Collateral, Contagion and Clearing

PRELIMINARY

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Abstract

In a network of connected financial institutions (FIs), pairs of FIs hedge portfolio risks using over-the-counter contracts, becoming exposed to counterparty risk. Defaults within pairs can propagate through the network. With central clearing, collateral is fungible across pairs, allowing high protection at low collateral costs, provided that enough pairs choose to clear. FIs' choices determine collateral requirements by central counterparties as well as contagion risk, feeding back on each pair's incentive to clear. Equilibria differing in the extent of clearing may co-exist, and are Pareto-ranked. Margin requirements on non-cleared contracts reduce defaults within pairs but may discourage clearing, exacerbating systemic risk.

1 Introduction

One central premise of systemic risk regulation is that financial institutions (FIs) do not fully internalize the system-wide consequences of the risks they choose to bear. In over-the-counter (OTC) derivatives markets, FIs trade bilateral contracts that generate counterparty exposures. Mutual exposures and informational effects can notoriously cause defaults in a given bilateral relationship to spill over to otherwise uninvolved institutions, turning idiosyncratic episodes into systemic events (Brunnermeier and Oehmke [2013], Benoit, Colliard, Hurlin, and Pérignon [2017]). Central clearing can attenuate contagion: by novating trades to a central counterparty (CCP) and mutualizing losses across members, FIs can insulate from counterparty risk.

This paper's main contribution is to embed clearing choice in a model in which the terms of OTC contracts and the fraction of contracts cleared jointly determine *(i)* the severity of

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default contagion, *(ii)* the CCP's collateral demand as well as *(iii)* its expected losses. Doing so makes the feedback between aggregate clearing and the private value of clearing explicit: as more contracts are cleared, the residual OTC segment becomes safer and members' individual contribution to CCP's default fund decrease, the two forces having opposite effects on the relative attractiveness of remaining OTC versus clearing. Our stylized model captures these features in a tractable way, generating new insights for both positive and normative questions. Who clears? What drives risks for CCPs? And how does regulation reshape incentives and equilibrium systemic risk?

Empirically, following a rapid expansion after the 2008 financial crisis, clearing rates have plateaued in many derivatives classes, consistent with weak private incentives to voluntarily clear for the remaining participants.[1] Complementary to existing theories of voluntary clearing, our model suggests an intuitive mechanism: when a larger share of contracts is centrally cleared, contagion is lower, therefore counterparty risk embedded in OTC contracts becomes less severe—placing a natural limit on the private benefits of clearing. At the same time, while clearing implies counterparty risk protection, its net benefits are dampened by CCPs' collateral demand and the extent to which clearing members will have to contribute to its losses, both of which are higher when clearing is low. This is a source of strategy complementarity in clearing decisions, creating coordination effects that can explain why the extent of central clearing in a given asset class may vary substantially across jurisdictions. Turning to policy, we revisit a common rationale for OTC margin requirements, namely that higher minimum margins both make OTC safer and push activity toward clearing. Our theory highlights that, by containing risk within the OTC segment, margin requirements can in fact reduce FIs' private incentives to clear. We characterize situations in which introducing (or tightening) minimum margins leads to less clearing, paradoxically increasing defaults and exacerbating systemic risk.

Preview of the model and results.—The model features a finite number of identical pairs of FIs and a CCP. Each pair is composed of a protection buyer and a protection seller. Within a pair, the buyer has negative exposure and the seller has equal but positive exposure to the same risk factor. FIs are subject to a minimum pledgeable income constraint, and default if the constraint is violated at an interim period.[2] Defaults are inefficient because they make all FI's assets worthless, including those whose returns are not pledgeable to external investors. This gives buyers a motive to enter contracts with sellers to hedge their negative exposures. However, entering such a contract exposes the buyer to counterparty risk, since we assume that, in a "bad" state of the world, one seller chosen at random may default exogenously. To capture an interconnected financial system, we assume that defaults may spill over exogenously

[1]See the most recent report from the Bank for International Settlements https://www.bis.org/publ/otc_hy2512.htm.

[2]The assumption captures in reduced form a situation whereby FIs have short-term debt that is owned by dispersed investors and must be rolled-over and are subject to a moral hazard problem, e.g. the returns on their asset depends on unobservable costly effort.

from one pair to another. These unmodeled links are such that FIs form a circular network. To protect from counterparty risk, contracts can specify margins that sellers must post in protected accounts, acting as collateral for the buyer in case of seller's default. To post margins, sellers have to liquidate their assets prematurely, forgoing part of their returns. Pairs choose the terms of their OTC contract (transfers conditional on realized exposures and margins to be posted beforehand) and simultaneously decide whether to clear the contract with a CCP (becoming a CCP "member"), or remain OTC. The CCP replaces member pairs in their respective promised transfers, and collects from members the minimum amount of collateral necessary to be solvent in every state. In case of defaults, the CCP taps into the fund and redistributes the remainder to members in equal proportion. We characterize the equilibria of the game in which pairs choose contracts and make clearing decisions that are jointly optimal for each pair, given the other pairs strategy profile.

We begin with a benchmark in which FIs operate in isolation, corresponding to the case in which the exogenous spillover probability is set to zero. Because at most one seller can default exogenously, the amount of collateral collected by the CCP is fixed at the buyer's maximal exposure. Therefore, per-member contribution to the CCP's fund is lower the higher is the number of clearing pairs, which we refer to as the "size" of the CCP.[3] On the other hand, a pair joining the CCP raises the probability that a default among members occurs, increasing its expected losses. We derive a condition under which the first effect dominates, making the value of clearing increase in the size of the CCP. In this case, FIs may coordinate on at most two equilibria: one in which no pair clears, and one in which all pairs do. When the latter equilibrium exists, it Pareto-dominates the former. We note that, in the benchmark, the introduction of minimum margin requirements for OTC contracts unambiguously decreases the relative value of remaining OTC, promoting clearing.

Next, we switch on contagion across pairs. The default of a seller in an OTC contract can induce the buyer's default when its realized exposure is larger than the privately chosen margin; in turn, this can trigger default of the seller in the next adjacent OTC pair. Such a cascade of defaults stops necessarily when it hits a seller in a clearing pair. The extent to which a pair is exposed to contagion risk depends on the profile of contracts entered by the preceding OTC pairs. We prove that, under the same condition that makes the value of clearing increase in CCP size in the benchmark without cross-pairs contagion, equilibria can only take one particular form: the FIs' strategy profile forms a series of identical repeated chains of adjacent OTC pairs terminating with an individual clearing pair. Equilibria can

[3]In reality, most CCPs adopt the "cover 2" model, a standard requiring to maintain sufficient financial resources to withstand the simultaneous default of their two largest members under extreme but plausible market conditions. In realistic risk setups, this would also result in individual contributions that decrease in size. Other sources of economies of scale in central clearing derive from improved diversification that results in more efficient loss mutualization. This feature is absent in our model because in each contract the seller must pledge all its positive exposure to the buyer to avoid the buyer's default.

then be classified based on the number of pairs participating in clearing. Equilibria with lower participation feature longer OTC chains, which imply a larger maximal number of simultaneous defaults, a natural measure of systemic risk. Under a restriction on primitives that limits the sensitivity of privately optimal margins to the seller's default probability, we also show that risks build up along OTC chains. This helps us further qualify the sense in which low participation equilibria are riskier: in them, the expected number of defaults is lower and CCP members are more likely to default. In some cases, the low participation equilibria are such that the CCP experiences higher expected losses, despite most defaults originating outside of it. We revisit the role of minimum margin requirements in this richer environment. Compared to the benchmark with no cross pairs contagion, margin requirements on OTC contracts do not affect the value of staying OTC uniformly. While pairs that are closer to clearing members only bear the cost of the policy, those that are more exposed to OTC chains benefit from lower contagion, reducing their motive to clear. We construct examples in which, compared to the laissez-fair, an economy with margin requirements in place features lower clearing and thus higher systemic risk.

Contribution.— We contribute to three strands of literature.

Several works highlight contagion externalities in financial networks. Closest to our analysis are Zawadowski [2013], Caballero and Simsek [2013] and Acharya and Bisin [2014]. We contribute to such class of models by embedding the decision to centrally clear.

The decision to centrally clear is at the core of the analysis in Antinolfi et al. [2022] and DErasmo et al. [2025]. The first model shares with ours the view that clearing is motivated by the incentive to insure more effectively against counterparty risk. Unlike their setup, we are concerned with systemic risk, and thus we model contagion explicitly. The second model features bankers that are atomistic hence their decision whether or not to clear does not impact the bank's ability to absorb losses and avoid defaults to spread. Their motive to centrally clear is entirely driven by regulatory-induced costs of remaining OTC. Our model thus uniquely combines risk-protection-induced voluntary clearing and endogenous systemic risk.

Finally, Biais et al. [2016], Huang and Zhu [2024], Kuong and Maurin [2024] and Kubitza and Oehmke [2025] propose theories that cast light on the contractual and organizational design that characterize modern CCPs. We sidestep many of the important issues emerging from such works, to focus our analysis on the strategic incentives to clear in a network of connected FIs. Our contribution is to endogenize the costs and benefits of clearing, which are the outcomes of participation decisions and risk-taking outside the CCP.

Roadmap.—The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes optimal contracts and describes a benchmark with no scope for cross-pairs contagion. Section 4 contains the main results. Section 5 concludes.

2 The Model

Agents and Assets.—The economy lasts four periods, indexed by $t = 0, 1, 2, 3$. There are N pairs of risk-neutral FIs, a single central clearing counterparty (CCP) and only one good, cash. FIs' payoff amounts to the cash they have at $t = 3$, and there is no discounting across periods. Each pair is composed of a protection buyer and a protection seller.

The protection buyer in pair i owns an asset that pays $x+y-z\,(y+\varepsilon_i)$ if held until $t = 3$ and zero otherwise. The variables x and y are positive constants, z is a binary variable taking values $\{0, 1\}$ with equal probability, and ε_i a random variable distributed according to the measure G admitting a continuous density g with full support $[0, \bar{\varepsilon}] \subset \mathbb{R}$. Importantly, x represents the component of the returns from the asset that is not pledgeable to external investors.

The protection seller in pair i owns two assets. The first is a divisible "project", each fraction which pays a per-unit return R if held until $t = 3$, L if liquidated at $t = 0$ or $t = 1$, and zero otherwise. The payoff from the non-liquidated part of the project cannot be pledged to external investors. The second asset pays $z\varepsilon_i$ if held until $t = 3$ and zero otherwise, and its returns can be pledged to external investors.

We interpret z as "risk-factor" to which the buyer is negatively exposed and the seller is positively exposed. In the state $z = +1$, the buyer's pledgeable income becomes negative, to an extent that depends on the realization of the "shock" ε_i. The fact that the seller's returns have the opposite sensitivity to ε_i makes the seller the natural counterparty from which the buyer may seek insurance. The facts that z and G do not depend on i—that is all pairs are exposed to the same risk factor, and their exposures are drawn from a common distribution—add tractability to the model but are not essential to derive the main results of the paper.

Defaults.—FIs may default only at $t = 2$, for three reasons. First, to introduce counterparty risk into a seller-buyer relationship, we assume that with probability q, a large shock hits one protection seller at random. When it does, the seller defaults. Second, to capture in reduced form the fact that FIs are subject to refinancing risk, we assume that any FI defaults at $t = 2$ if its total pledgeable income falls (strictly) below zero. Finally, to introduce the possibility of contagion across pairs, we assume that if the buyer in pair i defaults, the seller in pair $i+1$ also defaults with probability ρ. To improve tractability, we also assume that the network resulting from these unmodeled intra-pair links is circular: formally, this occurs when the default of the buyer in pair N induces the default of the seller in pair 1. An institution's default renders all its assets worthless.

We can briefly anticipate the role of each element. The constraint that FIs' pleadgeable income must not fall below zero means that, around that threshold, the buyer's payoff is non-convex, motivating their demand for insurance and thus creating scope for valuable contracts between sellers and buyers; Counterparty risk will give a role to margins, which will act as collateral in case of seller's default; The fact that contagion may occur between pairs implies that, depending on the FIs decisions, idiosyncratic risks can become systemic.

Information and Contracts.—The risk factor z realizes and is publicly observed at $t = 1$, whereas the exposures ε_i realize and are publicly observed at $t = 2$. A contract between the seller and the buyer in pair i specifies: *(i)* (net) transfers made by the seller and the buyer at $t = 3$, which can be functions of all available information and are denoted $T_i^s(\varepsilon_i, z)$ and $T_i^b(\varepsilon_i, z)$ and *(ii)* a margin $M_i(z)$ to be posted by the seller at $t = 1$. Without loss of generality, we do not allow the buyer to post any margin. Posting margins means depositing M_i units of cash into a protected segregated margin account. If a seller defaults at $t = 2$, it may be unable to meet its liability towards the buyer at $t = 3$, in which case the latter receives the amount that is deposited in the margin account. Upon a seller's default, the buyer thus receives: $\min\left\{T_i^s(\varepsilon_i, z) - T_i^b(\varepsilon_i, z), M_i(z)\right\}$. If a seller defaults at $t = 2$ and she is owed a positive net transfer from the buyer, the contract is canceled. Contract terms are only observable by the parties involved. Finally, we impose the restriction that a seller cannot promise a net payment above its remaining pledgeable income after posting margins.

To simplify the exposition, we assume (and later verify) that the seller does not liquidate its project in excess of what is required to meet margin calls at $t = 1$. When this is the case, the balance sheet of the seller in every state (ε_i, z) is fully pinned down by the contract terms, that is by the functions $M_i(z)$, $T_i^s(\varepsilon_i, z)$ and $T_i^b(\varepsilon_i, z)$.

Clearing.—At $t = 0$, FIs can choose whether or not to clear the contract with the CCP. We refer to the alternative decision as to "stay over-the-counter (OTC)". When a contract is cleared, the CCP replaces the buyer and the seller in their respective obligations towards each other—summarized by the transfer schedule $\left\{T_i^s(\varepsilon_i, z), T_i^b(\varepsilon_i, z)\right\}$. In case of clearing, margins are not the FIs' choice, but are determined and collected by the CCP at $t = 1$. In particular, we assume that the CCP's collects just enough margins so that is is always going to be able to repay its obligations at $t = 3$. In case that margins are used to pay some of the CCP's obligations, the remaining funds are redistributed in equal amount to all members.

Summary of the timing and equilibrium.— We summarize the timing of events. At $t = 0$, each pair simultaneously decides whether to enter into a contract, the terms of the contract, and wheather to clear the contract via the CCP. At $t = 1$ the binary risk factor z realizes and is publicly observed. FIs meet margin calls according to what the contracts or the CCP require. At $t = 2$, all FI's exposures become observable. Sellers may be hit by a large shock and hence default. FIs who are not hit by the large shock must satisfy the minimum pledgeable income constraint in order to survive until period $t = 3$, and default otherwise. Defaults of buyers may spillover to the next adjacent sellers. In period $t = 3$, all assets' payoffs realize, all contracts are settled and margins posted and not used by the CCP are redistributed.

Denote with $\sigma_i \in \{c, o\}$ pair i's decision whether to clear or stay OTC. We require that clearing decisions form a pure strategy Nash equilibrium and that, within pairs, contracts are written to maximize joint expected surplus given all other pairs' decisions, subject to the constraint that buyers and sellers receive a payoff weakly above their zero outside option.

Definition 1. *A strategy profile is described by a collection* $\{\sigma_i, T_i, M_i\}_{i=1}^N$. *An equilibrium is such that:*

(i). *Each pair maximizes their joint payoffs given the strategy profile.*

(ii). *Contracts satisfy Buyers and Sellers participation constraint.*

3 Analysis

In this section, we characterize general features of the optimal contracts that are signed in equilibrium and we derive the equilibrium under a benchmark scenario in which there is no contagion across pairs.

3.1 Optimal insurance

Consider pair i and assume that it chooses to stay OTC. For any given contract, if the seller does not default at the beginning of $t = 2$, the buyer's pledgeable income is:

$$y - z(y + \varepsilon_i) + T^s(\varepsilon_i, z) - T^b(\varepsilon_i, z)$$

If the seller defaults, the buyer's pledgeable income is:

$$y - z(y + \varepsilon_i) + \max\left\{0, \min\left\{T_i^s(\varepsilon_i, z) - T_i^b(\varepsilon_i, z), M_i(z)\right\}\right\}$$

The seller's pledgeable income at $t = 2$ is:

$$z\varepsilon_i + M_i + T^b(\varepsilon_i, z) - T^s(\varepsilon_i, z)$$

Observe that when $z = +1$ a large enough shock ε_i can cause the buyer's pledgeable income to fall below zero, leading to a default. Since, for all $T^s(\varepsilon_i, 1) \leq \varepsilon_i$, the seller's pleadgeable income is non-negative, it must therefore be that $T^s(\varepsilon_i, 1) - T^b(\varepsilon_i, 1) \geq \varepsilon_i$. This means that in the states in which the seller does not default, the buyer is insured and survives any shock ε_i. Using the costraint that $T^s(\varepsilon_i, 1) \leq T^b(\varepsilon_i, 1) + \varepsilon_i$, the result below follows.

Lemma 1. *Irrespective of the pair's clearing decision, in every optimal contract,* $T^b(\varepsilon_i, z = 1) = 0$ *and* $T^s(\varepsilon_i, z = 1) = \varepsilon_i$.

3.2 Optimal margins in OTC contracts

By Lemma 1, the buyer in pair i defaults only when its seller defaults, $z = +1$ and $\varepsilon_i > M_i$. The optimal margin balances off the safety benefits of margins with the costs associated to inefficient asset liquidation. To derive cleaner expressions, it is useful to define the wedge w as:

$$w := \frac{R}{L} - 1$$

which measures the cost of liquidating the seller's project prematurely in terms of relative forgone returns.

Consider an equilibrium in which pair i chooses to stay OTC, and denote with p_i the probability that, when $z = +1$, the seller defaults in the candidate equilibrium. We refer to the buyer's *counterparty risk* as the probability that its seller's default causes the buyer to default when $z = +1$. We have that:

$$\text{buyer's counterparty risk} = qp_i \left(1 - G\left(M_i\right)\right)$$

The next result characterizes the optimal margin M_i as a function of p_i.

Lemma 2. *In an OTC contract, the optimal M_i is non-decreasing in q, in p_i and in the ratio $\frac{x}{w}$. If the density $g(\varepsilon)$ is strictly decreasing, M_i satisfies:*

$$M_i = \begin{cases} 0 & \textit{if} \quad g(0) \leq \frac{1-qp_i}{qp_i}\frac{w}{x} \\ \\ g^{-1}\left(\frac{1-qp_i}{qp_i}\frac{w}{x}\right) & \textit{if} \quad g(0) > \frac{1-qp_i}{qp_i}\frac{w}{x} \geq g(L) \\ \\ L & \textit{if} \quad \frac{1-qp_i}{qp_i}\frac{w}{x} < g(L) \end{cases}$$

Some comments are in order. First, observe that there would be no role for margins in the absence of counterparty risk—that is $M_i = 0$ if $q = 0$. Indeed, costly margins are part of the contract only for sufficiently large qp_i, which is the probability of default of seller i. Second, the marginal benefit of increasing M_i is given by the change in the buyer's exposure to counterparty risk, that is $qp_i g(M_i)$, times the marginal value of a unit of funds at the buyer's default threshold, which is equal to the size of the payoff discontinuity x. The marginal cost of increasing M_i is the cost of inefficient liquidation w, which is borne by sellers in all states in which they do not default, that is with probability $1 - qp_i$. A decreasing g implies concavity of the problem.

3.3 Margining in the CCP

By Lemma 2, the CCP's maximal liability towards its members is $\overline{\varepsilon}$. Observe that since the CCP is solvent in all states, in any candidate equilibrium there is at most one clearing member defaulting. Define $s := \sum_i 1_{\{\sigma_i = c\}}$. We refer to s as the "size" of the CCP. It follows that the amount of collateral the CCP requires sellers to post at $z = 1$, denoted with M^c, is:

$$M^c = \frac{\overline{\varepsilon}}{s}$$

The CCP's individual collateral requirement is decreasing in the number of clearing members. While in reality variation margins are purely pass-through and initial margin depends on the member's own risk (not on how many members there are), we interpret M^c as a holistic clearing requirement that includes the member's default-fund share in addition to initial margin. Because default funds are sized to cover extreme but plausible losses—commonly the simultaneous default of the two largest members—the total target does not rise one-for-one with membership. Consequently, as participation increases, the per-member default-fund allocation declines. In our framework, this pattern is a direct implication of negatively correlated exogenous defaults across FIs.

If seller i is a member and defaults, the CCP uses its funds to pay the buyer ε_i. The residual $\overline{\varepsilon} - \varepsilon_i$ is redistributed equally across all members. Just like it occurs in the context of OTC contracts, the collateral that is paid back to the defaulted seller can be thought of as contributing to default costs.

3.4 Benchmark: isolated pairs ($\rho = 0$)

The model encompasses an enviroment with no scope for contagion across pairs, which is obtained by setting $\rho = 0$. To characterize the equilibrium, denote with V_i^o and V_i^c pair i's expected payoff from staying OTC and clearing their contract respectively. Because OTC pairs operate in isolation, their payoff from choosing to stay OTC does not depend on all other pairs' chosen profile $\{M_j\}_{j \neq i}$ and $\{\sigma_j\}_{j \neq i}$. By contrast, the value of clearing via a CCP depends on the profile of clearing decisions $\{\sigma_j\}_{j \neq i}$. In particular, in the current benchmark, a sufficient statistic is the size of the CCP, s, which affects both the CCP's margin requirements as well as the expected contribution of each member when a CCP must pay a defaulted seller's obligations. While a larger size reduces the former, it also increases the latter. The result below characterizes the net effect.

Lemma 3. *A larger CCP size increases the value of clearing ($V_i^c(s)$ increasing) if*

$$w \geq \frac{q}{N-q} \frac{\mathbb{E}\varepsilon}{\overline{\varepsilon}} \qquad (\star)$$

Intuitively, higher margins are more costly the higher is w, which measures the opportunity cost of liquidating the long-term project, whereas the expected payment that the CCP makes to replace a defaulted member's obligation is independent of it. Thus, a sufficiently large w guarantees that the benefits of spreading the default fund contribution across more members outweighs the higher expected costs due to more frequent defaults inside the CCP. We will mantain condition $(\star)$ throughout the rest of the analysis.

Assumption 1. *Condition $(\star)$ holds.*

Observe that the only advantage that joining the CCP confers pertains to the fungibility of collateral among members, which makes it possible for FIs to insulate against counterparty risk at lower collateral costs. For this advantage to exist, a necessary condition is that more than one pair chooses to clear. Moreover, under A1, the decision to join the CCP becomes a strategic complement irrespective of the strategy profile (that is, for any size s). These considerations lead to the following result.

Proposition 1. *Assume that $\rho = 0$. Then:*

(i). There always exists an equilibrium with $s = 0$ (no participation).

(ii). If $V^c(N) \geq V^o$, there is an equilibrium with $s = N$ (full participation).

(iii). The $s = N$ equilibrium, when it exists, Pareto-dominates the $s = 0$ equilibrium.

The existence of an equilibrium without clearing is due to a classic coordination failure. In this setup, the economies of scale are not due to the well-known multilateral netting benefits of clearing (see Menkveld and Vuillemey [2021]) but instead from collateral fungibility alone.

4 Main Results

We can now analyze the general setup with $\rho > 0$. When contagion across pairs is possible, the choices of each pair impose externalities onto the system beyond the economies of scale in the CCP and the whole strategy profile affects both functions V^o and V^c. In particular, assume that pair i chooses to stay OTC, and note that:

$$p_{i+1} = \underbrace{\frac{1}{N}}_{\text{direct default}} + \underbrace{\rho\left[1 - G\left(M_i\right)\right] p_i}_{\text{contagious default}}$$

If pair i chooses to clear instead, buyer i is fully protected from seller's default, blocking potential for downstream contagion ($p_{i+1} = \frac{1}{N}$). In light of this, a useful variable to keep track of is the pair's "distance" from the closest upstream clearing pair.

$$\text{distance to CCP} = d_i := \text{number of preceding consecutive OTC pairs}$$

Intuitively, d corresponds to the number of possible contagion paths that can hit the seller in the focal pair.[4]

Indeed, the distance to CCP will be a sufficient statistic to compute a pair's equilibrium payoff given a strategy profile. This is formalized below.

Lemma 4. *For any profile* $(\mathcal{M}, \mathcal{S})$ *and any two pairs* i *and* j*:*

$$V_i^c(d; \mathcal{M}, \mathcal{S}) = V_j^c(d; \mathcal{M}, \mathcal{S})$$

and

$$V_i^o(d; \mathcal{M}, \mathcal{S}) = V_j^o(d; \mathcal{M}, \mathcal{S})$$

In light of the above, we can drop subscripts and denote with $V^c(d; \mathcal{M}, \mathcal{S})$ and $V^o(d; \mathcal{M}, \mathcal{S})$ the expected payoffs to a pair a distance d from the CCP, choosing to clear or remain OTC respectively.

While the function $V^c(d; \mathcal{M}, \mathcal{S})$ depends on d, and thus not all pairs who choose to clear receive the same expected payoff, we can derive a result analogous to Lemma 3 which will reveal useful for the analysis. To do so, note that the strategy profile $\mathcal{S}$ partitions the set $\{1, 2, ..., N\}$ into a sequence of chains $\{\mathcal{O}_1, \mathcal{C}_1, \mathcal{O}_2, \mathcal{C}_2, ...\}$. We define the chain lenghts with $L_n^o := |\mathcal{O}_n|$ and $L_n^c := |\mathcal{C}_n|$.

Lemma 5. *For for any two* $\left(\mathcal{S}, \mathcal{S}'\right)$ *where* $\mathcal{S}'$ *equals* $\mathcal{S}$ *except,* $L_n^{c'} = L_n^c + 1$ *for some* n*:*

$$V^c\left(d; \mathcal{M}, \mathcal{S}'\right) > V^c(d; \mathcal{M}, \mathcal{S})$$

Intuitively, when one OTC pair at distance $d = 0$ chooses to clear (one clearing chain becomes longer), it produces the same effects of a joining pair in the benchmark analyzed in the previous section. Our parameter restriction $(\star)$ implies that the net effect is beneficial to all existing members of the CCP. The results above drastically restrict the set of candidate equilibria.

Proposition 2. *In any equilibrium:*

(i). If multiple chains $\mathcal{O}_n$ *exist, they have the same lenght.*

(ii). If $s < N$*, it holds that* $L_n^c \leq 1$

The implication of the statement above is that there are only three possible types of equilibria: two candidate equilibria in which all pairs make the symmetric decision to clear or remain

[4] To formalize the distance d one can indicate, for every pair i, its predecessor by defining the function $u(i) = i - 1$ for all $i > 1$, with $u(1) = N$ due to the circularity of the network. Define $u^k(i)$ the k-th iteration of $u(i)$. Then, we have $d_i := \max\left\{d \in \{0, 1, .., N-1\} : \sigma_{u^k(i)} = o \text{ for all } k \leq d\right\}$.

OTC, and a set of "hybrid" equilibria in which the profile $\mathcal{S}$ is characterized by identical repeated chains of adjacent OTC pairs terminating with an individual clearing pair. Intuitively, consecutive clearing pairs cannot be part of an equilibrium with partial participation because the next adjacent OTC pair, by deviating, would induce net benefits to all clearing pairs under condition ($\star$).

Figure 1 illustrates the reason why consecutive pairs cannot clear in an equilibrium with partial participation.

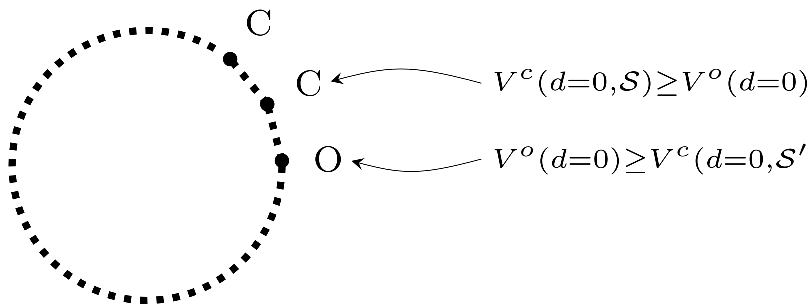

Figure 1: The picture illustrates the necessary conditions for two adjacent pairs not to deviate from their equilibrium clearing decisions. By Lemma 5, when ($\star$) holds the value of clearing for a pair at distance d increases when the strategy profile is modified by making one clearing chain longer. Hence the two conditions shown cannot hold simultaneously.

In light of Proposition 2, the equilibrium CCP size s is a sufficient statistic to describe clearing decisions. A given size s pins down, for a given N, the number and lenght of OTC chains in equilibrium. Clearly, a smaller s implies longer OTC chains, leading to the remark below.

Remark 1. *In an equilibrium with CCP size s, there are $\frac{N}{S}$ OTC chains of lenght $L^o = \frac{N}{s} - 1$. The largest number of simultaneous defaults that can occur on equilibrium path is $2\frac{N}{s} - 1$. Hence equilibria with lower clearing feature worse "tail" events.*

To obtain a more comprehensive picture, we must examine what occurs within the OTC chain and understand how risk propagates when endogenous contracting within the chain is taken into account. Denote with p_d the default probability in the bad state conditional on $z = +1$ for the seller in a pair at distance d from the CCP. Note that $p_0 = \frac{1}{N}$. For $d > 1$, we have:

$$p_d = \underbrace{\frac{1}{N}}_{\text{direct default}} + \underbrace{\rho\left[1 - G\left(M_d\right)\right] p_d}_{\text{contagious default}}$$

Observe that $p_1 \geq p_0$ and thus $M_1 \geq M_0$ with strict inequality as long as $G\left(M_1\right) < 1$. In words, a pair at distance $d = 1$ is exposed to contagion and therefore chooses higher protection. In principle, such endogenous response may lead to oscillations in exposures along a chain, that is a non-monotone profile of default probabilities p_d. Below, we identify a simple restriction on the model's primitives that rules this possibility out.

Lemma 6. *Assume that g is strictly decreasing and that $g(0) > \frac{1-qp}{qp}\frac{w}{x} \geq g(L)$ for all $p \in \left[\frac{1}{N}, 1\right]$ (interior optimal margins). If for all $\varepsilon \in [0, L]$*

$$-\frac{g'(\varepsilon)}{g(\varepsilon)^2}(1 - G(\varepsilon)) \geq \frac{1}{1-q} \qquad (\star\star)$$

then, in any equilibrium, $p_d > p_{d-1}$ and $M_d > M_{d-1}$ for all d, and $V^o(d)$ is decreasing.

The sufficient condition in $(\star\star)$ ensures that privately optimal margins do not respond too strongly to counterparty risk. Along an OTC chain, the fact that pairs at greater distance from the CCP can be hit by more contagion paths implies a larger probability of default. This has a number of implications, formalized below.

Proposition 3. *If the conditions identified in Lemma 6 hold, equilibria with lower clearing:*

(i). are Pareto-dominated by those with higher clearing,

(ii). feature higher expected number of defaults,

(iii). feature higher default risk for sellers in the CCP.

4.1 Margin requirements

We now investigate the role of minimum margin requirements on OTC contracts. We assume that a margin requirement $\underline{M}$ applies to all pairs, that is, in all equilibria under the policy, it must hold that $M_d \geq \underline{M}$ for all OTC pairs at distance d. The result below highlights how margin requirements, whose rationale is to contain systemic risk, may have the unintended effect of discouraging central clearing, defeating their primary purpose.

Proposition 4. *For any N and any $\rho > 0$ there exist distributions G, parameters (q, R, L) and a margin requirement $\underline{M}$ such that*

(i). After the policy, the margin requirement binds for some d.

(ii). Compared to the laissez faire, the equilibrium under the requirement features strictly lower clearing.

Our constructive proof hinges on the fact that, when contagion is present ($\rho > 0$), for profiles such that $M_d > M_{d-1}$, a margin requirement that binds for an OTC pair at distance $d-1$ need not bind for pair at distance d. Formally, denote with $M_d(\underline{M})$ the optimal margin choice for pair at distance d given the minimum requirement is in place. Observe that, if the positive distance $\underline{M} - M_{d-1}$ is sufficiently small, $M_d(\underline{M}) > \underline{M}$. In such a region, a marginal increase in $\underline{M}$ reduces exposure to contagion and hence counterparty risk for a pair at distance d. The benefit of being preceded by a safer OTC chain can increase the value of remaining OTC, to an extent that pairs who would choose to clear in the laissez faire equilibrium prefer to remain OTC under the policy.

5 Conclusions

This paper develops a tractable framework in which the decision to centrally clear OTC derivatives is jointly determined with contract terms, shaping the equilibrium degree of default contagion in the financial system. As more contracts are cleared, CCP per-member default fund contributions decrease, but at the same time the OTC network becomes safer because cascades of defaults are more likely to be interrupted by clearing members. These two forces move the private value of clearing in opposite directions. We offer a disciplined analysis of how these natural forces interact.

By modeling contagion explicitly, we can describe systemic risk outside the CCP and risk borne by the CCP. Under conditions, low-clearing equilibria can simultaneously display higher risk in the OTC segment and higher expected losses in the CCP, even though the majority of defaults originate outside the cleared segment.

The policy implications follow directly from these incentive effects. In the benchmark without cross-pair contagion, minimum margins on OTC contracts unambiguously tilt incentives toward clearing by making bilateral trading more costly. Once contagion is present, however, the same policy has heterogeneous effects across institutions depending on their exposure to OTC chains. As a result, margin requirements on non-cleared derivatives can, in equilibrium, reduce clearing participation and raise systemic risk.

The model can serve as a starting point to evaluate, for example, the two-way feedback between CCPs' organizational design and systemic risk, or to cast light on the regulation of for-profit CCPs and related issues.

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