Leveraging to Lend: A Theory of Lax Credit

PRELIMINARY. PLEASE DO NOT DISTRIBUTE.

Ulf Axelson and Igor Makarov*

March 8, 2026

ABSTRACT

A pervasive feature of financial intermediation is the use of third-party deal-by-deal capital in addition to balance sheet capital to finance investment activity. This paper studies how third-party capital and decentralized fundraising jointly shape competition, screening, and welfare in intermediated credit markets. We show that third-party capital serves a novel economic function: it allows informed intermediaries to separate screening decisions from surplus sharing, thereby intensifying competition and reducing the cost of capital to firms. Yet it also amplifies a dynamic adverse selection externality, resulting in excessively lax screening and overinvestment. Consequently, although entrepreneurs benefit from intermediary third-party leverage, restricting third-party financing can raise social welfare.

JEL Codes: D44, D82, G10, G20.

---

[*] London School of Economics. Houghton Street, London WC2A 2AE, UK. Ulf Axelson: u.axelson@lse.ac.uk. Igor Makarov: i.makarov@lse.ac.uk. We thank seminar participants at EDHEC, the LSE, Toulouse School of Economics, and the University of Essex for helpful comments.

1. Introduction

Financial intermediaries play a central role in channeling savings toward productive investment. At the core of this function is the intermediary's ability to overcome informational frictions—screening borrowers, conducting due diligence, and monitoring projects that uninformed investors cannot evaluate on their own. The economy-wide consequences of how intermediaries perform this role, and how their funding structures shape their incentives to perform it well, have been a subject of intense scrutiny since the Global Financial Crisis.

A striking feature of intermediated finance is the prevalence of third-party capital. Commercial and shadow banks routinely sell portions of originated loans to outside investors. Private equity sponsors finance leveraged buyouts with substantial deal-level debt, and venture capital investors increasingly rely on venture debt facilities. Lead arrangers in syndicated lending markets retain only a fraction of the loans they originate, distributing the remainder to participant banks. In each of these settings, the intermediary combines its own capital with third-party financing that is often raised on a deal-by-deal basis from investors who rely on the intermediary's screening decision.

A second salient feature is the decentralized nature of fundraising. Most primary capital markets—especially those catering to small and medium-sized firms—function as search markets in which firms seeking financing approach informed investors one at a time, moving on to the next if rejected. Intermediaries competing for deal flow in these markets need to offer financing contracts that reflect not only their private information about the project but also the adverse selection induced by the entrepreneur's interaction with other informed intermediaries. We show in Axelson and Makarov (2025) that this market structure, even in the absence of search and screening costs, hinders competition among investors.

This paper develops a unified framework for studying these two features jointly—third-party capital and decentralized fundraising. Unlike much of the existing literature, we do not assume that third-party capital is cheaper than the intermediary's own capital for unmodeled reasons. Rather, the relative cost of the two sources of financing is determined endogenously by the interplay of competition, information, and adverse selection. Our framework allows us to study why intermediaries choose the funding structures they do, how those choices interact with screening incentives and competition, what the welfare consequences of intermediary leverage are, and whether regulation that restricts third-party capital can improve outcomes.

We study a model in which an entrepreneur with a project of unknown quality approaches a sequence of informed investors. Investors compete through directed search by posting the terms on which they are willing to finance the entrepreneur and the entrepreneur visits investors based on these offers. Each investor, upon being approached, conducts due diligence at no cost, yielding a private signal about the project's quality. Based on this signal, the investor either agrees to finance the project at the posted terms or rejects it. The search continues until the entrepreneur either finds an investor who is willing to finance the project or runs out of options and abandons the project. We assume that the length of time an entrepreneur has been on the market seeking financing is observable to investors. This informational regime may be supported either because of formal market structures, such as a credit registry recording past credit checks on the entrepreneur, or informal mechanisms, where investors obtain information through word of mouth. Financing terms specify both the interest rate charged by the informed investor and the fraction of the investment that will be raised from competitive, uninformed capital markets.

We characterize the unique equilibrium of this sequential fundraising game and establish three main results. First, we show that third-party capital plays an important role: it allows to separate the problem of setting the optimal screening standards from the problem of surplus sharing between informed investors and the entrepreneur, and therefore, serves as a mechanism that improves competition among investors.

Without third-party capital, a lending contract between the entrepreneur and the informed investor takes the form of a simple debt contract, in which the interest rate charged by the investor affects both the informed investor's rent and the marginal project to be financed.[1] This creates a fundamental tension: a high interest rate increases the probability of financing but also increases informed investors' rent. As a result, as we show in Axelson and Makarov (2025), investors earn substantial rents, which remain bounded away from zero even as the number of investors grows without bound.

In contrast, third-party capital allows the entrepreneur and the visited investor to maximize their joint surplus in each financing round. The financing contract specifies a pair (δ, r^I), where δ is the fraction of the investment financed by uninformed capital markets and r^I is the informed investor's return on his retained portion $1-\delta$. When an investor accepts the project, he raises the fraction δ from uninformed investors at a competitive rate r^D that reflects the information revealed by the investor's willingness to finance the project. The total cost of capital to the entrepreneur is a weighted average of the informed and uninformed rates.

The interest rate r^I and the retained share $1-\delta$ play two distinct roles in the

---

[1] We show that in our model, restricting the analysis to a simple interest rate contract is without loss of generality.

financing contract. The interest rate r^I determines which projects are going to be financed, while the fraction δ determines the split of surplus between the entrepreneur and the informed investor. Since the informed investor only supplies $1-\delta$ share of the capital, it is the interest rate r^I he earns on his capital that governs his financing decision.

Uninformed investors take the financing decision as given. Since they are competitive, they earn no rents, unlike informed investors who earn inframarginal rents. Raising capital from uninformed investors allows the informed investor to commit to dilute his informational rents without distorting screening incentives, and therefore be more competitive in the market.

We show that as the number of investors increases, aggregate investor rents vanish in contrast to the case without third-party capital, where rents stay bounded away from zero. We also show that screening standards rise and the intermediary's retained share increases as the entrepreneur progresses through the sequence of investors, reflecting the progressive deterioration of perceived credit quality and reduced competition from remaining investors.

Practitioners often cite that they need to deliver a return above a certain hurdle rate to their investors. We do not impose this constraint on the intermediary in our model, but by using third-party capital, intermediaries are able to reduce their rent while maintaining a high return on their capital.

It is worth emphasizing that the competitive advantage of tapping third-party capital in our model does not arise from third-party investors having inherently lower funding costs. Different from the models where intermediaries monitor firms, it is important that the amount of third-party financing is committed before intermediaries observe their signals. If the intermediary were to sell loans after financing the project, it would be unable to do so because of standard no-trade arguments.

In any given financing round, investors who can commit to use the third-party capital will be able to outbid investors who could only use their own capital. Once some investors gain access to third-party capital, others face a stark choice: secure outside funding themselves or be shut out of deals.

This logic echos a famous remark by Citigroup's then-CEO Chuck Prince, who on the eve of the financial crisis told the Financial Times: "As long as the music is playing, you've got to get up and dance." He subsequently clarified in his testimony before the U.S. Congress that his qoute "was about leveraged lending. It had nothing to do with the mortgage business. It had nothing to do with the CDO business." While his remark is often cited as emblematic of the reckless risk-taking that precipitated the Global Financial Crisis, Prince was articulating precisely the competitive pressure our model formalizes: banks that could tap third-party capital were winning deal flow, and any institution that refused to match them risked losing its franchise.

Our second main result shows that despite the fact that each bilateral negotiation maximizes stage surplus, the overall equilibrium features overly lax credit and investment relative to the social optimum and the solution without third-party capital. This makes the parallel to Prince's remark and the global financial crisis even more complete.

The source of overinvestment in our model is the decentralized nature of the fundraising process. While the posted terms are observable to all market participants, the entrepreneur and the visited investor are free to renegotiate them. This renegotiation is private and therefore unobservable to other investors. As a result, other investors' beliefs are formed based on equilibrium expectations rather than actual contract terms. Since the realized contract does not affect remaining investors' beliefs about project quality, the entrepreneur does not internalize the impact of her choices across rounds. In particular, she does not internalize that choosing a contract with a low screening standard in the current round worsens the adverse selection faced by future investors. The result is lending standards that are optimal in each financing round given investors' beliefs but collectively too lax, leading to excessive financing of bad projects.

The dynamic adverse selection externality is also present when investors lack access to third-party capital. However, in that case, overinvestment is mitigated by the fact that increasing the probability of financing requires the entrepreneur to leave rents to investors, which is costly. We show that the entrepreneur is better off when informed investors have access to third-party capital, but that the resulting equilibrium features lower social surplus and greater overinvestment.

Finally, we show that regulation limiting third-party financing can be welfare-improving. With restricted third-party capital, screening standards at the stage level are set above the efficient threshold, but because of the dynamic adverse selection externality, this has a positive effect on overall welfare.

Our paper is related to several bodies of work. First, it contributes to the literature on the effect of loan sales on the supply of credit. Many papers in this literature—for example, Besanko and Kanatas (1993), Holmstrom and Tirole (1997), and Parlour and Plantin (2008)—assume that banks have an advantage over non-bank lenders in monitoring borrowers and reducing moral hazard. As in our paper, loan sales to outside investors in these models can lead to worse performance of originated loans. However, the economic mechanisms differ. Banks in these models are perfectly competitive and earn no rents; loan sales reduce their incentives to monitor firms after origination, exacerbating the moral hazard problem and worsening loan performance ex post. In contrast, in our paper intermediaries are imperfectly competitive and earn informational rents. The use of third-party capital allows intermediaries to commit to reducing these rents and thereby compete more aggressively for deal flow. Because the entrepreneur does not internalize the impact of her choices across rounds, the resulting increase in competitiveness leads to overinvestment.

A related literature studies optimal contracting with loan sales and moral hazard Gorton and Pennacchi (1995), Hartman-Glaser et al. (2012), Vanasco (2017).

Our paper is also related to Inderst and Mueller (2006) and Daley et al. (2020), who consider settings in which banks have access to a screening technology. However, their focus differs from ours. Inderst and Mueller (2006) examine the role of security design when lenders make inefficient accept-or-reject decisions following screening. Daley et al. (2020) explore the effect of credit ratings on loan origination.

2. Setup

The model has three sets of risk-neutral agents. First, a firm that seeks financing to start a new project. Second, a set $\{1, ..., N\}$ of expertise investors, who can screen the project by observing a private signal before deciding whether to extend financing. Third, a set of competitive, uninformed investors who provide external funding at a competitive rate, which we refer to as the capital market. In our main analysis, we refer to the firm as the entrepreneur and assume that the firm has no other assets or financial resources.

The project requires one unit of investment, and can be of two types: good ($\theta = G$) or bad ($\theta = B$). The good project pays $1 + X$, while the bad projects returns 0. The type of a particular project is initially unknown with investors sharing the same unconditional prior $P(G)$ that the project is good.

2.1. Fundraising Market

Fundraising takes place in a sequential market, in which the entrepreneur visits investors in sequence until she either gets financed or runs out of investors to approach. Investors observe the entrepreneur's time on the market t, or equivalently, the number of times $t - 1$ the entrepreneur has been rejected. Investors compete via directed search: At the beginning of each round t, before observing their signals, the $N - t + 1$ remaining investors (since one investor departs each round) publicly post the terms on which they are willing to finance the entrepreneur. The entrepreneur then picks which investor to approach. The chosen investor then performs his screening test; if the entrepreneur is accepted, the firm is financed at the agreed terms and the game ends. If the entrepreneur is rejected, the investor leaves the game and the entrepreneur moves to the next round.[2]

A posted financing contract from investor i in round t is a pair $\{\delta_{i,t}, r^I_{i,t}\}$, where $\delta_{i,t}$ is the fraction of the project that will be financed by competitive, uninformed investors, and $r^I_{i,t}$ is the intermediary's required net return on his own investment $1-\delta_{i,t}$ if the project is good. The investor prearranges the third-party funding: before observing his signal, he secures a commitment from uninformed investors to provide $\delta_{i,t}$ at an interest rate $r^D_{i,t}$, contingent on the project being financed. If the investor's signal leads him to approve the project, the prearranged third-party funding is drawn down at the agreed rate. The total interest rate paid by the entrepreneur if the project is good is then given by

$$r_{i,t} = (1-\delta_{i,t})r^I_{i,t} + \delta_{i,t}r^D_{i,t}. \tag{1}$$

The capital $1-\delta$ financed directly by the informed investor could be either his own wealth, or ex ante balance sheet capital raised from competitive uninformed investors at the start of the game and backed by the whole pool of investments made by the intermediary. For clarity of exposition in the main analysis below, we will assume that the informed investors have deep pockets and so can finance their activity out of their own wealth. However, as we discuss in Section 6, it makes no difference to our results whether the informed investor instead acts as an intermediary by raising ex ante balance sheet capital from outside investors. What is important is the distinction between third party deal-by-deal capital, which crucially will not be perfectly aligned with the pay off of the informed investor, and capital that is perfectly aligned with the informed investor. In Section 6 we show that by issuing a combination of debt and equity balance sheet capital ex ante, the intermediary can raise financing at a competitive rate without creating any conflicts of interest.

We show below that it is without loss of generality to focus on the simple financing contracts described above rather than more general mechanisms. Table 1 maps the stylized contract to two canonical real-world financing arrangements.

To ensure posted terms are credible, we require that financing contracts are renego-

---

[2] In particular, we do not allow the entrepreneur to "shop around" by presenting an accepted agreement to other investors in the hope of obtaining better financing terms. This assumption of exclusivity is important and represents one of the defining properties of sequential markets. Since we abstract from any search costs and costs associated with generating information, allowing the entrepreneur to take accepted offers to other investors without forfeiting them would make the resulting mechanism resemble a competitive centralized market place, which we study in detail in Axelson and Makarov (2023).

tiation proof. Once the entrepreneur visits an investor posting $\{\delta, r^I\}$, but before the investor performs his screening test, the investor can propose a new contract $\{\delta', r'^I\}$. The entrepreneur can then choose to stick with the old contract or accept the new contract. Any such renegotiation is not observed by the other investors. A contract is renegotiation proof if there is no such contract that makes all parties strictly better off.

Table 1: *Mapping the Stylized Contract to Real-World Financing Arrangements*

Model	**Private Equity (LBO)**	**Syndicated Lending**
δ	LP capital + deal-level debt	Syndicated portion (85–95%)
$1-\delta$	GP commitment (1–5%)	Lead arranger's hold level (5–15%)
r^I	Carried interest (20%) + management fee	Yield on retained portion + arrangement fees
r^D	LP net return / cost of debt	Participant banks' yield

Notes: The table maps the stylized financing contract $\{\delta, r^I\}$ to two canonical real-world arrangements. In each setting, δ represents the fraction financed by relatively uninformed investors, $1-\delta$ is the intermediary's retained "skin in the game," r^I is the intermediary's requiredreturn on the retained portion, and r^D is the competitive return paid to outside investors.

2.2. Signals

Investors have free access to a screening technology. When an investor makes an investigation, he gets a private signal that is informative about the project type. Conditional on the project type θ, signals are drawn identically and independently on $[0,1]$ with differentiable conditional densities $f_G(s)$ and $f_B(s)$ satisfying the strict maximum likelihood ratio property (MLRP):

Assumption 1:

$$\forall s > s', \quad \frac{f_G(s)}{f_B(s)} > \frac{f_G(s')}{f_B(s')}.$$

Assumption 1 ensures that higher signals are better news than lower signals.[3] We also assume that $f_B(1) > 0$, and that the likelihood ratio $f_G(1)/f_B(1)$ at the most

---

[3] The assumption of strict MLRP is for simplicity. It allows us to focus on pure strategy equilibria. All results go through under the weaker assumptions that signals satisfy weak MLRP: $\forall s \geq s',\ f_G(s)/f_B(s) \geq f_G(s')/f_B(s')$.

optimistic signal realization $s = 1$ is bounded and equal to $\lambda < \infty$. These assumptions ensure that the observation of a single signal can never rule out the possibility of the project being bad, while an observer of all signals will be able to learn the project type perfectly as the number of investors goes to infinity.

We also impose two assumptions that ensure the investment decision is non-trivial for each investor. First, to exclude the trivial scenario when the project is never financed, we assume that the project is positive NPV conditional on the top signal of a single investor:

Assumption 2: $P(G|S = 1)X > P(B|S = 1)$.

Second, we assume that the project is negative NPV ex ante:

Assumption 3: $P(G)X < P(B)$.

Assumption 3 ensures the entrepreneur needs to raise financing from informed investors rather than go to the uninformed capital market, and is not essential for our results. Assumptions 2 and 3 together imply the existence of $\underline{s} \in (0, 1)$ such that the project breaks even at $S = \underline{s}$.

Finally, we assume that any private information the entrepreneur herself may have is independent of investor signals conditional on the true type of the project. As we explain below, under this assumption private information held by the entrepreneur does not affect our analysis—the entrepreneur will always act *as if* the project is good.

2.3. Equilibrium

We look for a symmetric equilibrium in which at the beginning of each round t, remaining investors have common ex ante beliefs about project quality and post the same contract menu $\{\delta_t, r_t^I\}$.

Investor's common beliefs at the beginning of a round t, before their private signals have been observed, are described by the likelihood ratio $z_{t-1} = \frac{P(G|\text{reached round } t)}{P(B|\text{reached round } t)}$ of project type for an entrepreneur who has failed to get financing in the previous $t-1$ rounds. The conditional probabilities of the project being of type G or B given z_{t-1} are:

$$P(G|z_{t-1}) = \frac{z_{t-1}}{1 + z_{t-1}}, \qquad P(B|z_{t-1}) = \frac{1}{1 + z_{t-1}}. \tag{2}$$

Given z_{t-1}, investor i's strategy in round t is a choice of whether to participate or wait until the next round, and if he participates, the investor's financing fraction

$1 - \delta_t \in [0, 1]$ and interest rate $r_t^I \in [1, X]$, and a signal threshold $\hat{s}_t \in [0, 1]$ such that if visited, the investor will extend financing when $S_i \geq \hat{s}_t$.[4]

The entrepreneur's strategy is a choice of which of the remaining investors to visit based on their postings, and if the visit results in a financing offer (possibly after a secret renegotiation), whether to accept this offer or not.

The equilibrium requires that: investors screen optimally given the terms they post, the capital market is competitive, beliefs are Bayesian, and posted terms are viable and renegotiation proof. Formally, we define a symmetric equilibrium as a set $\{\delta_t, r_t^I, r_t^D, \hat{s}_t, z_{t-1}\}_{t=1}^N$ such that all investors follow the same strategies and have the same beliefs, and such that the following holds:

1. Individual rationality at visits: Financing thresholds $\hat{s}_t$ maximize a visited investor's profits given r_t^I, δ_t and z_{t-1}
2. Uninformed investors in the capital market break even at interest rate r_t^D given r_t^I, δ_t and z_{t-1}.
3. Belief consistency: Beliefs z_{t-1} about credit quality are consistently updated using Bayes' rule on the equilibrium path.
4. Individual rationality at the posting stage:
   (a) Investors prefer posting a renegotiation-proof contract (r_t^I, δ_t) over waiting for the next round, and the entrepreneur prefers to accept offer (r_t^I, δ_t) over waiting for the next round.
   (b) There is no alternative renegotiation-proof posting $({r'}_t^I, \delta'_t) \neq (r_t^I, \delta_t)$ such that the entrepreneur prefers to visit the deviating investor.

3. Stage game

We start by considering a particular round t before analyzing the full dynamic equilibrium. We first demonstrate that any visited investor will indeed use a threshold strategy under which he finances the project if and only if his private signal is sufficiently high. We then characterize the set of renegotiation-proof contracts to which any posted contract in round t must belong. We show that within any round, the renegotiation-proof contract sets the screening threshold to maximize net joint surplus, but leaves the split of surplus between the investor and the entrepreneur—determined by the fraction of third-party capital δ_t—to be determined by the equilibrium.

---

[4] We show in the next subsection that it is indeed optimal for investors to use cut-off strategies when choosing whether to finance or not.

3.1. Threshold strategies

Consider an investor who is visited in round t and a contract $\{r_t^I, \delta_t\}$ that the entrepreneur and investor have agreed upon, possibly after renegotiation. After observing his signal $S = s$, the investor will agree to finance the project if

$$P(G|z_{t-1}, S = s)r_t^I - P(B|z_{t-1}, S = s) \geq 0, \tag{3}$$

which, using Bayes' law and rearranging, can be written as

$$\frac{f_G(s)}{f_B(s)} \geq \frac{1}{z_{t-1}r_t^I}. \tag{4}$$

The right-hand side of Equation (4) is the expected losses to gains ratio before the signal is observed. The likelihood ratio $f_G(s)/f_B(s)$ for the private signal must exceed this threshold for the investor to break even.

Since the likelihood ratio $\frac{f_G(s)}{f_B(s)}$ is strictly increasing by MLRP, the investor will finance the project for all signals above a threshold $\hat{s}_t$ defined by the break-even condition:

$$\frac{f_G(\hat{s}_t)}{f_B(\hat{s}_t)} = \min\left[\frac{1}{z_{t-1}r_t^I}, \lambda\right]. \tag{5}$$

The case $\frac{f_G(\hat{s}_t)}{f_B(\hat{s}_t)} = \lambda$ corresponds to setting the threshold at the highest signal $S = 1$, so that no financing takes place. As shown below, this can occur if a sufficiently large number of rejections reduces credit quality z_{t-1} to the point where the project is not positive NPV even conditional on the highest possible signal $S = 1$.

Having established that the investor uses a threshold rule, we can equivalently parametrize the contract by the threshold $\hat{s}_t$ itself, expressing all interest rates as functions of it. This parametrization simplifies the analysis of the renegotiation-proof contract set. The informed investor's required interest rate $r_t^I(\hat{s}_t)$ necessary to implement threshold $\hat{s}_t$ is given by inverting Equation (5):

$$r_t^I(\hat{s}_t) = \frac{1}{z_{t-1}}\frac{f_B(\hat{s}_t)}{f_G(\hat{s}_t)}. \tag{6}$$

The interest rate r_t^D demanded by uninformed investors who observe that the informed investor has accepted the project at $r_t^I(\hat{s}_t)$ is given by their break-even condition:

$$P(G|z_{t-1}, S \geq \hat{s}_t) r_t^D(\hat{s}_t) - P(B|z_{t-1}, S \geq \hat{s}_t) = 0, \tag{7}$$

which using Bayes' law can be written as

$$r_t^D(\hat{s}_t) = \frac{1}{z_{t-1}} \frac{1 - F_B(\hat{s}_t)}{1 - F_G(\hat{s}_t)}. \tag{8}$$

MLRP implies that $r_t^D(\hat{s}_t) < r_t^I(\hat{s}_t)$, so the informed investor earns a mark-up spread reflecting his informational rents. The investor return $r_t^I(\hat{s}_t)$ is set such that the informed investor breaks even at the marginal signal $S = \hat{s}_t$, and makes inframarginal rents when $S > \hat{s}_t$. Uninformed investors break even on average when the project is financed, but make expected losses for marginal signals.

The total promised interest rate $r_t(\delta_t, \hat{s}_t)$ paid by the entrepreneur given the financing threshold $\hat{s}_t$ is

$$r_t(\delta_t, \hat{s}_t) = \delta_t r_t^D(\hat{s}_t) + (1 - \delta_t) r_t^I(\hat{s}_t). \tag{9}$$

The total interest rate $r_t(\delta_t, \hat{s}_t)$ decreases in the amount of third-party capital δ_t and in the financing threshold $\hat{s}_t$.

3.2. Renegotiation-proof contracts

We now derive conditions for a contract $\{\delta_t, r_t^I(\hat{s}_t)\}$ to be renegotiation proof. Note that since the $N - t$ remaining investors after round t do not observe the terms of a secretly renegotiated contract, their aftermarket beliefs z_t about credit quality do not depend directly on any such renegotiation. For a contract to be renegotiation proof, there must therefore be no alternative contract that makes both the investor and the entrepreneur better off, holding the aftermarket beliefs z_t fixed.

Our analysis is simplified by the fact that the entrepreneur receives no payoff if the project is bad. Denote the entrepreneur's utility conditional on the project being good by $v_t^G(\delta_t, \hat{s}_t)$.[5] This utility satisfies

$$v_t^G(\delta_t, \hat{s}_t) = (1 - F_G(\hat{s}_t))(X - r_t(\delta_t, \hat{s}_t)) + F_G(\hat{s}_t)V_{t+1}^G, \tag{10}$$

where $V_{t+1}^G(z_t)$ is the entrepreneur's continuation value if rejected, conditional on the project being good, which depends only on the aftermarket beliefs z_t.

Denote the investor's expected utility from a contract $\{\delta_t, r_t^I(\hat{s}_t)\}$ by $R_t(\delta_t, \hat{s}_t)$. It satisfies

$$\begin{aligned} R_t(\delta_t, \hat{s}_t) &= (1 - \delta_t)\left[P(G|z_{t-1})(1 - F_G(\hat{s}_t))r_t^I(\hat{s}_t) - P(B|z_{t-1})(1 - F_B(\hat{s}_t))\right] \\ &= (1 - \delta_t)P(G|z_{t-1})(1 - F_G(\hat{s}_t))\left(r_t^I(\hat{s}_t) - r_t^D(\hat{s}_t)\right) = (1 - \delta_t)P(B|z_{t-1})\phi(\hat{s}_t), \end{aligned} \tag{11}$$

where $\phi(\hat{s}_t)$ is the part of the investor's informational rent that depends only on the threshold $\hat{s}_t$:

$$\phi(\hat{s}_t) = (1 - F_G(\hat{s}_t))\frac{f_B(\hat{s}_t)}{f_G(\hat{s}_t)} - (1 - F_B(\hat{s}_t)). \tag{12}$$

Intuitively, $\phi(\hat{s}_t)$ measures the gap between the informed investor's expected return (which is computed at the marginal signal) and the uninformed market's expected return (which averages over all accepted signals), which reflects the informed investor's inframarginal information rents.

A contract $\{\delta_t, r_t^I(\hat{s}_t)\}$ with the associated threshold $\hat{s}_t$ is renegotiation proof if it maximizes the entrepreneur's payoff among all contracts that the investor would accept. Formally, it solves the following problem:

$$\begin{aligned} V_t^G \equiv &\max_{\delta_t, \hat{s}_t} v_t^G(\delta_t, \hat{s}_t) \\ &\text{s.t. } R_t(\delta_t, \hat{s}_t) \geq R_t. \end{aligned}$$

where R_t is the investor's reservation utility in period t. Substituting the expression for $r_t(\delta_t, \hat{s}_t)$ from Equations (6), (8), and (9), together with the expression for $R_t(\delta_t, \hat{s}_t)$

---

[5] The entrepreneur's belief about credit quality z_{t-1}^E may, in principle, differ from the investors' beliefs z_{t-1}. But since neither the entrepreneur's prior $P(G|z_{t-1}^E)$ nor the investor's prior $P(B|z_{t-1})$ are affected by the contract terms, this difference does not affect the analysis—the entrepreneur will always act *as if* the project is good.

from Equation (11), we can rewrite this maximization problem as

$$V_t^G = \max_{\delta_t, \hat{s}_t} (1 - F_G(\hat{s}_t))(X - V_{t+1}^G) - \frac{1}{z_{t-1}}(1 - F_B(\hat{s}_t)) - \frac{1}{z_{t-1}}(1 - \delta_t)\phi(\hat{s}_t) \tag{13}$$

$$\text{s.t.} \quad \frac{1}{z_{t-1}}(1 - \delta_t)\phi(\hat{s}_t) \geq \frac{R_t}{P(G|z_{t-1})}. \tag{14}$$

Note that the investor's participation constraint must bind in any solution to this problem: if it did not, the entrepreneur could increase δ_t, thereby raising the amount of third-party capital and increasing her surplus. Multiplying the maximand and the participation constraint by $P(G|z_{t-1})$, we get that a renegotiation-proof contract must maximize the net joint surplus, while providing the investor with his reservation utility:

$$\max_{\delta_t, \hat{s}_t} P(G|z_{t-1})(1 - F_G(\hat{s}_t))(X - V_{t+1}^G) - P(B|z_{t-1})(1 - F_B(\hat{s}_t)) \tag{15}$$

$$\text{s.t.} \quad P(B|z_{t-1})(1 - \delta_t)\phi(\hat{s}_t) = R_t. \tag{16}$$

Since the net joint surplus does not depend on δ_t, a renegotiation-proof contract sets screening threshold $\hat{s}_t$ to maximize the net joint surplus, which occurs when the project's payoff net of the entrepreneur's continuation value just breaks even given investor beliefs. The associated interest rate $r_t^I(\hat{s}_t)$ is given by $r_t^I = X - V_{t+1}^G$. The following lemma characterizes the renegotiation-proof contract.

Lemma 1: *Given investor beliefs z_{t-1} and z_t, a contract $\{\delta_t, r_t^I\}$ is renegotiation proof if and only if*

$$r_t^I = X - V_{t+1}^G,$$

so that the associated threshold s_t^* satisfies

$$\frac{f_G(s_t^*)}{f_B(s_t^*)} = \frac{1}{z_{t-1}(X - V_{t+1}^G)}. \tag{17}$$

Renegotiation proofness itself does not restrict how this surplus is divided between the investor and the entrepreneur; the split is determined by the fraction of third-party capital δ_t. Given investors' reservation utility R_t, δ_t is set so that the investor's participation constraint binds:

$$1 - \delta_t = \frac{R_t}{P(B|z_{t-1})\phi(\hat{s}_t)}. \tag{18}$$

In the next section, we determine R_t, z_{t-1}, and the continuation value V_{t+1}^G endogenously in equilibrium.

4. Full dynamic equilibrium

We now merge the stage-game results into a full dynamic equilibrium. In equilibrium, the entrepreneur's continuation value V_t^G becomes endogenous and depends on the beliefs of unvisited investors, summarized by z_{t-1}. The analysis has three steps. First, we specify how beliefs evolve across rounds using Bayes' rule. Second, we solve the last round in closed form. Third, we use backward induction to characterize earlier rounds, showing that the equilibrium is unique and that screening becomes progressively stricter as competition intensifies.

4.1. Belief consistency

From Bayes' law, consistency of beliefs requires that if $\hat{s}_t$ is an equilibrium financing threshold, the common equilibrium prior z_t of unvisited investors at the start of period $t+1$ evolves recursively as

$$z_t = z_{t-1} \frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)}. \tag{19}$$

Equation (19) captures how unvisited investors update their beliefs when the project remains unfinanced after period t, given their belief that the investor approached in period t offers financing if and only if $S \geq \hat{s}_t$, and the entrepreneur accepts financing if it is offered. Equation (19) shows that the perceived credit quality z_t declines as the entrepreneur remains in the market longer, since $F_G(s)/F_B(s) < 1$ by MLRP.

In general, the perceived credit quality could become so low that the project can no longer be financed given the investors screening technology. To streamline the exposition, we impose an additional regularity condition on the signal distribution that ensures that all informed investor are visited in equilibrium. This assumption is also sufficient to guarantee the uniqueness of equilibrium. Define the function H as

$$H(s) = \frac{f_B(s)}{f_G(s)} \frac{F_G(s)}{F_B(s)}. \tag{20}$$

Assumption 4: *The function H strictly decreases on the interval* $[0, 1]$.

4.2. Last Round

In the last round $t = N$, provided that $\lambda z_{N-1} X > 1$,, i.e., financing is possible for sufficiently high signals, the last investor faces no competition and extracts all surplus, by setting $\delta_N = 0$ and $r_N^I = X$. This leaves the entrepreneur with zero continuation value: $V_N^G(z_{N-1}) = 0$. The investor's utility from this contract is given by

$$R_N = P(G|z_{N-1})(1 - F_G(\hat{s}_N))X - P(B|z_{N-1})(1 - F_B(\hat{s}_N)) = \frac{\phi(\hat{s}_N)}{1 + z_{N-1}}, \tag{21}$$

where the function ϕ is defined in Equation (12) and the screening threshold $\hat{s}_N$ solves

$$\frac{f_G(\hat{s}_N)}{f_B(\hat{s}_N)} = \frac{1}{z_{N-1}X}. \tag{22}$$

4.3. Round $t < N$

For $t < N$, competition among investors shifts the bargaining power in favor of the entrepreneur. Using the expression for the evolution of beliefs (19), we can write the first-order condition (17) for the entrepreneur's choice of screening threshold $\hat{s}_t$ as

$$H(\hat{s}_t) = z_t \left(X - V_{t+1}^G(z_t) \right), \tag{23}$$

$$z_{t-1} = z_t \frac{F_B(\hat{s}_t)}{F_G(\hat{s}_t)}. \tag{24}$$

Using Equation (13), the evolution of the entrepreneur's reservation utility $V_t^G(z_{t-1})$ is given by

$$X - V_t^G(z_{t-1}) = F_G(\hat{s}_t) \left(X - V_{t+1}^G(z_t) \right) + \frac{1}{z_{t-1}}(1 - F_B(\hat{s}_t)) + \frac{R_t}{P(G|z_{t-1})}, \tag{25}$$

where R_t is the surplus that needs to be given to an investor conditional on reaching round t. Since the investor participation constraint binds in every period, investors earn the same expected profits across all rounds.

The probability of reaching round $t+1$, conditional on reaching round t is equal to

$$P(G|z_{t-1})F_G(\hat{s}_t) + P(B|z_{t-1})F_B(\hat{s}_t) = F_B(\hat{s}_t)\frac{1 + z_t}{1 + z_{t-1}}, \tag{26}$$

where we used Equations (2) and (19). Therefore,

$$R_t = F_B(\hat{s}_t)\frac{1 + z_t}{1 + z_{t-1}} R_{t+1}. \tag{27}$$

Using equations (23), (25), and (27), the equilibrium can be solved using backward induction, starting from the last period. The above equations define a map from z_{N-1} to z_0. We show in the proof of Proposition 1 that this map is strictly monotone and defines a unique equilibrium.

Proposition 1: *Suppose Assumptions 1-4 hold. Then, there exists a unique equilibrium. The entrepreneur visits all informed investors. Screening thresholds* $\hat{s}_t$ *and the fraction of the informed financing* $1-\delta_t$ *increase with the number of visited investors and go to 1 as* N *goes to infinity. The aggregate investor rent goes to zero as* N *goes to infinity.*

Proof: See the Appendix.

Using the expression for the retained capital (18) and the connection between investors surplus in periods t and $t+1$ (27), we can express the retained capital in period t as a function of the retained capital in period $t+1$:

$$1-\delta_t = (1-\delta_{t+1})F_B(\hat{s}_t)\frac{\phi(\hat{s}_{t+1})}{\phi(\hat{s}_t)}. \tag{28}$$

Since $F_B(\hat{s}_t) < 1$ and $\phi(\hat{s}_{t+1})/\phi(\hat{s}_t) < 1$, the retained capital in period t is less than the retained capital in period $t+1$. This happens because of two effects: First, because the probability of reaching round $t+1$ is lower than the probability of reaching round t, the total rent that the investor gets in period $t+1$ is higher than the rent that the investor gets in period t. Second, because screening thresholds increase with t, informational rent in period $t+1$ is lower than the informational rent in period t. To compensate for the lower informational rent, but higher total rent, investors retain more capital in period $t+1$.

4.4. Overinvestment

Social surplus is determined by which projects get financed, which in turn depends on screening thresholds $\{\hat{s}_t\}_{t=1}^N$ (which happens when at least one of the signals is above the threshold). Given thresholds $\{\hat{s}_t\}_{t=1}^N$, a good and bad project get financed with probability $P(G|z_0)\left(1-\prod_{t=1}^N F_G(\hat{s}_t)\right)$ and $P(B|z_0)\left(1-\prod_{t=1}^N F_B(\hat{s}_t)\right)$, respectively.

Therefore, social social surplus is given by

$$W(\{\hat{s}_t\}_{t=1}^N, z_0) = P(G|z_0)\left(1-\prod_{t=1}^N F_G(\hat{s}_t)\right)X - P(B|z_0)\left(1-\prod_{t=1}^N F_B(\hat{s}_t)\right). \tag{29}$$

Consider a social planner who maximizes welfare (29) by setting the thresholds $\{\hat{s}_t\}_{t=1}^N$. We show in Axelson and Makarov (2025) that under Assumption 4, the socially optimal screening policy is to use the same screening threshold in all rounds. The optimal screening threshold is where the project just breaks even conditional on the highest signal among investors being at the threshold:

$$P(G|\max_{t\leq N} S_t = \hat{s}_P)X - P(B|\max_{t\leq N} S_t = \hat{s}_P) = 0. \tag{30}$$

In contrast, Proposition 1 shows that in equilibrium, screening thresholds $\hat{s}_t$ increase with the number of rounds. Our main result in this section is to show that the market equilibrium features overinvestment:

Proposition 2: *There is overinvestment for any $N > 1$ relative to the social planner solution.*

Proof: See the Appendix.

Even though the entrepreneur and investors maximizes social surplus in each financing round, this does not imply the efficiency of the full dynamic equilibrium. When agreeing on the round-t contract, the entrepreneur treats her continuation value V_{t+1} as given and independent of her choice of which investor to approach.

If investors in subsequent rounds could observe the renegotiated contracts in earlier rounds, the entrepreneur would internalize the impact of her choices on future adverse selection. In that case, she would opt for contracts that imply higher screening thresholds. However, since investors do not directly observe the renegotiated contracts, their beliefs are formed based on equilibrium expectations. As a result, the entrepreneur and investors settle for lower screening thresholds, which are associated with a higher probability of acceptance.

5. Restricted third-party capital

To understand the role of third-party capital, suppose third-party capital is restricted such that δ_t is constrained to be below $\bar{\delta}_t < 1$, where $\bar{\delta}_t < 1$. The case of no third-party financing corresponds to $\bar{\delta}_t = 0$.

We show two properties of renegotiation-proof contracts when third-party financing is restricted: First, thresholds will generally be above the stage surplus-maximizing threshold s_t^*—i.e., there is underinvestment relative to the stage surplus-maximizing contract. Second, investor may earn rents above their reservation utility.

Going through the same steps as in the analysis in Section 4, a contract $\{r_t^I, \delta_t\}$ with the associated threshold $\hat{s}_t$ is renegotiation proof if $\{\hat{s}_t, \delta_t\}$ solves

$$\max_{\delta_t, \hat{s}_t} \, (1 - F_G(\hat{s}_t))(X - V_{t+1}^G) - \frac{1}{z_{t-1}}(1 - F_B(\hat{s}_t)) - \frac{1}{z_{t-1}}(1 - \delta_t)\phi(\hat{s}_t) \tag{31}$$

$$\text{s.t.} \quad \frac{1}{z_{t-1}}(1 - \delta_t)\phi(\hat{s}_t) \geq \frac{R_t}{P(G|z_{t-1})}, \tag{32}$$

$$\delta_t \; \leq \bar{\delta}_t. \tag{33}$$

Note that in the absence of the investor's participation constraint, the entrepreneur always prefer to set $\delta_t = \bar{\delta}_t$ to minimize investor rents. Denote by r_t^E the interest rate that maximizes the entrepreneur's payoff when $\delta_t = \bar{\delta}_t$. The associated threshold $\hat{s}_t(r_t^E)$ solves

$$\hat{s}_t(r_t^E) = \arg\max_{\hat{s}_t} \, (1 - F_G(\hat{s}_t))(X - V_{t+1}^G) - \frac{1}{z_{t-1}}(1 - F_B(\hat{s}_t)) - \frac{1}{z_{t-1}}(1 - \bar{\delta}_t)\phi(\hat{s}_t). \tag{34}$$

Comparing Equation (34) with Equation (17), we see that when $\bar{\delta}_t < 1$, so that third-party capital is constrained, the entrepreneur's preferred threshold $\hat{s}_t(r_t^E)$ is strictly higher than the stage surplus-maximizing threshold $\hat{s}_t^*$, or, equivalently, $r_t^E < X - V_{t+1}^G$.

After using the maximum amount of third-party financing to reduce investor rents, the entrepreneur prefers to reduce investor rents further by lowering the interest rate below $X - V_{t+1}^G$, despite the resulting loss of surplus from raising the financing threshold. This adjustment is costly, however. As a result, even if the investor's participation constraint is slack at $\delta_t = \bar{\delta}_t$ and $\hat{s}_t = \hat{s}_t(r_t^E)$, the entrepreneur will not want to reduce the rate below r_t^E.

Thus, in contrast to the case with $\bar{\delta}_t = 1$, the minimum investor rent $\underline{R}_t$ in a renegotiation-proof contract is strictly positive and given by

$$\underline{R}_t = (1 - \bar{\delta}_t)P(B|z_{t-1})\phi(\hat{s}_t(r_t^E)). \tag{35}$$

The following lemma characterizes renegotiation proof contracts when third-party capital is restricted:

Lemma 2: *Suppose third-party capital is restricted such that $\delta_t \leq \bar{\delta}_t < 1$. Renegotiation-proof contracts belong to the following set:*

- $r_t^I = X - V_{t+1}^G$ *and* $\delta_t \in [0, \bar{\delta}_t]$, *with* $\hat{s}_t = \hat{s}_t^*$ *and* $R_t = (1 - \delta_t)P(B|z_{t-1})\phi(\hat{s}_t^*)$

- $\delta_t = \bar{\delta}_t$ *and* $r_t^I \in [r_t^E, X - V_{t+1}^G)$, *with corresponding threshold* $\hat{s}_t \in [\hat{s}_t(r_t^E), s_t^*)$ *and investor rent* $R_t \in [\underline{R}_t, (1-\bar{\delta}_t)P(B|z_{t-1})\phi(s_t^*))$.

With restrictions on third-party financing, the amount of stage surplus is tied to the amount of rents captured by the investor. In the extreme case, where no third party financing is available ($\bar{\delta}_t = 0$), stage surplus can only be maximized if the investor is given all the rents. As a consequence, if the entrepreneur has enough bargaining power (e.g. through competition), she prefers a contract with a lower investor interest rate and hence underinvestment ($\hat{s}_t < \hat{s}_t^*$). But relative to using third-party capital, this is a costly way to reduce rents, so the entrepreneur always prefers to give away some surplus to investors.

5.1. General mechanisms

We have focused on simple interest rate contracts in the above analysis for ease of exposition. In principle, the investor could propose a more general mechanism at the renegotiation stage, and would do so if that improves on the original contract. In particular, a general mechanism consists of an incentive-compatible menu of contracts that the investor would pick from after observing his signal, where each contract stipulates payments contingent on both success, failure, and no investment, and can include randomization on whether the project is financed or not. It is easy to see that with third-party financing, there is no loss of generality to focus on simple interest rate contracts, since these contracts maximize stage surplus and can split surplus arbitrarily.

With restrictions on third-party financing, we have shown that the stage-maximizing surplus is not always achievable with simple interest rate contracts, and that surplus cannot be split arbitrarily. If a more general mechanism not using third-party financing could solve these issues, our theory of third-party capital would be more limited in scope. However, as we show in the Appendix, this is not the case— restricting ourselves to simple interest rate contracts in which the entrepreneur gets paid only in case of success is in fact without loss of generality, under either of the following two conditions:

- Fly-by-night condition: The entrepreneur cannot get a payoff if the project is not started.
- Interim participation: The investor is free to walk away from the mechanism after observing his signal.

The fly-by-night condition can be microfounded by assuming that there is a large enough pool of unserious entrepreneurs who never have a successful project but can masquerade as serious entrepreneurs in the hope of getting paid off if contracts contain payments in case of no financing or in case of failure. The interim participation condition assumes that the investor cannot commit ex ante to enter into a mechanism that has negative expected pay off at the interim stage when he observes his signal. What either of these conditions rules out is a contract without third-party financing that can mimick the third party financing solution by stipulating a transfer from the investor to the entrepreneur in case of no financing; such a contract would allow for the stage surplus to be maximized with an arbitrary split of surplus, just as third-party financing does.

5.2. No third-party financing

When third-party financing is not available, the renegotiation-proof contract solves:

$$\max_{\hat{s}_t} \, (1 - F_G(\hat{s}_t))(X - V_{t+1}^G - r_t^I(\hat{s}_t)), \tag{36}$$

$$\text{s.t. } \phi(\hat{s}_t) \geq \frac{R_t}{P(B|z_{t-1})}, \tag{37}$$

where $r_t^I(\hat{s}_t)$ is given by Equation (6). Because the investor's participation constraint may no longer bind, investors may earn rents above their reservation utility. We show in Axelson and Makarov (2025) that without third-party financing, the rents are disproportionately captured by investors who are visited early. Unlike in the case with third-party financing, the rents stay bounded away from zero even as the number of visited investors increases to infinity.

Third-party capital allows investors to commit to reduce their informational rents, and therefore, effectively increases comptetion among investors. In any given round, an investors who can commit to use the third-party capital will be able to outbid investors who could only use their own capital. Once some investors gain access to third-party capital, others face a stark choice: secure outside funding themselves or be shut out of deals.

This logic echos a famous remark by Citigroup's then-CEO Chuck Prince, who on the eve of the financial crisis told the Financial Times: "As long as the music is playing, you've got to get up and dance." Prince was articulating precisely the competitive pressure our model formalizes: banks that could tap third-party capital like wholesale funding markets or structured credit were winning deal flow, and any institution that refused to match them risked losing its franchise.

We would like to emphasize that the competitive advantage of tapping third-party capital in our model does not come from ad-hoc assumptions about cheaper funding costs like it is common in the literature (e.g., Parlour and Plantin (2008) and Daley et al. (2020)). Rather, it comes from the fact that third-party capital allows investors to commit to reduce their informational rents and therefore, be more competitive in the market.

Proposition 3 below shows that while the entrepreneur is better off if there is third-party financing, there is overinvestment relative to the social optimal and the solution without third-party financing.

Proposition 3: *Suppose Assumptions 1-4 hold, and* $N = 2$*. Then, the entrepreneur is better off if there is third-party financing. However, there is overinvestment relative to the social optimal and the solution without third-party financing.*

Proof: See the Appendix.

As before, the reason for overinvestment is that the entrepreneur does not internalize the impact of her choices in the first period on future adverse selection. As a result, she chooses a screening threshold that is optimal in this round, but is lower than the social optimal. When third-party capital is constrained, the entrepreneur opts for a higher threshold in the first round because a low threshold correposds to a high interest rate and leaves too much rent to investors. While we prove the proposition for $N = 2$, we conjecture that it continues to hold for any N. We show in the next section that regulation limiting the amount of third-party capital can be welfare improving.

5.3. Regulation (tbc)

6. Intermediary capital structure: Balance sheet versus third-party financing (tbc)

For ease of exposition, we have assumed above that informed investors have deep pockets and can invest freely out of their own wealth. Our preferred interpretation of the informed investors in our model is as financial intermediaries, and the real-world applications we discuss all refer to intermediary settings. We now show that none of our results depend on this assumption. This is an important distinction relative to the literature building on the influential paper by Holmstrom and Tirole (1997), in which the personal wealth of intermediaries plays a critical role.

Assume now that informed investors have no personal wealth and require an average amount $1 - \delta$ of capital for each entrepreneur they finance. Suppose the maximum number of entrepreneurs an informed investor can encounter during a given time frame is M, where we think of the time frame as being long enough for M to be large. We also assume that the outcomes of entrepreneurial projects are independently distributed.

At the start of the time frame, the intermediary raises $M(1-\delta)$ of capital into a fund from uninformed capital market participants by issuing a security backed by the fund's pooled total cash flows X at the end of the time frame. The total cash flow consists of the interest and principal payments made by any entrepreneur financed by the fund, plus any remaining uninvested capital.

Consider a simple security consisting of a debt piece with face value D and an equity piece giving security holders a fraction α of all cash flows in excess of D. The payoffs $w(X)$ of this security are given by

$$w(X) = \begin{cases} X, & \text{if } X \leq D, \\ D + \alpha(X - D), & \text{if } X > D. \end{cases} \tag{38}$$

The informed investor retains a "carry" of $1-\alpha$ of all profits in excess of the face value D.

Note that as long as the informed investor expects total cash flows X to exceed D, he is perfectly aligned with these ex ante security holders: for any entrepreneur he encounters, his stake and the security holders' stake are linear in the payoffs of the investment. Hence, he will act as if he is investing his own wealth. Furthermore, as M increases, as long as the face value D is no larger than the raised amount $M(1-\delta)$, the law of large numbers together with the independence assumption imply that the probability of X exceeding $M(1-\delta)$ converges to one. (For pure equity, where $D = 0$, this holds for any M.)

Hence, by creating a sufficiently large pool, uninformed fund investors can be assured that the intermediary will act as if he is investing his own wealth, as we have assumed throughout the paper. The intermediary can therefore raise the fund at a competitive rate.

Furthermore, by setting $D = M(1-\delta)$, fund investors ensure that a "fly-by-night" operator who is unable to make any profitable investments receives no payoff from simply sitting on the raised capital. We imposed the "fly-by-night" condition on entrepreneurs in Section 5.1 to show that a simple debt contract between the entrepreneur and the informed investor is indeed optimal.

Note the critical difference between this ex ante pooled balance sheet capital and the ex post third-party deal-by-deal capital. Balance sheet capital serves to finance an intermediary with limited personal wealth; assuring alignment of incentives with these ex ante investors is essential. For third-party capital, in contrast, it is critical that the intermediary and the third-party capital providers are *not* aligned once the intermediary observes his private signal. This misalignment is precisely what allows the intermediary to commit to a lower screening threshold than would otherwise be optimal.

7. Conclusion

This paper studies how third-party capital and decentralized fundraising jointly shape competition, screening, and welfare in intermediated finance. In our model, an entrepreneur approaches informed investors sequentially, and each investor can combine retained capital with competitively priced outside funding. Third-party capital allows informed investors to dilute their informational rents without distorting the margin on which financing decisions are made. As a result, access to outside funding intensifies competition for deal flow and reduces the cost of capital to firms.

However, the privately optimal use of third-party capital has adverse consequences for aggregate welfare. Because contract renegotiations are unobservable to other market participants, entrepreneurs do not internalize that accepting laxer screening standards in the current financing round worsens the adverse selection problem faced by future investors. The resulting equilibrium features lending standards that are individually rational in each financing round but collectively too permissive, leading to overinvestment. This dynamic adverse selection externality is more severe when intermediaries have access to third-party capital than when they do not, providing a rationale for regulatory restrictions on intermediary leverage.

These results speak directly to debates about leverage, loan sales, and the design of regulation in intermediated credit markets. In our framework, the appeal of third-party capital does not arise because outside funding is exogenously cheap, but because it serves as a competitive instrument that helps intermediaries win deal flow. This mechanism helps explain why institutions may feel compelled to lever up even when doing so worsens aggregate outcomes. More broadly, the paper highlights that the welfare consequences of intermediary leverage cannot be understood without accounting for market structure, information frictions, and competition jointly.

References

Axelson, U. and Makarov, I. (2023). Informational black holes in financial markets. *Journal of Finance*, 78:3099–3140.

Axelson, U. and Makarov, I. (2025). Sequential credit markets. *Journal of Financial Economics*.

Besanko, D. and Kanatas, G. (1993). Credit market equilibrium with bank monitoring and moral hazard. *The Review of Financial Studies*, 6(1):213–232.

Daley, B., Green, B., and Vanasco, V. (2020). Securitization, ratings, and credit supply. *The Journal of Finance*, 75(2):1037–1082.

Gorton, G. B. and Pennacchi, G. G. (1995). Banks and loan sales: Marketing nonmarketable assets. *Journal of Monetary Economics*, 35(3):389–411.

Hartman-Glaser, B., Piskorski, T., and Tchistyi, A. (2012). Optimal securitization with moral hazard. *Journal of Financial Economics*, 104(1):186–202.

Holmstrom, B. and Tirole, J. (1997). Financial intermediation, loanable funds, and the real sector. *The Quarterly Journal of Economics*, 112(3):663–691.

Inderst, R. and Mueller, H. M. (2006). Informed lending and security design. *The Journal of Finance*, 61(5):2137–2162.

Parlour, C. A. and Plantin, G. (2008). Loan sales and relationship banking. *The Journal of Finance*, 63(3):1291–1314.

Vanasco, V. (2017). The downside of asset screening for market liquidity. *The Journal of Finance*, 72(5):1937–1982.

Appendix

Proof of Proposition 1

We first show that the market never breaks down. Suppose contrary to the claim in the proposition that there is an $n < N$ such that financing is not possible after round n. Because $N > n$, there are more than one investor present in round n, and therefore their reservation value is zero. Hence, investors will compete by setting δ_n arbitrary close to 0. In the limit, the screening threshold $\hat{s}_n$ solves

$$\max_{\hat{s}_n} P(G|z_{n-1})(1 - F_G(\hat{s}_n))X - P(B|z_{n-1})(1 - F_B(\hat{s}_n)), \tag{A1}$$

or

$$\frac{f_G(\hat{s}_n)}{f_B(\hat{s}_n)} = \frac{1}{z_{t-1}X}. \tag{A2}$$

Since financing is possible in round n, $\hat{s}_n < 1$, and therefore, $\lambda z_{t-1}X > 1$. But then using the definition of the function H given by Equation (20), we have

$$z_t X = \frac{F_G(\hat{s}_n)}{F_B(\hat{s}_n)} z_{t-1}X = H(\hat{s}_n)\frac{f_G(\hat{s}_n)}{f_B(\hat{s}_n)} z_{t-1}X. \tag{A3}$$

By Assumption 4, the function $H(s)$ is strictly decreasing and attains its minimum of $1/\lambda$ at $s = 1$. Therefore, $\lambda z_t X > 1$, which contradicts that round n is the last financing round.

Next, we show that there exists a unique equilibrium. We start with deriving the explicit expression for the expected investor's surplus in round t.

Lemma A1:

$$R_t = P(B|z_{t-1})\left(\prod_{n=t}^{N-1} F_B(\hat{s}_n)\right)\phi(\hat{s}_N). \tag{A4}$$

Proof: Interating Equation (27) forward and using the expression for the expected profits in the last period (21), yields equation (A4).

Q.E.D.

Equations (25) and (A4) imply that

$$X - V_t^G = F_G(\hat{s}_t)(X - V_{t+1}^G) + \frac{1 - F_B(\hat{s}_t)}{z_{t-1}} + \frac{1}{z_{t-1}}\left(\prod_{n=t}^{N-1} F_B(\hat{s}_n)\right)\phi(\hat{s}_N) \tag{A5}$$

Iterating Equation (A5) forward, we obtain

$$z_{t-1}(X - V_t^G) = z_{N-1} X \prod_{n=t}^{N-1} F_B(\hat{s}_n) + \left(1 - \prod_{n=t}^{N-1} F_B(\hat{s}_n)\right) + (N-t)\left(\prod_{n=t}^{N-1} F_B(\hat{s}_n)\right)\phi(\hat{s}_N). \tag{A6}$$

Using Equations (17) and (19), we can rewrite the above equation as

$$1 - H(\hat{s}_{t-1}) = \left(\prod_{n=t}^{N-1} F_B(\hat{s}_n)\right)\left(1 - z_{N-1}X - (N-t)\phi(\hat{s}_N)\right), \quad t = 2, \dots, N. \tag{A7}$$

By Assumption 4, $1 - H(s)$ is a strictly increasing function. Therefore, given thresholds $\hat{s}_\tau$, $\tau = t, \dots N$, there is at most one solution for $\hat{s}_{t-1}$. Note that the right-hand side of Equation (A7) increases in t, and attains its minimum at $t = 2$. Therefore, for Equations (A7) to have a solution, it is necessary and sufficient that

$$1 - z_{N-1}X - (N-2)\phi(\hat{s}_N) > 0, \quad z_{N-1} > (\lambda X)^{-1}. \tag{A8}$$

Using Equation (22), we can see that the above condition is satisfied for z_{N-1} sufficiently close to $(\lambda X)^{-1}$. Since $1 - H(s)$ is a strictly increasing function and the right-hand side of Equation (A7) increases in t, screening thresholds $\hat{s}_t$ are strictly increasing in t and strictly decreasing in z_{N-1}. Finally, since

$$z_{t-1} = \frac{F_B(\hat{s}_t)}{F_G(\hat{s}_t)} z_t, \tag{A9}$$

and $\frac{F_B(\hat{s}_t)}{F_G(\hat{s}_t)}$ is strictly decreasing in $\hat{s}_t$, all z_t strictly increase in z_{N-1}. Therefore, there is a unique map from z_{N-1} to z_0. Hence, there is a unique equilibrium.

Lemma A1 and Equation (11) imply that

$$(1 - \delta_t) = \left(\prod_{n=t}^{N-1} F_B(\hat{s}_n)\right)\phi(\hat{s}_N)/\phi(\hat{s}_t). \tag{A10}$$

Therefore, $(1 - \delta_t)$ strictly increases with t.

It remains to show that aggregate investor rent goes to zero. Since the market never breaks down,

$$\lim_N \hat{s}_N = 1. \tag{A11}$$

Taking the Taylor expansions, we obtain

$$\phi(\hat{s}_N) = c(1-\hat{s}_N)^2 + O\left((\hat{s}_N - 1)^3\right), \quad c > 0. \tag{A12}$$

Thus, if $\lim_N N(1-\hat{s}_N) = b$, $b < \infty$, then

$$\lim_N N\phi(\hat{s}_N) = 0, \tag{A13}$$

and therefore, aggregate investor rent goes to zero. Thus, we need to show that $\lim_N N(1-\hat{s}_N) < \infty$. Since the last investor breaks even at the maximal interest rate, we have

$$\prod_{t=1}^{N} \frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)} = \frac{H(\hat{s}_N)}{z_0 X}. \tag{A14}$$

Therefore,

$$\sum_{t=1}^{N} \ln\left(\frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)}\right) < \infty. \tag{A15}$$

Since both f_G and f_B are continuous functions, there exist limits

$$\lim_{s\to 1} \frac{1-F_G(s)}{1-s} = f_G(1),$$
$$\lim_{s\to 1} \frac{1-F_B(s)}{1-s} = f_B(1).$$

Therefore, for the sum in Equation (A15) to be finite, it must be that $\lim_N N(1-\hat{s}_N) = b < \infty$.

Q.E.D.

Proof of Proposition 2

We assume H is strictly decreasing. A feature shared by the market setups we look at is that the last investor breaks even at the maximal interest rate, as does the social planner:

$$\prod_{t=1}^{N} \frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)} = \frac{H(\hat{s}_N)}{z_0 X}. \tag{A16}$$

Take two sets of thresholds $\{\hat{s}_t\}_{t=1}^N$ and $\{\hat{s}'_t\}_{t=1}^N$ satisfying Equation (A16). We say that $\{\hat{s}_t\}_{t=1}^N$ is *steeper* than $\{\hat{s}'_t\}_{t=1}^N$ if the following condition is satisfied:

Definition 1: *Steepness: $\{\hat{s}_t\}_{t=1}^N$ is steeper than $\{\hat{s}'_t\}_{t=1}^N$ if there is a T such that*

- *$\hat{s}_t < \hat{s}'_t$ for all $t < T$ and $\hat{s}_t > \hat{s}'_t$ for all $t > T$.*
- *Both $\hat{s}_t$ and $\hat{s}'_t$ are increasing for $t \leq T$, and $\hat{s}'_t \geq \hat{s}'_T$ for $t > T$.*

Take two sets of thresholds $\{\hat{s}_t\}_{t=1}^N$ and $\{\hat{s}'_t\}_{t=1}^N$ satisfying Equation (A16), with $\{\hat{s}_t\}_{t=1}^N$ steeper than $\{\hat{s}'_t\}_{t=1}^N$. We show that if H is strictly decreasing and $\{\hat{s}'_t\}_{t=1}^N$ is increasing, then $\{\hat{s}_t\}_{t=1}^N$ is less efficient and features more overinvestment than $\{\hat{s}'_t\}_{t=1}^N$.

Note that since H is decreasing and $\hat{s}_N > \hat{s}'_N$ from the definition of steepness, Equation (A16) implies that

$$\prod_{t=1}^N \frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)} < \prod_{t=1}^N \frac{F_G(\hat{s}'_t)}{F_B(\hat{s}'_t)}.$$

Hence, from MLRP, there is a $\delta > 0$ such that

$$\prod_{t=1}^N \frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)} = \prod_{t=1}^N \frac{F_G(\hat{s}'_t - \delta)}{F_B(\hat{s}'_t - \delta)}.$$

Note that $\{\hat{s}_t\}_{t=1}^N$ is steeper than $\{\hat{s}'_t - \delta\}_{t=1}^N$, with some $T' \leq T$ such that $\hat{s}_t < \hat{s}'_t - \delta$ for $t < T'$ and $\hat{s}_t > \hat{s}'_t - \delta$ for $t > T'$.

Next, we prove that social surplus is higher in the scenario where all screening thresholds are equal to $\hat{s}'_t - \delta$ rather than $\hat{s}_t$, and that screening thresholds $\hat{s}'_t - \delta$ feature less overinvestment than $\hat{s}_t$. Finally, we show that using thresholds $\hat{s}'_t$ in turn features less overinvestment than thresholds $\hat{s}'_t - \delta$, completing the proof.

Consider the following perturbation: For some $m \geq T'$ such that $\hat{s}_m > \hat{s}'_m - \delta$ and some $n \leq T'$ such that $\hat{s}_n < \hat{s}'_n - \delta$, increase $\hat{s}_n$ and decreases $\hat{s}_m$ holding $\prod_{t=1}^N \frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)}$ constant. Note that social surplus (29) can be written as

$$\begin{aligned} W\left(\{\hat{s}_t\}_{t=1}^N\right) &= \frac{z_0}{1+z_0}\left(1 - \prod_{t=1}^N F_G(\hat{s}_t)\right) X - \frac{1}{1+z_0}\left(1 - \prod_{t=1}^N F_B(\hat{s}_t)\right) \\ &= \frac{z_0 X - 1}{1+z_0} + \frac{1}{1+z_0}\left(\prod_{t=1}^N F_G(\hat{s}_t)\right)\left[\frac{\prod_{t=1}^N F_B(\hat{s}_t)}{\prod_{t=1}^N F_G(\hat{s}_t)} - z_0 X\right]. \end{aligned} \tag{A17}$$

Since the perturbation by construction holds $\frac{\prod_{t=1}^N F_B(\hat{s}_t)}{\prod_{t=1}^N F_G(\hat{s}_t)}$ constant, the change in social surplus from the perturbation is given by

$$\frac{dW}{d\hat{s}_n} = \frac{1}{1+z_0}\frac{d\prod_{t=1}^N F_G(\hat{s}_t)}{d\hat{s}_n}\left[\frac{\prod_{t=1}^N F_B(\hat{s}_t)}{\prod_{t=1}^N F_G(\hat{s}_t)} - z_0 X\right] = \frac{z_0 X}{1+z_0}\frac{d\prod_{t=1}^N F_G(\hat{s}_t)}{d\hat{s}_n}\left[\frac{1}{H(\hat{s}_N)} - 1\right], \tag{A18}$$

where the last equality follows from Equation (A16). Since $H(\hat{s}_N) < 1$, it follows that $1/H(\hat{s}_N) - 1 > 0$.

Note that

$$\frac{d\prod_{t=1}^N F_G(\hat{s}_t)}{d\hat{s}_n} = \left[f_G(\hat{s}_n)F_G(\hat{s}_m) + \frac{d\hat{s}_m}{d\hat{s}_n} f_G(\hat{s}_m)F_G(\hat{s}_n)\right]\prod_{t\neq n,m} F_G(\hat{s}_t). \tag{A19}$$

By construction,

$$\frac{F_G(\hat{s}_n)F_G(\hat{s}_m(\hat{s}_n))}{F_B(\hat{s}_n)F_B(\hat{s}_m(\hat{s}_n))} = C,$$

for some constant C. Therefore,

$$\frac{d\hat{s}_m}{d\hat{s}_n} = -\frac{f_G(\hat{s}_n)F_G(\hat{s}_m)}{f_G(\hat{s}_m)F_G(\hat{s}_n)}\frac{1-H(\hat{s}_n)}{1-H(\hat{s}_m)}. \tag{A20}$$

Substituting (A20) into (A19), we obtain

$$\frac{d\prod_{t=1}^N F_G(\hat{s}_t)}{d\hat{s}_n} = f_G(\hat{s}_n)F_G(\hat{s}_m)\left[1 - \frac{1-H(\hat{s}_n)}{1-H(\hat{s}_m)}\right]\prod_{t\neq n,m} F_G(\hat{s}_t) > 0, \tag{A21}$$

where the last inequality follows from $\hat{s}_n < \hat{s}_m$, H decreasing, and $H < 1$ for $s > 0$. Hence, the perturbation increases the probability $\prod_{t=1}^N F_G(\hat{s}_t)$ of rejecting good projects. Equation (A18) then implies that the perturbation increases social surplus by reducing overinvestment.

We can continue doing these perturbations until each threshold is equal to $\hat{s}'_t - \delta$, which shows that social surplus is higher in the scenario where all screening thresholds are equal to $\hat{s}'_t - \delta$ rather than $\hat{s}_t$, and that screening thresholds $\hat{s}'_t - \delta$ induce less overinvestment than $\hat{s}_t$.

The last step is to show that going from thresholds $\hat{s}'_t - \delta$ to thresholds $\hat{s}'_t$ decreases overinvestment. Note that

$$
\begin{aligned}
-(1+z_0)\frac{dW\left(\{\hat{s}'_t-\varepsilon\}\right)}{d\varepsilon} &= \sum_{t=1}^{N}\left[f_B(\hat{s}'_t-\varepsilon)\prod_{n\neq t}^{N}F_B(\hat{s}'_n-\varepsilon)-z_0Xf_G(\hat{s}'_t-\varepsilon)\prod_{n\neq t}^{N}F_G(\hat{s}'_n-\varepsilon)\right]\\
&= \prod_{n=1}^{N}F_B(\hat{s}'_t-\varepsilon)\sum_{t=1}^{N}\left[\frac{f_B(\hat{s}'_t-\varepsilon)}{F_B(\hat{s}'_t-\varepsilon)}-\frac{f_G(\hat{s}'_t-\varepsilon)}{F_G(\hat{s}'_t-\varepsilon)}z_0X\frac{\prod_{n=1}^{N}F_G(\hat{s}'_n-\varepsilon)}{\prod_{n=1}^{N}F_B(\hat{s}'_n-\varepsilon)}\right].
\end{aligned} \tag{A22}
$$

We need to show that this is positive as ε goes from δ to 0. By MLRP,

$$
\frac{\prod_{n=1}^{N}F_G(\hat{s}'_n-\varepsilon)}{\prod_{n=1}^{N}F_B(\hat{s}'_n-\varepsilon)}
$$

decreases in ε. Equation (A16) implies that when $\varepsilon=0$, the above expression is equal to $H(\hat{s}'_N)$. Therefore, a sufficient condition for the derivative to be positive is that

$$
\sum_{t=1}^{N}\left[\frac{f_B(\hat{s}'_t-\varepsilon)}{F_B(\hat{s}'_t-\varepsilon)}-\frac{f_G(\hat{s}'_t-\varepsilon)}{F_G(\hat{s}'_t-\varepsilon)}\frac{f_B(\hat{s}'_N)}{f_G(\hat{s}'_N)}\frac{F_G(\hat{s}'_N)}{F_B(\hat{s}'_N)}\right]\geq 0,
$$

i.e.,

$$
\sum_{t=1}^{N}\frac{f_G(\hat{s}'_t-\varepsilon)}{F_G(\hat{s}'_t-\varepsilon)}\left[H(\hat{s}'_t-\varepsilon)-H(\hat{s}'_N)\right]\geq 0. \tag{A23}
$$

This is guaranteed if $\hat{s}'_N$ is the highest threshold in $\{\hat{s}'_t\}_{t=1}^{N}$.

Q.E.D.

Proof of Proposition 3:

We first prove that the entreprener recives a larger surplus when third-party financing is available than when it is not. When $N=2$, the threshold $s_2(s_1)$ solves

$$
\frac{f_B(s_2(s_1))}{f_G(s_2(s_1))}\frac{F_B(s_1)}{F_G(s_1)}=z_0X. \tag{A24}
$$

Since both f_B/f_G and F_B/F_G are strictly decreasing, $s_2(s_1)$ is strictly decreasing in s_1.

When third-party financing is available, the screening threshold $\hat{s}_1$ and $\hat{s}_2$ solve

$$
\frac{f_B(\hat{s}_1)}{f_G(\hat{s}_1)}=z_0X, \tag{A25}
$$

$$
\frac{f_B(\hat{s}_2)}{f_G(\hat{s}_2)}\frac{F_B(\hat{s}_1)}{F_G(\hat{s}_1)}=z_0X. \tag{A26}
$$

Therefore, $\hat{s}_2 > \hat{s}_1$. We show in Axelson and Makarov (2025) that without third-party financing, the entrepreneur's preferred screening threshold s_1^* solves

$$\psi(s_1^*) = z_0 X, \tag{A27}$$

where

$$\psi(s) = \frac{f_B(s)}{f_G(s)} - \frac{1 - F_G(s)}{f_G(s)} \left(\frac{f_B(s)}{f_G(s)} \right)'. \tag{A28}$$

Since f_B/f_G is strictly decreasing, $s_1^* > \hat{s}_1$. Without third-party financing, the investor's participation constraint (32) becomes

$$\phi(s_1) \geq F_B(s_1)\phi(s_2), \tag{A29}$$

where ϕ is defined in Equation (12). Since F_B in an increasing function, the above constraint is easier to satisfy the smaller s_1 is, and the larger s_2 is. In particular, the constraint is not binding for $s_1 = \hat{s}_1$ and $s_2 = s_2(\hat{s}_1)$.

If the investor's participation constraint is binding for $s_1 = s_1^*$ and $s_2 = s_2(s_1^*)$, then as we show in Axelson and Makarov (2025), the screening threshold $\bar{s}_1$ solves

$$\phi(\bar{s}_1) = F_B(\bar{s}_1)\phi(s_2(\bar{s}_1)). \tag{A30}$$

Because $s_2(s_1)$ is strictly decreasing in s_1, Equation (A30) has a unique solution for $\bar{s}_1$. Since $s_1^* > \hat{s}_1$, and the constraint is not binding for $s_1 = \hat{s}_1$, we have $\bar{s}_1 > \hat{s}_1$ and $\bar{s}_2 = s_2(\bar{s}_1) < s_2(\hat{s}_1) = \hat{s}_2$.

Next, we proove that the investtor's rent in the last period increases in the screening threshold $\hat{s}_1$. Then it follows that the investor's rent in the last period is smaller when third-party financing is available than when it is not.

Lemma A2: *The investor's rent in the last period increases in the screening threshold $\hat{s}_1$.*

Proof: We need to show that

$$\frac{\phi(s_2(s_1))}{1 + \frac{F_G(s_1)}{F_B(s_1)} z_0}$$

increases in s_1. Direct computation shows that

$$\ln \left(\frac{\phi(s_2(s_1))}{1 + \frac{F_G(s_1)}{F_B(s_1)} z_0} \right)' = \frac{\phi'(s_2(s_1)) s_2'(s_1)}{\phi(s_2(s_1))} - \frac{\left(\frac{F_G(s_1)}{F_B(s_1)} z_0 \right)'}{1 + \frac{F_G(s_1)}{F_B(s_1)} z_0}. \tag{A31}$$

Differentiating Equation (A24) with respect to s_1, we obtain

$$s_2'(s_1) = \frac{f_G(s_1)}{F_G(s_1)}(1 - H(s_1))\frac{f_B(s_2(s_1))}{f_G(s_2(s_1))} / \left(\frac{f_B(s_2(s_1))}{f_G(s_2(s_1))}\right)'. \tag{A32}$$

Note that

$$\left(\frac{F_G(s)}{F_B(s)}z_0\right)' = \frac{f_G(s)}{F_B(s)}(1 - H(s))z_0, \tag{A33}$$

$$\phi'(s) = (1 - F_G(s))\left(\frac{f_B(s)}{f_G(s)}\right)'. \tag{A34}$$

Substituting (A32), (A33), and (A34) into (A31), we obtain

$$\ln\left(\frac{\phi(s_2(s_1))}{1 + \frac{F_G(s_1)}{F_B(s_1)}z_0}\right)' = f_G(s_1)(1 - H(s_1))\left(\frac{(1 - F_G(s_2))\frac{f_B(s_2)}{f_G(s_2)}\frac{1}{F_G(s_1)}}{\phi(s_2)} - \frac{\frac{z_0}{F_B(s_1)}}{1 + \frac{F_G(s_1)}{F_B(s_1)}z_0}\right) =$$
$$= \frac{f_G(s_1)(1 - H(s_1))}{\phi(s_2)\left(1 + \frac{F_G(s_1)}{F_B(s_1)}z_0\right)}\left((1 - F_G(s_2))\frac{f_B(s_2)}{f_G(s_2)}\frac{1}{F_G(s_1)} + \frac{z_0}{F_B(s_1)}(1 - F_B(s_s))\right) > 0. \tag{A35}$$

Q.E.D.

Since the investor's rent in the last period is smaller when third-party financing is available than when it is not, the entrepreneur always earns zero suprplus in the last period, and the entrepreneur maximizes joint surplus in the first period when third-party financing is available, it follows that her surplus is larger when third-party financing is available than when it is not.

Finally, we show that the probability of financing is larger when third-party financing is available than when it is not. We first notice that social surplus is maximized when $s_2(\hat{s}_P) = \hat{s}_P$, where $\hat{s}_P$ is the screening threshold that maximizes social surplus defined in Equation (30). For $s_1 < \hat{s}_P$, we have $s_2(s_1) > \hat{s}_P$. Therefore, the profile $\{s_1, s_2(s_1)\}$ is steeper than the profile $\{\hat{s}_P, \hat{s}_P\}$. By Proposition 2, it follows that the probability of financing is larger for the profile$\{s_1, s_2(s_1)\}$ than for the profile $\{\hat{s}_P, \hat{s}_P\}$.

Consider $s_1 > \hat{s}_P$. In this case, it holds that $\hat{s}_1 \leq s_2(s_1) < \hat{s}_P < s_1$. From the definition of $s_2(s_1)$, we have

$$\frac{f_G(s_2(s_2(s_1)))}{f_B(s_2(s_2(s_1)))}\frac{F_B(s_1)}{F_G(s_1)}H(s_2(s_1)) = 1. \tag{A36}$$

We claim that it must be that $s_2(s_2(s_1)) < s_1$. To see this, suppose on the contrary that $s_2(s_2(s_1)) \geq s_1$. Then by MLRP, we have

$$1 = \frac{f_G(s_2(s_2(s_1)))}{f_B(s_2(s_2(s_1)))} \frac{F_B(s_1)}{F_G(s_1)} H(s_2(s_1)) > \frac{H(s_2(s_1))}{H(s_1)} > 1. \tag{A37}$$

Thus, we arrived at a contradiction. Therefore, it must be that $s_2(s_2(s_1)) < s_1$.

By Proposition 2, it follows that the probability of financing is larger for the profile $\{\hat{s}_1, \hat{s}_2\}$ than for the profile $\{s_2(s_1), s_2(s_2(s_1))\}$. Since $s_2(s_2(s_1)) < s_1$, it follows that the probability of financing is larger for the profile $\{s_2(s_1), s_2(s_2(s_1))\}$ than for the profile $\{s_1, s_2(s_1)\}$. Thus, we have shown that the probability of financing is larger when third-party financing is available than when it is not.

Q.E.D.

Proof that a renegotiation-proof mechanism is a simple debt contract

Here we show here that the interest rate offers we use are in fact optimal mechanisms.[6] Suppose the entrepreneur enters period t, where the common belief of the project being good is π_t at the start of the period. There may be a posted contract that serves as the outside option for both parties; here we study the set of contracts that are efficient given some reservation utility R for the investor. A posted contract which is proof to secret renegotiation must belong to this set. We will assume that the incoming posted contract is renegotiation proof in this sense; the mechanism chosen in the secret renegotiation does not effect the continuation value of the entrepreneur.

A general direct mechanism satisfying the entrepreneur's limited liability constraint and budget balancing in a bilateral setting is an incentive-compatible set

$$\{\mu(s), r(s), c_0(s), c_N(s)\}$$

such that if the investor reports signal s, the project is financed with probability $\mu(s) \in [0,1]$, in which case, the payment to the investor gross of the investment is $1 + r(s) \leq 1 + X$ if the project is Good, and the investor makes a transfer $c_0(s) \geq 0$ to the entrepreneur if the project is Bad. Finally, $c_N(s) \geq 0$ is the transfer from the investor to the entrepreneur in case of no financing (with probability $1 - \mu(s)$).

Note that when we allow for $c_N(s) > 0$ or $c_0(s) > 0$, we can no longer ignore the possibility of the project being bad when calculating the entrepreneur's utility V_t,

---

[6] Remark 1: The proof can be extended to show that when there are positive cash flows for bad projects (but lower than the investment amount so that the project is negative NPV), risky debt is the optimal security design in the take-it-or-leave-it offer. For example, suppose gross cash flows are $X+1$ in the good state and $L < 1$ in the bad state. A mechanism would then be $\{\mu(s), r_X(s), c_N(s), r_0(s)\}$ where $r_0(s) \leq L$ is the payment the investor gets if the project is bad. Using similar steps as in proof below, one can show that in an optimal mechanism, $r_0(s) = L$.

since the cash transfer comes also when the project is bad. However, when solving the entrepreneur's maximization problem, we will start off by assuming that cash transfers are zero for all future dates $j > t$, and will show that if this is the case, then cash transfers will optimally be set to zero also at date t. Since the assumption that future cash transfers are zero holds automatically in the last round $t = N$, it then follows by induction that this is true for all t.

Under the assumption that $c_0(s) = c_N(s) = 0$ for all future time periods and all s, the entrepreneur's continuation utility V_{t+1} is zero conditional on the project being bad. Conditional on the project being good, V_{t+1} also does not depend on any action or signal in period t, only on the number of remaining investors and their beliefs (which in turn also do not depend on any action or signal in period t directly.)

Denote by $\underline{s}$ the signal at which a one-period project just breaks even:

$$\frac{\pi_0}{1-\pi_0}\frac{f_G(\underline{s})}{f_B(\underline{s})} = \frac{1}{X}. \tag{A38}$$

We first show the following result:

Proposition A1: *In any renegotiation proof, ex-post rational bilateral mechanism,* $c_N(s) = c_0(s) = 0$.

Proof: For any mechanism $m = \{\mu(s), r(s), c_N(s), c_0(s)\}$, write the net social surplus contingent on signal $S = s$ as $\mu(s)W(s)$ where $W(s)$ is given by

$$W(s) = P(G|s)(1 + X - V^G_{t+1}) - 1, \tag{A39}$$

and

$$P(G|s) = \frac{\pi_t f_G(s)}{\pi_t f_G(s) + (1-\pi_t) f_B(s)}. \tag{A40}$$

is the probability an investor with signal s attaches to the project being good given the common prior π_t.

Denote the unconditional probability density for signal s by $f(s)$,

$$f(s) = \pi_t f_G(s) + (1-\pi_t) f_B(s).$$

Let $U(s, s')$ be the utility to an investor with signal s who reports s' and gets the contract $\{\mu(s'), r(s'), c_0(s'), c_N(s')\}$. Denote by R the ex ante reservation utility of the investor for participating in the round.

Any ex-post rational renegotiation-proof bilateral mechanism must solve the following problem for some R:

$$\max_{\substack{\mu:[0,1]\to[0,1],r:[0,1]\to[0,X],\\ c_0:[0,1]\to\Re_+,c_N:[0,1]\to\Re_+}} \int_0^1 \left(\mu(s)W(s) - U(s,s)\right) f(s)ds \tag{A41}$$

s. t.

$$U(s,s) \geq 0 \quad \forall s \in [0,1], \tag{IRI}$$

$$U(s,s) \geq U(s,s') \quad \forall s, s' \in [0,1], \tag{IC}$$

$$\int_0^1 U(s,s)f(s)ds \geq R. \tag{PC}$$

Here, (IRI) is the ex post individual rationality constraint of the investor after observing his signal, (IC) is the incentive compatibility condition, and (PC) is the ex-ante participation constraint of the investor.

The problem maximizes the entrepreneur's surplus subject to the participation constraint of the investor and the two ex post individual rationality constraints; different values of R maps out the efficient frontier, so any renegotiation-proof contract must lie on this frontier. We also note that R must lie in $[0, \bar{R}]$, where $\bar{R}$ is the maximal rents the investor can get while satisfying the Entrepreneur's ex ante participation constraint (which we omit—once we set $R \leq \bar{R}$, the entrepreneur's ex ante participation constraint will turn out to be slack):

$$\bar{R} = \int_{\hat{s}_{FB}}^1 W(s)f(s)ds. \tag{A42}$$

where $\hat{s}_{FB}$ is the first-best threshold signal for starting the project, given implicitly by

$$W(\hat{s}_{FB}) = 0 \quad \Leftrightarrow \quad \frac{f_G(\hat{s}_{FB})}{f_B(\hat{s}_{FB})} = \frac{1-\pi_t}{\pi_t} \frac{1}{X - V_{t+1}^G}. \tag{A43}$$

We establish the proof in a sequence of lemmas. We first note that the investor's ex post rationality constraint implies that there will be no investment or transfers for investor signals below $\underline{s}$ defined in Equation (A38):

Lemma A3: *For $s \leq \underline{s}$, we have $\mu(s) = c_N(s) = c_0(s) = 0$, and $U(s,s) = 0$.*

Proof: From the entrepreneur's limited liability constraint, we have $r(s) \leq X$ and c_0 and c_N nonnegative. When $s \leq \underline{s}$, the investor therefore cannot break even on any investment. The only contract that does not violate the investor's ex post individual rationality constraint (IRI) is then $\mu(s) = c_N(s) = c_0(s) = 0$, and $U(s,s) = 0$.

Q.E.D.

Next, note that the investor's utility $U(s,s')$ of reporting signal s' when he observes s is given by

$$U(s,s') = \mu(s')\left[P(G|s)r(s') - (1-P(G|s))(1+c_0(s'))\right] - ((1-\mu(s'))c_N(s') \tag{A44}$$

$$= P(G|s)\tilde{R}(s') - [\mu(s') + \tilde{c}(s')], \tag{A45}$$

where

$$\tilde{R}(s') = \mu(s')\left(r(s') + 1 + c_0(s')\right), \tag{A46}$$

$$\tilde{c}(s') = (1-\mu(s'))c_N(s') + \mu(s')c_0(s'). \tag{A47}$$

Thus, we can view the contract as an expected up-front payment $\tilde{c}(s')$ to the entrepreneur and an expected investment cost $\mu(s')$, and an expected gross return $\tilde{R}(s')$ to the investor if the project is good. Lemma A4 simplifies the maximization problem by expressing the rents to investors as a function of $\mu + \tilde{c}$ only:

Lemma A4: *Any incentive compatible, ex post rational mechanism has* $[\mu(s) + \tilde{c}(s)]$ *increasing, with* $\mu(s) + \tilde{c}(s) = 0$ *for* $s \leq \underline{s}$, *and the rents to investors are given by*

$$\int_{\underline{s}}^{1} U(s,s)f(s)ds = (1-\pi_t)\int_{\underline{s}}^{1} \phi(s)d[\mu(s) + \tilde{c}(s)], \tag{A48}$$

where $\phi(s)$ *is defined in Equation* (12).

Proof: The incentive constraints (IC) are given by

$$P(G|s)\tilde{R}(s) - [\mu(s) + \tilde{c}(s)] \geq P(G|s)\tilde{R}(s') - [\mu(s') + \tilde{c}(s')] \quad \forall s, s' \in [0,1]. \tag{A49}$$

Combining the (IC) conditions for an investor with signal s and an investor with signal $s' < s$, we obtain

$$P(G|s)[\tilde{R}(s) - \tilde{R}(s')] \geq [\mu(s) + \tilde{c}(s)] - [\mu(s') + \tilde{c}(s')] \geq P(G|s')[\tilde{R}(s) - \tilde{R}(s')]. \tag{A50}$$

Equation (A50) shows that both $\tilde{R}$ and $\mu + \tilde{c}$ are increasing in s. Letting s' go to s we get

$$P(G|s)\frac{d\tilde{R}(s)}{d[\mu(s) + \tilde{c}(s)]} = 1. \tag{A51}$$

Thus, the derivative $\frac{d\tilde{R}(s)}{d[\mu(s)+\tilde{c}(s)]}$ exists and is equal to $P(G|s)^{-1}$, even if the denominator itself may jump. Using (A51) we obtain

$$\tilde{R}(s) = \int_{\underline{s}}^{s} P(G|s')^{-1} d[\mu(s') + \tilde{c}(s')], \tag{A52}$$

where the integral is in general the Riemann–Stieltjes integral. Using (A44), (A52), and change in the order of integration, we have

$$\begin{aligned}
&\int_{\underline{s}}^{1} U(s,s)f(s)ds = \int_{\underline{s}}^{1} P(G|s)\tilde{R}(s)f(s)ds - \int_{\underline{s}}^{1} [\mu(s) + \tilde{c}(s)]f(s)ds = \\
&= \int_{\underline{s}}^{1} P(G|s)\left(\int_{\underline{s}}^{s} P(G|s')^{-1} d[\mu(s') + \tilde{c}(s')]\right) f(s)ds - \int_{\underline{s}}^{1} \left(\int_{\underline{s}}^{s} d[\mu(s') + \tilde{c}(s')]\right) f(s)ds = \\
&= \int_{\underline{s}}^{1} P(G|s)^{-1}\left(\int_{s}^{1} P(G|s')f(s')ds'\right) d[\mu(s) + \tilde{c}(s)] - \int_{\underline{s}}^{1} \left(\int_{s}^{1} f(s')ds'\right) d[\mu(s) + \tilde{c}(s)] = \\
&= \int_{\underline{s}}^{1} \left(\pi_t \frac{(1 - F_G(s))}{P(G|s)} - (\pi_t(1 - F_G(s)) + (1 - \pi_t)(1 - F_B(s)))\right) d[\mu(s) + \tilde{c}(s)] = \\
&= (1 - \pi_t)\int_{\underline{s}}^{1} f_B(s)\left(\frac{1 - F_G(s)}{f_G(s)} - \frac{1 - F_B(s)}{f_B(s)}\right) d[\mu(s) + \tilde{c}(s)].
\end{aligned}$$

Q.E.D.

The next lemma shows that there are no cash transfers in an optimal mechanism:

Lemma A5: *In any optimal mechanism, without loss of generality, we can set $\tilde{c} = c_N = c_0 = 0$ for all s and $\mu(s) = 0$ for $s \leq \hat{s}_{FB}$.*

Proof: Using Lemma A4, we can write the maximization problem (A41) as

$$\max_{\substack{\mu:[0,1]\to[0,1],\\ \tilde{c}:[0,1]\to\Re_+}} \int_{\underline{s}}^{1} \mu(s)W(s)f(s)ds - (1-\pi_t)\int_{\underline{s}}^{1} \phi(s)d[\mu(s)+\tilde{c}(s)] \tag{A53}$$

$$s.t. \quad \mu(\underline{s}) + \tilde{c}(\underline{s}) = 0, \tag{IRI'}$$

$$\mu(s)+\tilde{c}(s) \geq \mu(s')+\tilde{c}(s') \quad \forall s > s', \tag{IC'}$$

$$(1-\pi_t)\int_{\underline{s}}^{1} \phi(s)d[\mu(s)+\tilde{c}(s)] \geq R. \tag{PC'}$$

By strict MRLP, $\phi(s)$ is positive and strictly decreasing:

$$\frac{d\phi(s)}{ds} = \frac{d\left(f_B(s)/f_G(s)\right)}{ds}\left(1-F_G(s)\right) < 0,$$

with $\phi(0) = \frac{f_B(0)}{f_G(0)} - 1 > 0$ and $\phi(1) = 0$. Also note that $W(s)f(s)$ is strictly negative for $\underline{s} \leq s < \hat{s}_{FB}$ and strictly positive for $s > \hat{s}_{FB}$. If $\hat{s}_{FB} = 1$, the market breaks down and there is no financing. Hence, in what follows, we assume that $\hat{s}_{FB} < 1$.

Since $\phi(1) = \frac{d\phi(1)}{ds} = 0$, any optimal mechanism must have $\mu(1) = 1$, i.e., it is always optimal to get financing from an investor with the highest possible signal. Therefore, we will restrict attention to mechanisms in which $\mu(1) = 1$.

Now suppose a feasible mechanism has $\tilde{c}(s) > 0$ over some set in $[0,1]$, or $\mu(s) > 0$ for some $s < \hat{s}_{FB}$, and compare to an alternative mechanism with $\tilde{c}^* = 0$ and $\mu^*(s)$ such that $\mu^*(s) = 0$ for $s < \hat{s}_{FB}$, where surplus is negative, and $\mu^*(s) = \max_{s' \in [\hat{s}_{FB}, s]} \mu(s')$ for $s \geq \hat{s}_{FB}$. That is, $\mu^*(s)$ is the smallest non-decreasing function (weakly) larger than $\mu(s)$ for the signals where the surplus is positive.

Note that μ^* generates weakly higher surplus than μ. We next show that investors capture weakly higher rents under $(\tilde{c}, \mu)$ than under $(\tilde{c}^*, \mu^*)$, i.e., that

$$\int_{\underline{s}}^{1} \phi(s)\left\{d[\mu(s)+\tilde{c}(s)] - d\mu^*(s)\right\} \geq 0.$$

Note that by construction, $\mu(s) + \tilde{c}(s) \geq \mu^*(s)$ for all s. This follows from the fact that $\mu(s) + \tilde{c}(s)$ is a non-decreasing function and $\tilde{c}(s) \geq 0$, and $\mu^*(s)$ is the smallest non-decreasing function (weakly) larger than $\mu(s)$. Intergating by parts, we obtain

$$\int_{\underline{s}}^{1} \phi(s)\left\{d[\mu(s)+\tilde{c}(s)] - d\mu^*(s)\right\} = -\int_{\underline{s}}^{1} \left(\mu(s)+\tilde{c}(s) - d\mu^*(s)\right) d\phi(s) \geq 0. \tag{A54}$$

Since we have assumed $\tilde{c}$ is strictly positive somewhere, it must either be that $\mu(s)+\tilde{c}(s) > \mu^*(s)$ for some s, or that $\mu^*(s) > \mu(s)$ for some s. In the first case, investor rents are strictly higher with mechanism $\{\mu(s), c_0(s), c_N(s)\}$, while in the second case, surplus is strictly lower.

Finally, note that we can replace μ^* with a mechanism $\mu^{**} \geq \mu^*$ such that $\mu^{**} = 0$ for $s \in [0, \hat{s}_{FB}]$ and such that investors earn the same rents as under the mechanism $\{\mu(s), c_0(s), c_N(s)\}$. This follows from Equation (A54) which shows that for two mechanisms μ' and μ'', $\mu'(\hat{s}_{FB}) = \mu''(\hat{s}_{FB}) = 0$ and $\mu'(1) = \mu''(1) = 1$, $\mu'' > \mu'$), investors rents are higher under the mechanism μ'' and under μ'. Since the investor's utility from mechanism $\{\mu, c_0, c_N\}$ is bounded above by $\bar{R}$, and $\bar{R}$ is achievable by setting $\mu^{**}(s) = 1$ for all $s \geq \hat{s}_{FB}$, there exists such a $\mu^{**} \geq \mu^*$ that gives the same utility to the investor as in the original mechanism. By construction, μ^{**} generates higher surplus than μ^*, and hence than $\{\mu(s), c_0(s), c_N(s)\}$. Hence, there exists a mechanism that Pareto dominates $\{\mu(s), c_0(s), c_N(s)\}$, so it cannot be renegotiation proof.

Q.E.D.

Note that since $V_{T+1} = 0$, the proof holds for period T, so that the continuation utility V_T is positive only if the project is good. Since the proof shows that the same holds for any period t such that V_{t+1} is only positive if the project is good, by induction, the assumption at the beginning of the proof is justified.

Q.E.D.

Setting $c_N = c_0 = 0$, we have that $\mu(s)$ is increasing with $\mu(s) = 0$ for $s < \hat{s}_{FB}$, and $\mu(1) = 1$. Hence, we can think of μ as a cumulative distribution function on $[\hat{s}_{FB}, 1]$ and can write the social surplus as

$$\int_{s=\hat{s}_{FB}}^{1} \mu(s)W(s)f(s)ds = -\int_{s=\hat{s}_{FB}}^{1} \mu(s)d\left(\int_{s}^{1} W(s')f(s')ds'\right) =$$
$$= \int_{s=\hat{s}_{FB}}^{1} \big(\pi_t(1-F_G(s))(X - V_{t+1}^G) - (1-\pi_t)(1-F_B(s))\big)\, d\mu(s).$$

As a result, we can write the Entrepreneur's utility from a mechanism μ as

$$\int_{s=\hat{s}_{FB}}^{1} \mu(s)W(s)f(s)ds - (1-\pi_t)\int_{s=\hat{s}_{FB}}^{1} \phi(s)d\mu(s) =$$
$$= \int_{s=\hat{s}_{FB}}^{1} \left[X - V_{t+1}^G - \frac{1-\pi_t}{\pi_t}\frac{f_B(s)}{f_G(s)}\right]\pi_t(1-F_G(s))d\mu(s).$$

Using Lemma A4, any renegotiation-proof mechanism then solves the following maximization problem for some $R \leq \bar{R}$:

$$\max_{\mu \in M[\hat{s}_{FB},1]} \int_{s=\hat{s}_{FB}}^{1} \left[X - V_{t+1}^{G} - \frac{1-\pi_t}{\pi_t} \frac{f_B(s)}{f_G(s)} \right] \pi_t (1 - F_G(s)) d\mu(s) \tag{A55}$$

$$s.t. \quad (1-\pi_t) \int_{s=\hat{s}_{FB}}^{1} \phi(s) d\mu(s) \geq R.$$

where we denote by $M[\hat{s}_{FB}, 1]$ a set of Borel measures on $[\hat{s}_{FB}, 1]$. Define $\hat{s}^*$ as

$$\hat{s}^* = \arg\max_{s} (1 - F_G(s)) \left(X - V_{t+1}^{G} - \frac{1-\pi_t}{\pi_t} \frac{f_B(s)}{f_G(s)} \right) \tag{A56}$$

If $(1-\pi_t)\phi(\hat{s}^*) \geq R$, i.e., the participation constraint of the investor is not binding at the maximal value of the integrand, then the solution to problem (A55) is to set $\mu(s) = 0$ for $s < \hat{s}^*$, and $\mu(s) = 1$ for $s \geq \hat{s}^*$. In other words, the optimal mechanism is an interest rate offer

$$r = \frac{1-\pi_t}{\pi_t} \frac{f_B(\hat{s}^*)}{f_G(\hat{s}^*)}$$

such that investors accept it if and only if their signal is above threshold $\hat{s}^*$.

If the participation constraint is violated at $\hat{s}^*$, the problem becomes

$$\max_{\mu \in M[\hat{s}_{FB},1]} \int_{\hat{s}_{FB}}^{1} w(s) d\mu(s) \tag{A57}$$

$$s.t. \quad (1-\pi_t) \int_{\hat{s}_{FB}}^{1} \phi(s) d\mu(s) = R,$$

where

$$w(s) = \pi_t (1 - F_G(s))(X - V_{t+1}^{G}) - (1-\pi_t)(1 - F_B(s)). \tag{A58}$$

Proposition A2 provides the sufficient condition for the solution to problem (A59) to be concentrated at single point, which in turn, translates into a mechanism being a debt contract.

Proposition A2: *Suppose $w \circ \phi^{-1}$ is a strictly concave function on $[\phi(\hat{s}^*), \phi(\hat{s}_{FB})]$, where functions w and ϕ are defined in Equations (A58) and (12), and thresholds $\hat{s}_{FB}$ and $\hat{s}^*$ in (A43) and (A56). Then any renegotiation proof, ex-post rational bilateral mechanism is a debt contract, extended if and only if the investor breaks even conditional on his signal.*

Proof: By making a substitution $u = \phi(s)$, we can rewrite problem (A59) as

$$\max_{\mu \in M[0,\phi(\hat{s}_{FB})]} \int_0^{\phi(\hat{s}_{FB})} w \circ \phi^{-1}(u) d\mu(u) \tag{A59}$$
$$s.t. \quad (1-\pi_t) \int_0^{\phi(\hat{s}_{FB})} u d\mu(u) = R,$$

Let u^* be such that $(1-\pi_t)u^* = R$. For u^* to the solution to problem (A59), it must that that for any positive ω_1, ω_2, δ_1, and δ_2 such that

$$\omega_1(u^* - \delta_1) + \omega_2(u^* + \delta_2) = (\omega_1 + \omega_2)u^*,$$

it holds that

$$\frac{\omega_1}{\omega_1 + \omega_2} w \circ \phi^{-1}(u^* - \delta_1) + \frac{\omega_2}{\omega_1 + \omega_2} w \circ \phi^{-1}(u^* + \delta_2) < w \circ \phi^{-1}(u^*).$$

The above inequality follows if the function $w \circ \phi^{-1}(s)$ is a strictly concave.

Q.E.D.

The proof of Proposition A2 shows that the requirement for $w \circ \phi^{-1}(s)$ to be concave is also essentially a necessary condition. Proposition A3 provides sufficient conditions for $w \circ \phi^{-1}(s)$ to be strictly concave.

Assumption 5: *For $s \geq \underline{s}$,*

$$\frac{d\left(\frac{f_G(s)}{1-F_G(s)}\right)}{ds} \geq 0, \tag{A60}$$

and

$$\frac{d^2 \log[f_G(s)/f_B(s)]}{ds^2} \leq \frac{\frac{f_G(s)}{f_B(s)}}{\frac{f_G(s)}{f_B(s)} - \frac{f_G(\underline{s})}{f_B(\underline{s})}} \left(\frac{d \log[f_G(s)/f_B(s)]}{ds}\right)^2. \tag{A61}$$

Assumption 5 is a relatively mild regularity condition. The first assumption states that the inverse hazard ratio $\frac{f_G(s)}{1-F_G(s)}$ when the project is good is non-decreasing, which is a condition that the density of signals doesn't suddenly drop too quickly in the right tail of the signal distribution. The second assumption rerquires that the log likelihood ratio $\log[f_G/f_B]$ is not too convex. It is quite a weak assumption satisfied by most regular signal distributions satisfying MLRP. It is significantly weaker than logconcavity, which says $\frac{d^2 \log[f_G(s)/f_B(s)]}{ds^2} < 0$.

Proposition A3: *Suppose Assumption 5 holds. Then the function* $w \circ \phi^{-1}(s)$ *is strictly concave.*

Proof: We need to show that $\frac{w'(s)}{\phi'(s)}$ is increasing for $s \in [\hat{s}_{FB}, \hat{s}^*]$. This will follow if

$$w''(s)\phi'(s) - w'(s)\phi''(s) > 0. \tag{A62}$$

Using the definition of $\hat{s}_{FB}$ in Equation (A43), we have that

$$X - V^G_{t+1} = \frac{1-\pi_t}{\pi_t} \frac{f_B(\hat{s}_{FB})}{f_G(\hat{s}_{FB})}. \tag{A63}$$

We use this relationship to substitute for $X - V^G_{t+1}$ in the below derivations of W', W'', ϕ', and ϕ'' :

$$w'(s) = -\pi_t f_G(s)(X - V^G_{t+1}) + (1-\pi_t) f_B(s) = -(1-\pi_t) f_B(s) \left[\frac{\frac{f_G(s)}{f_B(s)} - \frac{f_G(\hat{s}_{FB})}{f_B(\hat{s}_{FB})}}{\frac{f_G(\hat{s}_{FB})}{f_B(\hat{s}_{FB})}} \right] \tag{A64}$$

$$\begin{aligned} w''(s) &= -\pi_t f'_G(s)(X - V^G_{t+1}) + (1-\pi_t) f'_B(s) = \\ &= -(1-\pi_t) f_B(s) \left[\frac{f'_G(s)}{f_G(s)} - \frac{f'_B(s)}{f_B(s)} + \left[\frac{\frac{f_G(s)}{f_B(s)} - \frac{f_G(\hat{s}_{FB})}{f_B(\hat{s}_{FB})}}{\frac{f_G(\hat{s}_{FB})}{f_B(\hat{s}_{FB})}} \right] \frac{f'_G(s)}{f_G(s)} \right] = \\ &= -(1-\pi_t) f_B(s) \left[\left(\ln\left(\frac{f_G(s)}{f_B(s)} \right) \right)' + \left[\frac{\frac{f_G(s)}{f_B(s)} - \frac{f_G(\hat{s}_{FB})}{f_B(\hat{s}_{FB})}}{\frac{f_G(\hat{s}_{FB})}{f_B(\hat{s}_{FB})}} \right] \frac{f'_G(s)}{f_G(s)} \right] \end{aligned} \tag{A65}$$

$$\phi'(s) = -f_B(s) \frac{1-F_G(s)}{f_G(s)} \left(\ln\left(\frac{f_G(s)}{f_B(s)} \right) \right)' < 0 \tag{A66}$$

$$\phi''(s) = f_B(s) \left(\ln\left(\frac{f_G(s)}{f_B(s)} \right) \right)' \left[1 + \frac{1-F_G(s)}{f_G(s)} \left(\left(\ln\left(\frac{f_G(s)}{f_B(s)} \right) \right)' - \frac{d \ln \left[\left(\ln\left(\frac{f_G(s)}{f_B(s)} \right) \right)' \right]}{ds} \right) \right]. \tag{A67}$$

Therefore, $w''\phi' - w'\phi'' \geq 0$ is equivalent to

$$\frac{\frac{f_G(\hat{s}_{FB})}{f_B(\hat{s}_{FB})}}{\frac{f_G(s)}{f_B(s)} - \frac{f_G(\hat{s}_{FB})}{f_B(\hat{s}_{FB})}} \left(\ln\left(\frac{f_G(s)}{f_B(s)}\right)\right)' + \frac{f'_G(s)}{f_G(s)} \geq \tag{A68}$$

$$-\frac{f_G(s)}{1 - F_G(s)} - \left(\ln\left(\frac{f_G(s)}{f_B(s)}\right)\right)' + \frac{d \ln\left[[\left(\ln\left(\frac{f_G(s)}{f_B(s)}\right)\right)'\right]}{ds}, \tag{A69}$$

i.e.,

$$\frac{d \ln\left[\left(\ln\left(\frac{f_G(s)}{f_B(s)}\right)\right)'\right]}{ds} \leq \frac{\frac{f_G(s)}{f_B(s)}}{\frac{f_G(s)}{f_B(s)} - \frac{f_G(\hat{s}_{FB})}{f_B(\hat{s}_{FB})}} \left(\ln\left(\frac{f_G(s)}{f_B(s)}\right)\right)' + \frac{f_G(s)}{1 - F_G(s)} + \frac{f'_G(s)}{f_G(s)}. \tag{A70}$$

Note that

$$\frac{d\left(\frac{f_G(s)}{1-F_G(s)}\right)}{ds} = \frac{f_G(s)}{1 - F_G(s)} \left[\frac{f'_G(s)}{f_G(s)} + \frac{f_G(s)}{1 - F_G(s)}\right]. \tag{A71}$$

Hence, the assumption that $\frac{f_G(s)}{1-F_G(s)}$ is non-decreasing for $s \geq \underline{s}$ implies that (and is equivalent to)

$$\frac{f'_G(s)}{f_G(s)} + \frac{f_G(s)}{1 - F_G(s)} \geq 0 \quad \forall s \geq \underline{s}. \tag{A72}$$

A sufficient condition for $G''(s)\phi'(s) - \phi''(s)G'(s) \geq 0$ is then

$$\frac{d \ln\left[\frac{d \ln[f_G(s)/f_B(s)]}{ds}\right]}{ds} \leq \frac{\frac{f_G(s)}{f_B(s)}}{\frac{f_G(s)}{f_B(s)} - \frac{f_G(\hat{s}_{FB})}{f_B(\hat{s}_{FB})}} \frac{d \ln[f_G(s)/f_B(s)]}{ds}, \tag{A73}$$

i.e.,

$$\frac{d^2 \ln[f_G(s)/f_B(s)]}{ds^2} \leq \frac{\frac{f_G(s)}{f_B(s)}}{\frac{f_G(s)}{f_B(s)} - \frac{f_G(\hat{s}_{FB})}{f_B(\hat{s}_{FB})}} \left(\frac{d \ln[f_G(s)/f_B(s)]}{ds}\right)^2. \tag{A74}$$

Note that since $s \geq \hat{s}_{FB} \geq \underline{s}$, this is implied by Assumption 5.

Q.E.D.