, the carry fee and the hurdle rate. We leave open the question of what unmodeled factors resolve this indeterminacy. We speculate that it is related to two unmodeled effects. If investors are subject to liquidity concerns, so that they might choose to withdraw capital for reasons unrelated to managerial ability, managers might have a preference to limit the share of any one investor in the fund. That would provide a preference for a higher participation factor (and, consequently, a lower carry fee). A higher participation factor also limits the scale of the investment by the informed, which might be important if they do not want to be overly exposed to a single fund.

Conversely, a higher carry fee (and lower participation factor) might be optimal if it is needed to provide adequate incentives for the manager (we have ignored such agency costs in our model). Higher carry also has tax advantages for the manager, as carry is taxed as capital gain income whereas management fees are taxed as ordinary income.

In our current model, the informed investor invests a very small amount near the investment threshold. Such small investments seem unlikely, given fixed costs of allocating capital. It also seems unlikely that uninformed investors would find a small investment to be a credible signal of the manager's quality. In the appendix, we show an extension of the model with fixed costs of allocating capital, in which the informed investor makes a significant minimum investment if she invests at all. By making a large enough investment, thereby effectively signaling that she has committed a significant fraction of her limited resources to investigating, the informed investor makes the signal more credible.

6 Conclusion

In this article we demonstrate that a small set of parsimonious assumptions can explain the observed contracts in the delegated money management industry. Specifically we show that under the assumption of fully competitive capital markets and rational expectations, the industrial organization of both the private and public markets in money management can derived. In both markets, the equilibria are consistent with managers extracting all rents after the costs of information have been accounted for. Because it is suboptimal in public markets to pay for information, in those markets investors make zero alpha. In private markets, when it is optimal for investors to pay for information, some investors make positive alpha before the costs to acquire information are accounted for.

Our model identifies the crucial difference between public and private money management markets. The underlying investments in public markets are highly liquid, whereas private markets are characterized by illiquid underlying investments. We show that this difference explains the observed differences between the two markets. Because of the illiquidity of the underlying investments in private markets, some investors in those markets choose to acquire information about managerial ability in order to allocate capital more efficiently. This heterogeneity implies that the different investors invest under different contractual terms. In addition, to implement the equilibrium, the manager must commit to a size limit on the fund and the contract must contain both a performance fee ("carry") and a management fee.

Appendix

A Option to Shut Down

Consider the case when the manager manages his own money and assume that he has the option to shut down the fund at any time. With full information, the value of the fund is

$$\frac{1}{r}E\left[\left(\frac{\alpha^2}{4\beta}-\delta\right)^+\right],$$

as the manager will learn α immediately and only open the fund if it will be profitable.

If α is not learned upfront, then beliefs about α will update over time as returns are observed. In that case, there will be an optimal stopping time τ, measurable with respect to all prior information, that solves

$$\max_\tau E\left[\int_0^\tau e^{-rt}\pi_t dt\right] = \max_\tau E\left[\int_0^\tau e^{-rt}\left(\frac{\hat{\alpha}^2_{n_T(t)T}}{4\beta}-\delta\right)dt\right],$$

where $n_T(t) = \lfloor t/T \rfloor$ is the number of adjustment periods prior to t (that is, t/T rounded down to the nearest integer).

For any fixed T, having full information leads to a higher payoff since α is a mean preserving spread of $\hat{\alpha}_t$ and thus

$$E\left[\left(\frac{\alpha^2}{4\beta}-\delta\right)^+\right] \geq E\left[\left(\frac{\hat{\alpha}^2_{n_T(t)T}}{4\beta}-\delta\right)1\{t\leq\tau\}\right].$$

Moreover, holding τ fixed, the gap increases with T, since $n_T(t)$ is decreasing in T. Finally, less frequent rebalancing will reduce the efficiency of the optimal stopping decision, further increasing the benefit of information.

The maximal potential benefit from information is therefore attained when $T=\infty$. In that case, from Proposition 1, acquiring information is profitable if and only if

$$\frac{1}{r}E\left[\left(\frac{\alpha^2}{4\beta}-\delta\right)^+\right] - C \geq \frac{1}{r}E\left[\left(\frac{\hat{\alpha}^2_0}{4\beta}-\delta\right)^+\right]. \quad (52)$$

Note that condition (52) is equivalent to (5) if $\delta = 0$. When the RHS of (52) is positive, then this condition is weaker than (5).

B Extension: Minimum Informed Investment

In our optimal contract, the informed investor's stake, given by (13), approaches zero if α is close to $\alpha_I = \alpha_M$. In reality, a di minimus informed investment may be unrealistic — as well as unconvincing to uninformed investors. Indeed, it is likely that there are additional costs to the informed investor from deploying capital, and potentially from monitoring the manager to be sure the investment strategy is correctly implemented. There is also a potential opportunity cost of not being able to invest profitably elsewhere.

To incorporate this idea into the model, suppose that there is a fixed cost δ_i borne by the informed investor upon investing in the fund, and let δ_m be the fixed cost of the manager, so that the total cost of running the fund is $\delta = \delta_m + \delta_i$ and the total available surplus (31) is unchanged.

Let us continue to define $\alpha_M \equiv 2\sqrt{\beta\delta}$ to be the efficient investment threshold as in (7). Then, in the linear model, the optimal contract in this case will adjust as follows:

$$f_i = \frac{\alpha_M}{2} - \sqrt{\frac{\beta\delta_i k}{1-s}} \quad \text{and} \quad q_0 = \frac{f_i}{\beta}, \tag{53}$$

The informed investor will then invest according to

$$q_i(\alpha) = \left(\frac{\alpha/2 - f_i}{\beta k}\right) \mathbf{1}_{\{\alpha > \alpha_M\}}. \tag{54}$$

Then, at the investment threshold α_M, the minimum informed investment is

$$q_i(\alpha_M) = \sqrt{\frac{\delta_i}{\beta k(1-s)}}. \tag{55}$$

Note that we can use equation (55) to calibrate δ_i based on the minimum investment level for an "anchor" investor.

With these changes, the investment cap continues to provide the efficient level of capital,

$$\bar{q}(q_i(\alpha)) = q_0 + kq_i(\alpha) = \frac{f_i}{\beta} + \frac{\alpha/2 - f_i}{\beta} = \frac{\alpha}{2}, \tag{56}$$

and the minimum rent for the informed investor is equal to

$$\pi_i(\alpha_M) = q_i(\alpha_M)(1-s)(\alpha_M/2 - f_i) = \delta_i, \tag{57}$$

which also implies the manager earns $\alpha_M^2/(4\beta) - \delta_i = \delta_m$, as required.

Finally, the co-investment factor k must be set so that informed investor breaks even:

$$E\left[(\pi_i(\alpha) - \delta_i)^+\right] = c_T. \tag{58}$$

Compared to our earlier setting, compensating the informed investor for their opportunity cost will lead to a somewhat lower co-investment factor.

References

Barth, Daniel, Juha Joenväärä, Mikko Kauppila, and Russ Wermers, "A bias-free assessment of the hedge fund industry: A new evaluation of total assets, alphas, and the flow-performance relation," *OFR WP*, 2023, pp. 20–01.

Begenau, Juliane and Emil N. Siriwardane, "Fee Variation in Private Equity," *The Journal of Finance*, 2024, *79* (2), 1199–1247.

Berk, Jonathan B. and Richard C. Green, "Mutual Fund Flows and Performance in Rational Markets," *Journal of Political Economy*, 2004, *112* (6), 1269–1295.

Binfarè, Matteo, Gregory Brown, Robert Harris, and Christian Lundblad, "How does human capital affect investing? Evidence from university endowments," *Review of Finance*, 2023, *27* (1), 143–188.

Binfare, Matteo, RS Harris, GW Brown, and Christian T Lundblad, "How important are investment committees? Evidence from university endowments," in "the 2017 UNC Alternative Investments Conference" 2017.

Brown, Keith C, Lorenzo Garlappi, and Cristian Tiu, "Asset allocation and portfolio performance: Evidence from university endowment funds," *Journal of Financial Markets*, 2010, *13* (2), 268–294.

Fama, Eugene F., "The Behavior of Stock Market Prices," *Journal of Business*, January 1965, *38* (1), 34–105.

Gompers, Paul and Josh Lerner, "An analysis of compensation in the U.S. venture capital partnership," *Journal of Financial Economics*, 1999, *51* (1), 3–44.

Grossman, Sanford, "On the Efficiency of Competitive Stock Markets Where Trades Have Diverse Information," *Journal of Finance*, may 1976, *31* (2), 573–585.

Grossman, Sanford J. and Joseph E. Stiglitz, "On the Impossibility of Informationally Efficient Markets," *The American Economic Review*, jun 1980, *70* (3), 393–408.

Harris, Robert S, Tim Jenkinson, Steven N Kaplan, and Ruediger Stucke, "Has persistence persisted in private equity? Evidence from buyout and venture capital funds," *Journal of Corporate Finance*, 2023, *81*, 102361.

Hochberg, Yael V, Alexander Ljungqvist, and Annette Vissing-Jørgensen, "Informational holdup and performance persistence in venture capital," *The Review of Financial Studies*, 2014, *27* (1), 102–152.

Kaplan, Steven N.. and Antoinette Schoar, "Private Equity Performance: Returns, Persistence, and Capital Flows," *The Journal of Finance*, 2005, *60* (4), 1791–1823.

Kaplan, Steven N and Berk A Sensoy, "Private equity performance: A survey," *Annual Review of Financial Economics*, 2015, *7* (1), 597–614.

Korteweg, Arthur, "Risk and return in private equity," in "Handbook of the economics of corporate finance," Vol. 1, Elsevier, 2023, pp. 343–418.

Lerner, Josh and Antoinette Schoar, "The illiquidity puzzle: theory and evidence from private equity," *Journal of Financial Economics*, 2004, *72* (1), 3–40.

_ , _ , and Jialan Wang, "Secrets of the Academy: The Drivers of University Endowment Success," *Journal of Economic Perspectives*, September 2008, *22* (3), 207–22.

_ , _ , and Wan Wongsunwai, "Smart institutions, foolish choices: The limited partner performance puzzle," *The Journal of Finance*, 2007, *62* (2), 731–764.

LeRoy, Stephen F., "Efficient Capital Markets: Comment," *Journal of Finance*, mar 1976, *31* (1), 139–141.

Lintner, John, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *The Review of Economics and Statistics*, 1965, *47* (1), 13–37.

Mittal, Vrinda, "Desperate capital breeds productivity loss: Evidence from public pension investments in private equity," *Available at SSRN 4283853*, 2024.

Mossin, Jan, "Equilibrium in a Capital Asset Market," *Econometrica*, 1966, *34* (4), 768–783.

Muth, John F., "Rational Expectations and the Theory of Price Movements," *Econometrica*, jul 1961, *29* (3), 315–335.

Sharpe, William F, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance*, 1964, *19* (3), 425–442.

Treynor, Jack, "Toward a Theory of the Market Value of Risky Assets," 1961.