Human Skills in the Age of AI

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Abstract

Advances in artificial intelligence raise fundamental questions about how technology reshapes human skills. Modern AI systems may crowd out active human decision-making; unlike past technologies, such decisions are a key input for training and improving AI models. We develop a framework based on Markov matrices to study how AI substitutes for and augments different human skills, and how decision authority – whether the technology is assistive or autonomous – shapes these effects. We focus on two distinct skills: innovative skill, the ability to generate correct outcomes from incorrect inputs, and executional skill, the ability to reliably produce correct outcomes from correct inputs. We show that the presence of technology crowds out active human decision-making among individuals with low decision quality. Moreover, adoption patterns differ sharply across technologies: autonomous technologies complement innovative skills, whereas assistive technologies complement executional skills. Our model offers empirically testable relations among types and qualities of AI, their sensitivity to training data, human decision patterns, and overall welfare.

Key words: *Decision Authority, Integration, Innovation, Execution*

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1 Introduction

Technologies enable humans to complete many tasks more efficiently. As reliance on technology increases, however, human abilities to perform these tasks often deteriorate. For example, as students rely on calculators, their arithmetic skills weaken; as automatic transmissions become ubiquitous, fewer drivers know how to operate manual cars. While this pattern has long been observed, the resulting obsolescence of human executional skills was rarely a serious concern. As long as the final output is correct, it matters little whether it is produced by hand or by a machine.

This logic breaks down in the age of artificial intelligence. Modern AI systems rely critically on human inputs for training, and the tasks they perform are no longer purely executional. For instance, large language models such as ChatGPT learn to write by absorbing vast amounts of human-generated text. If individuals increasingly rely on such systems for writing and original human production declines, the technology itself may undergo structural change. This raises a fundamental question: when human skills are multidimensional, which skills are replaced by technology, and which are augmented? How does a technology's reliance on active human inputs shape this decomposition? And how do assistive technologies differ from autonomous technologies in their impact on active human decision-making?

To address these questions, we develop a framework based on Markov matrices to study substitution and augmentation between human decision-making and technology. Each agent chooses whether to adopt the technology and whether to rely on an active human decision rule or passively follow the technology's rule. Both human and technological decision rules are represented as Markov matrices over the action space. Motivated by the features of model training, we allow the technology provider to integrate its model with all active human decision rules. The platform charges a usage fee to maximize profit. This flexible integration of decision matrices allows us to capture, in a unified way, the distinction between assistive and autonomous technologies. When the technology's rule determines the final action, the technology is autonomous—even if it takes human input as an initial signal. By contrast, when humans retain final control, the technology is assistive.

Despite its simplicity, a binary action space—correct or incorrect—is rich enough to capture two distinct human skills. The ability to generate correct outcomes from incorrect inputs naturally corresponds to innovative skill. In contrast, the ability to reliably produce correct outputs from correct inputs reflects executional skill.

We first show that the presence of technology—whether assistive or autonomous—crowds out active human decision-making among individuals with the lowest decision quality, that is, those with weak innovative and executional skills. For these individuals, skills become most obsolete in the presence of technology. More counterintuitively, assistive technologies can crowd out more active human decision-making than autonomous technologies. With autonomous technologies, initial human input remains valuable when it improves the quality of the status-quo input. By contrast, assistive technologies generate the initial decision themselves, raising the bar for human intervention to be beneficial and thereby discouraging active human involvement.

Finally, we show that the division between users and non-users differs sharply across assistive and autonomous technologies. Because autonomous technologies make the final decision, their execution quality is paramount, and they complement humans with stronger innovative skills, who provide high-quality initial inputs. Assistive technologies, by contrast, derive value primarily from their innovative capability and thus complement humans with stronger executional skills, who implement the final decision. As a result, users of autonomous technologies tend to have higher innovative skills, whereas users of assistive technologies tend to have higher executional skills.

Related Literature

This paper is related to the classic literature on how technological change reshapes the demand for human skills. Canonical task-based frameworks emphasize that computerization substitutes for routine tasks while complementing non-routine cognitive activities, thereby reweighting the relative importance of different skill dimensions in production, for example, Autor et al. (2003), Goos et al. (2014), and Acemoglu and Restrepo (2022). More broadly, the literature on skill-biased technical change emphasizes that technological progress complements skilled labor and raises relative demand for higher skills, with implications for wage inequality and employment composition. Violante (2018) provides a comprehensive review of this literature.

A growing empirical literature on artificial intelligence documents substantial heterogeneity in its effects across workers and settings. Both Maasoum and Lichtinger (2025) and Brynjolfsson et al. (2025) document declines in employment among early-career workers in AI-exposed occupations, while employment for more experienced workers remains stable or increases. These empirical findings are consistent with our result that individuals with low skills along both dimensions optimally choose passive use, which makes them more susceptible to substitution.

In terms of topic, the closest to our paper is Ide and Talamàs (2025), which builds on the literature on knowledge hierarchies pioneered by Garicano (2000) and further developed in later work by Garicano and coauthors (e.g., Garicano and Rossi-Hansberg, 2004; Antràs et al., 2006; Fuchs et al., 2015). Ide and Talamàs (2025) model AI as a scalable agent capable of applying tacit knowledge at scale through compute. Embedding AI into a hierarchical production framework, they show that in competitive equilibrium, AI reorganizes occupational roles by substituting for humans with similar knowledge while complementing those with different knowledge levels, thereby reshaping labor allocation across the economy. Compared to their framework, we study endogenous AI technologies that learn and improve by integrating active human decision rules, as well as a technology provider's optimal design of usage fees to maximize profit while accounting for this reliance on human inputs for model enhancement. Besides, we introduce a multidimensional skill structure that distinguishes between execution and innovative skills, highlighting skill-specific effects of different AI technologies: assistive AI primarily complements execution skills and is therefore adopted by individuals with strong execution but weaker innovative ability, whereas autonomous AI mainly complements innovative skills.

Methodologically, this paper builds on the Markov-matrix framework developed in Zhong (2025) to study how AI substitutes for or augments multidimensional human skills. Zhong (2025) shows that, in multilayered decision processes, the optimal integration rule is to deploy higher-quality technologies (measured by error-correction capability normalized by new errors introduced) in later stages. We adapt this framework to analyze how AI shapes and is shaped by human decisions.

The remainder of the paper is organized as follows. Section 2 presents the model setup. Section 3 characterizes the equilibrium for both autonomous and assistive technologies, examining how technology providers set usage fees and how players self-select into active or passive decision makers. Section 4 analyzes the broader implications of technological progress on active choice, specifically demonstrating how autonomous systems can unexpectedly induce more active human decision-making than assistive ones. Section 5 concludes.

2 Model Setup

Players are infinitesimal and indexed by a type $s \in \mathbb{S}$, where $\mathbb{S}$ is a measurable type space. A player's type summarizes her decision-making characteristics. With a slight abuse of terminology, we also refer to $\mathbb{S}$ as the set of players. There is also a default type, $\mathcal{I} \in \mathbb{S}$, reflecting the decision characteristics associated with a passive decision.

A technology provider supplies a technology $t \in \mathbb{T}$. Each player chooses whether to adopt the technology and, conditional on that choice, whether to make an active decision using her own decision rule s or to rely on the default $\mathcal{I}$ in the decision-making process. Accordingly, the player set $\mathbb{S}$ can be partitioned into four subsets $\{\mathbb{U}_1, \mathbb{U}_0, \mathbb{N}_1, \mathbb{N}_0\}$, where $\mathbb{U}$ and $\mathbb{N}$ denote users and non-users of the technology, respectively, and the subscript 1 and 0 indicate active decision making (using s) and passive decision making (using $\mathcal{I}$).

Innovation v.s. Execution Skills Suppose there are two possible outcomes, viable and non-viable, which together form a binary state space. The ex ante probability of a viable outcome is p_0, yielding the prior distribution $\mathbf{p}_0 = (p_0, 1 - p_0)$ over the state space. We assume $p_0 < \frac{1}{2}$ so there is still sufficient

A decision-maker is characterized by two skill parameters: s_x, the probability of correctly executing a viable recommendation, and s_v, the probability of identifying errors and transforming a non-viable recommendation into a viable outcome. We refer to s_x as execution skill and s_v as innovation skill.

A player's type s is represented by the transition matrix

$$\mathcal{S} = \begin{pmatrix} s_x & 1 - s_x \\ s_v & 1 - s_v \end{pmatrix},$$

where the rows correspond to the initial state (viable or non-viable) and the columns correspond to the resulting state after the decision. In this matrix, s_x represents the probability of preserving a viable recommendation, while s_v represents the probability of converting a non-viable recommendation into a viable outcome. Applying this transition to the prior state distribution yields the posterior distribution $\mathbf{p}_0\mathcal{S}$ after the decision is made.

We assume that players are uniformly distributed over the right triangle defined by $0 \leq s_v \leq s_x \leq 1$ in the (s_x, s_v) space. For any player, execution skill s_x must be at least as high as innovation skill

s_v, reflecting the fact that it is generally easier to maintain a viability than to generate it from a non-viable recommendation.

Two metrics prove particularly useful for characterizing the transition of the state distribution: the skill difference $\delta_s = s_x - s_v$, and the invariant probability $\eta_s \equiv s_v/(s_v + 1 - s_x)$, which captures innovation skill s_x relative to execution error $1 - s_x$ and provides a one-dimensional measure of decision quality. Applying the decision rule described by the transition matrix shifts the prior probability of the viable state toward η_s at a convergence speed of $1 - \delta_s$.

The decision maker can passively accept the recommendation without modification. This corresponds to the default type

$$\mathcal{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

which leaves the state unchanged. We interpret this default type as passive acceptance of the status quo, with no execution errors and no innovation.

Technologies Technologies can be represented as $\mathcal{T} = \mathcal{T}(\mathcal{M}, \mathbb{U}_1 \cup \mathbb{N}_1)$, where $\mathcal{M}$ is a two-dimensional Markov matrix that encodes the technology's innovation and execution qualities. Intuitively, $\mathcal{T}$ can be "trained" using observable decision rules in the economy, specifically, the active decision-making rules of users and non-users ($\mathbb{U}_1 \cup \mathbb{N}_1$). A key assumption here is that technology can only be trained on *observable decision rules*. If a player abandons their own skill s and instead uses the passive decision $\mathcal{I}$ (i.e., the set $\mathbb{U}_0 \cup \mathbb{N}_0$), the potential training value associated with observing s is lost. This mechanism captures the possibility that technology use may *reduce active decision-making*, which in turn affects the training of the technology.

In general, the technology offered to users can be trained arbitrarily based on the active decision rules in the economy, $\mathbb{U}_1 \cup \mathbb{N}_1$. For concreteness, we adopt the following formulation as our primary specification

$$\mathcal{T} = E\big(\mathcal{S} \mid \mathcal{S} \in \mathbb{S}(\mathbb{U}_1 \cup \mathbb{N}_1)\big)\mathcal{M}, \tag{1}$$

where $\mathcal{M}$ provides a linear transformation of the average active decision rules based on the average type in $\mathbb{U}_1 \cup \mathbb{N}_1$. Here, $\mathcal{M}$ itself is a two-dimensional Markov matrix. The skill differences, δ_t and δ_m, and the invariant probabilities, η_t and η_m, are defined analogously to those of the player, but for the technology matrix $\mathcal{T}$ and the model $\mathcal{M}$. We assume that both the model $\mathcal{M}$ and the average player type $E(\mathcal{S} \mid \mathcal{S} \in \mathbb{S})$ have invariant probabilities exceeding p_0. This ensures that the technology represents an improvement over the initial situation.

We consider two types of technologies: autonomous and assistive. Autonomous technologies follow human directives yet exercise final control, while assistive technologies offer suggestions to human users, who retain ultimate decision-making authority. The differences in production and payoffs are detailed below.

Production and Payoffs The payoff associated with viable and non-viable final outcomes is normalized as $\mathbf{1} \equiv (1,0)^\top$. In the absence of technology, the posterior state distribution is $\mathbf{p}_0\mathcal{S}$ for active decision makers and $\mathbf{p}_0\mathcal{I}$ for passive decision makers. A player of type $s \in \mathbb{S}$ produces and receives a payoff

$$\begin{cases} Y(\mathcal{S}) = \mathbf{p}_0\mathcal{S}\mathbf{1} & \text{if in } \mathbb{N}_1 \\ Y(\mathcal{I}) = \mathbf{p}_0\mathcal{I}\mathbf{1} = p_0 & \text{if in } \mathbb{N}_0 \end{cases}. \tag{2}$$

where $\mathcal{S}$ and $\mathcal{I}$ denote the active and passive decision rules, respectively.

For autonomous technologies, the state distribution is first transformed by the human directive $\mathcal{S}$ or $\mathcal{I}$, followed by the technology $\mathcal{T}$. Production is then given by

$$\begin{cases} G(\mathcal{S}|\mathcal{T}) = \mathbf{p}_0\mathcal{S}\mathcal{T}\mathbf{1} & \text{if in } \mathbb{U}_1 \\ G(\mathcal{I}|\mathcal{T}) = \mathbf{p}_0\mathcal{I}\mathcal{T}\mathbf{1} = \mathbf{p}_0\mathcal{T}\mathbf{1} & \text{if in } \mathbb{U}_0 \end{cases}. \tag{3}$$

For assistive technologies, the technology matrix $\mathcal{T}$ first transforms the state distribution, after which human skill $\mathcal{S}$ or $\mathcal{I}$ is applied. Production is

$$\begin{cases} G(\mathcal{S}|\mathcal{T}) = \mathbf{p}_0\mathcal{T}\mathcal{S}\mathbf{1} & \text{if in } \mathbb{U}_1 \\ G(\mathcal{I}|\mathcal{T}) = \mathbf{p}_0\mathcal{T}\mathcal{I}\mathbf{1} = \mathbf{p}_0\mathcal{T}\mathbf{1} & \text{if in } \mathbb{U}_0 \end{cases}. \tag{4}$$

The monopolistic technology provider charges a uniform fee f to all users[1], so that a player's payoff when using the technology is given by $G(\cdot \mid \mathcal{T}) - f$.

[1] A uniform fee per use is more profitable for the provider than a fractional scheme that takes a fixed share of production.

Definition of Equilibrium The technology provider chooses f to maximize total profit

$$\max_{f} f\mu\left(\mathbb{U}_1 \cup \mathbb{U}_0\right). \tag{5}$$

For any given fee f, players rationally self-select into the four subsets $\{\mathbb{U}_1(f), \mathbb{U}_0(f), \mathbb{N}_1(f), \mathbb{N}_0(f)\}$ by choosing the option that yields the highest payoff:

$$\begin{cases} G(s|\mathcal{T}) - f & \text{if in } \mathbb{U}_1 \\ G(\mathcal{I}|\mathcal{T}) - f & \text{if in } \mathbb{U}_0 \\ Y(s) & \text{if in } \mathbb{N}_1 \\ Y(\mathcal{I}) & \text{if in } \mathbb{N}_0 \end{cases}. \tag{6}$$

An equilibrium consists of an optimal fee f^* that maximizes (5) and the corresponding player choices in technology usage and active/passive decisions $\{\mathbb{U}_1^*(f), \mathbb{U}_0^*(f), \mathbb{N}_1^*(f), \mathbb{N}_0^*(f)\}$.

3 Characterization of Equilibrium

In this section, we characterize player choices and the strategies of the technology provider, analyzing autonomous technologies in Section 3.1 and assistive technologies in Section 3.2.

3.1 Autonomous Technology

Autonomous technology receives human directives and executes actions without human final intervention. Within this framework, the human directive is represented by a state with distribution $\mathbf{p}_0\mathcal{S}$. The technology shifts the distribution toward its invariant distribution $(\eta_t, 1-\eta_t)$, producing

$$G(\mathcal{S}|\mathcal{T}) = \mathbf{p}_0\mathcal{S}\mathcal{T}\mathbf{1} = (1-\delta_t)\eta_t + \delta_t\mathbf{p}_0\mathcal{S}\mathbf{1}, \tag{7}$$

a weighted average of η_t and the output associated with the human directive, $\mathbf{p}_0\mathcal{S}\mathbf{1} = p_0 s_x + (1-p_0)s_v$. The technology's execution-innovation skill difference δ_t governs the relative weight of the human input in the final output.

The human player may also choose to act passively, in which case the technology operates according to the default directive $\mathcal{I}$, producing an output

$$G(\mathcal{I}|\mathcal{T}) = \mathbf{p}_0\mathcal{I}\mathcal{T}\mathbf{1} = (1-\delta_t)\eta_t + \delta_t p_0 \,. \tag{8}$$

While passive users of the technology produce a fixed output, active users generate an output that rises with their execution and innovation skills. However, this output responds less to player skills than for active decision makers who forgo the technology, who obtain $\mathbf{p}_0\mathcal{S}\mathbf{1}$ directly, without the moderating effect of the technology's quality.

A player's use of the technology is also influenced by the usage fee, which the provider sets to maximize its revenue. In principle, the technology provider could set the fee high enough that some low-skill players might prefer to rely on the default directive without using the technology. However, such a fee would be counterproductive, as it deters all players from using the technology. Lemma 1 formalizes this intuition.

Lemma 1 *The provider of the autonomous technology never sets a fee exceeding the difference in output between a passive user and a non-user of the technology,* $\mathbf{p}_0\mathcal{T}\mathbf{1} - p_0$*. Consequently, no player chooses to become a passive non-user,* $\mathbb{N}_0^* = \emptyset$*.*

Low-skill players benefit more from the technology than higher-skill players. The output difference between a user and a non-user of the technology is given by

$$G(\mathcal{S}|\mathcal{T}) - Y(\mathcal{S}) = (1-\delta_t)(\eta_t - \mathbf{p}_0\mathcal{S}\mathbf{1}) \tag{9}$$

for active decision makers, and

$$G(\mathcal{I}|\mathcal{T}) - Y(\mathcal{I}) = (1-\delta_t)(\eta_t - p_0) \tag{10}$$

for passive players. This difference is larger for lower-skill players. Intuitively, a fee that is too high for low-skill players, who benefit most from the technology, also discourages higher-skill players from using it.

Any usage fee below the threshold in Lemma 1 divides players into at most three groups: passive users of the technology ($\mathbb{U}_0$), active users of the technology ($\mathbb{U}_1$), and active non-users of the technology ($\mathbb{N}_1$). Specifically, players unable to improve on the default directive become passive users ($\mathbb{U}_0$), whereas those who can generate higher output than the technology net of the fee act independently as active non-users ($\mathbb{N}_1$). The following proposition characterizes this partition.

Proposition 1 *Let the usage fee satisfy $f \leq \mathbf{p}_0\mathcal{T}\mathbf{1} - p_0$. Then a player with access to autonomous technology chooses*

$$\begin{cases} \mathbb{U}_0, & \text{if } p_0 s_x + (1-p_0)s_v \leq p_0, \\ \mathbb{U}_1, & \text{if } p_0 < p_0 s_x + (1-p_0)s_v \leq \eta_t - \frac{f}{1-\delta_t}, \\ \mathbb{N}_1, & \text{if } p_0 s_x + (1-p_0)s_v > \eta_t - \frac{f}{1-\delta_t}. \end{cases} \tag{11}$$

Players choose their actions based on the weighted average of their execution and innovation skills, $p_0 s_x + (1-p_0)s_v$, as illustrated in Figure 1. Those with the lowest average skill choose $\mathbb{U}_0$, those with intermediate average skill choose $\mathbb{U}_1$, and those with the highest average skill choose $\mathbb{N}_1$. The threshold between $\mathbb{U}_1$ and $\mathbb{N}_1$ reflects the trade-off among the technology's output, players' own capability, and the technology usage fee. In comparison, the threshold separating $\mathbb{U}_0$ and $\mathbb{U}_1$ does not depend on the technology's quality or the provider's decision. Specifically, $\mathbb{U}_0$ comprises players whose average skill lies below p_0, whereas active decision makers in $\mathbb{U}_1 \cup \mathbb{N}_1$ are those with average skill exceeding p_0. As a result, any specification of the technology $\mathcal{T} = \mathcal{T}(\mathcal{M}, \mathbb{U}_1 \cup \mathbb{N}_1)$, including but not limited to the linear specification in (1), is independent of the provider's pricing strategy.

In equilibrium, the technology provider chooses the usage fee f to maximize revenue, equal to the fee multiplied by the measure of users, $\mu(\mathbb{U}_1 \cup \mathbb{U}_0)$. Equivalently, the provider can be viewed as choosing an adoption threshold θ, defined by $\mathbf{p}_0\mathcal{S}\mathbf{1} = \theta$, which separates users in $\mathbb{U}_1 \cup \mathbb{U}_0$ from non-users in $\mathbb{N}_1$. Under this parametrization, the fee associated with threshold θ is $f(\theta) = (1-\delta_t)(\eta_t - \theta)$. By Lemma 1, feasibility requires $f(\theta) \leq \mathbf{p}_0\mathcal{T}\mathbf{1} - p_0 = (1-\delta_t)(\eta_t - p_0)$, which implies $\theta \geq p_0$. The provider's problem therefore reduces to

$$\max_{\theta \in [p_0, 1]} (1-\delta_t)(\eta_t - \theta)\, \mu\big(\mathcal{S} \mid p_0 s_x + (1-p_0)s_v \leq \theta\big) \tag{12}$$

Although the execution–innovation skill gap of the technology, δ_t, proportionally scales the provider's payoff, it does not influence the optimal strategy. Proposition 2 characterizes the resulting optimal threshold θ and usage fee.

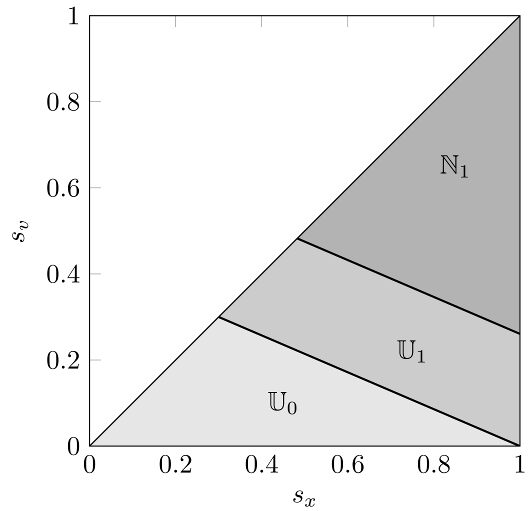

Figure 1: Player Choices with Autonomous Technologies

The figure depicts the decision regions for a player with access to autonomous technology in $\mathbb{S} = \{(s_x, s_v)|0 \leq s_v \leq s_x \leq 1\}$ with light, medium, and dark gray representing $\mathbb{U}_0$ (passive users), $\mathbb{U}_1$ (active users), and $\mathbb{N}_1$ (active non-users) respectively. Parameters are $p_0 = 0.3$ and $\eta_t = 0.9$, and the technology provider sets $f = 0.2952$.

Proposition 2 *The autonomous technology provider chooses the adoption threshold* $\mathbf{p}_0\mathcal{S}\mathbf{1} = \theta^*$, *with*

$$\theta^* = \max\left\{\frac{2 + \eta_t - \sqrt{(1-\eta_t)^2 + 3(1-p_0)}}{3}, p_0\right\}. \tag{13}$$

and sets the corresponding fee as $f^* = (1-\delta_t)(\eta_t - \theta^*)$.

When the technology is of reasonably high quality, $\eta_t > 3p_0/2$, the provider optimally chooses an adoption threshold $\theta^* > p_0$, leading to a positive measure of players actively using the technology ($\mathbb{U}_1$). This condition is satisfied in most reasonable scenarios, for instance, when the technology is model-free and reflects the average type of active decision-makers $\mathcal{T} = E\big(\mathcal{S} \mid \mathcal{S} \in \mathbb{S}(\mathbb{U}_1 \cup \mathbb{N}_1)\big)$. The case is illustrated in Figure 1, where the provider selects the optimal fee and adoption threshold in response to the exogenously specified technology $\eta_t = 0.9$.

Within this region, the optimal threshold θ^* rises with technology quality η_t. As the technology becomes more effective, players gain greater output from using it compared with acting independently. This improvement allows the provider to both increase the usage fee and attract a larger user base. Corollary 1 formalizes how θ^* responds to changes in η_t.

Corollary 1 *As the technology's invariant probability* η_t *rises, the optimal threshold* θ^* *increases.*

This improvement in technology prompts some active non-users $\mathbb{N}_1$ *to transition into active users* $\mathbb{U}_1$*.*

In the less common case in which the technology is not of sufficiently high quality relative to the initial situation, $\eta_t \leq 3p_0/2$, the provider sets $\theta^* = p_0$. In this case, players either passively use the technology ($\mathbb{U}_0$) or make active decisions without technology ($\mathbb{N}_1$), so the three-way partition collapses into a two-way partition.

3.2 Assistive Technology

In contrast to autonomous technology, assistive technology supports and advises human decision-making, while leaving final control and execution with the human. The technology's advice is represented by the distribution $\mathbf{p}_0\mathcal{T}$, and the resulting output is equal to the weighted average of the player's decision quality η_s and $\mathbf{p}_0\mathcal{T}\mathbf{1}$,

$$G(\mathcal{S}|\mathcal{T}) = \mathbf{p}_0\mathcal{T}\mathbf{1}s_x + (1-\mathbf{p}_0\mathcal{T}\mathbf{1})s_v \; = (1-\delta_s)\eta_s + \delta_s\mathbf{p}_0\mathcal{T}\mathbf{1}\,. \tag{14}$$

Because the technology improves the state distribution relative to the initial state, this output is always higher than that from active decision making without technology, which is given by

$$Y(\mathcal{S}) = p_0 s_x + (1-p_0)s_v \; = (1-\delta_s)\eta_s + \delta_s p_0\,. \tag{15}$$

Assistive technology also places a higher weight on execution skill s_x and a lower weight on innovation skill s_v. In other words, players with a high execution skill relative to the innovation skill benefits more from the adoption of assistive technology.

The human player may also follow the technology's advice without further intervention, in which case the output is $G(\mathcal{I}|\mathcal{T}) = \mathbf{p}_0\mathcal{T}\mathbf{1}$. Similar to autonomous technology, this action $\mathbb{U}_0$ always dominate passive decision without technology $\mathbb{N}_0$. Lemma 2 describes this result.

Lemma 2 *The provider of assistive technology never sets a fee above* $\mathbf{p}_0\mathcal{T}\mathbf{1} - p_0$*. Consequently, no player chooses to become a passive non-user,* $\mathbb{N}_0^* = \emptyset$*.*

Players whose weighted average of execution and innovation skills, $p_0 s_x + (1-p_0)s_v$, exceeds $\mathbf{p}_0\mathcal{T}\mathbf{1} - f$ will choose $\mathbb{N}_1$ over $\mathcal{U}_0$. However, unlike in the case of autonomous technology, this weighted average is no longer the sole determinant of players' actions. Instead, the relative contributions

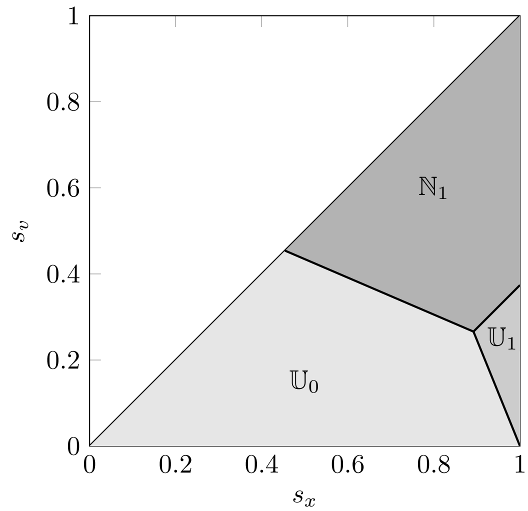

Figure 2: Player Choices with Assistive Technologies

The figure depicts the decision regions for a player with access to assistive technology in $\mathbb{S} = \{(s_x, s_v)|0 \leq s_v \leq s_x \leq 1\}$ with light, medium, and dark gray representing $\mathbb{U}_0$ (passive users), $\mathbb{U}_1$ (active users), and $\mathbb{N}_1$ (active non-users), respectively. Parameters are $p_0 = 0.3, \eta_m = 0.8, \delta_m = 0.5$, and the technology provider sets $f = 0.2578$.

of execution and innovation skills, captured by δ_s and η_s, also play a significant role. Specifically, players whose execution–innovation skill difference satisfies

$$\delta_s(\mathbf{p}_0\mathcal{T}\mathbf{1} - p_0) = G(\mathcal{S}|\mathcal{T}) - Y(\mathcal{S}) < f\,, \tag{16}$$

will choose $\mathbb{N}_1$ over $\mathbb{U}_1$. Meanwhile, players with a high innovation skill relative to their execution error, satisfying

$$(1 - \delta_s)(\eta_s - \mathbf{p}_0\mathcal{T}\mathbf{1}) = G(\mathcal{S}|\mathcal{T}) - G(\mathcal{I}|\mathcal{T}) \geq 0 \tag{17}$$

will choose $\mathbb{U}_1$ over $\mathbb{U}_0$. Proposition 3 summarizes the preceding analysis.

Proposition 3 *Let the usage fee satisfy* $f \leq \mathbf{p}_0\mathcal{T}\mathbf{1} - p_0$. *Then a player with access to assistive technology chooses*

$$\begin{cases} \mathbb{U}_0, & \textit{if } \eta_s \leq \mathbf{p}_0\mathcal{T}\mathbf{1} \textit{ and } p_0 s_x + (1-p_0)s_v \leq \mathbf{p}_0\mathcal{T}\mathbf{1} - f, \\ \mathbb{U}_1, & \textit{if } \eta_s > \mathbf{p}_0\mathcal{T}\mathbf{1} \textit{ and } \delta_s \geq \frac{f}{\mathbf{p}_0\mathcal{T}\mathbf{1} - p_0}, \\ \mathbb{N}_1, & \textit{if } p_0 s_x + (1-p_0)s_v > \mathbf{p}_0\mathcal{T}\mathbf{1} - f \textit{ and } \delta_s < \frac{f}{\mathbf{p}_0\mathcal{T}\mathbf{1} - p_0}. \end{cases} \tag{18}$$

The sets $\mathbb{U}_0$, $\mathbb{U}_1$, and $\mathbb{N}_1$ partition $\mathbb{S}$ radially, with three boundaries intersecting at a common point, as shown in Figure 2. The boundary between $\mathbb{U}_0$ and $\mathbb{N}_1$ is parallel to the boundaries in Figure 1. The boundary between $\mathbb{U}_0$ and $\mathbb{U}_1$ emanates from the point $(s_x, s_v) = (1, 0)$, which corresponds to players of type $s = \mathcal{I}$. The boundary between $\mathbb{U}_1$ and $\mathbb{N}_1$ has a 45-degree slope, separating players with high and low execution–innovation skill differences.

In this equilibrium, players with the highest innovation skill choose to make active decisions without using the technology, selecting $\mathbb{N}_1$. All remaining players use the technology. Among these technology users, players with sufficiently high execution skill choose to make active decisions, selecting $\mathbb{U}_1$, whereas the remaining players, who have relatively low skills in both dimensions, choose $\mathbb{U}_0$.

The technology provider's pricing strategy affects the composition of active decision makers, $\mathbb{U}_1 \cup \mathbb{N}_1$. A higher usage fee raises the skill thresholds required to make active decisions and, as a result, increases the average skill level within this training set. In turn, this enhances the technology, $\mathcal{T} = E\left(\mathcal{S} | \mathcal{S} \in \mathbb{U}_1 \cup \mathbb{N}_1\right) \mathcal{M}$.

We characterize the technology provider's revenue-maximization problem using the threshold θ, defined by the line $p_0 s_x + (1 - p_0) s_v = \theta$, which separates $\mathbb{N}_1$ from $\mathbb{U}_0$. The other boundary separating non-users and users of the technology, that is, between $\mathbb{N}_1$ and $\mathbb{U}_1$, is characterized by

$$\delta_s = \frac{\mathbf{p}_0 \mathcal{T}(\theta) \mathbf{1} - \theta}{\mathbf{p}_0 \mathcal{T}(\theta) \mathbf{1} - p_0}, \tag{19}$$

where $\mathcal{T}(\theta)$ emphasizes the dependence of $\mathcal{T}$ on θ. This threshold is decreasing in θ. Consequently, an increase in θ tightens both boundaries surrounding $\mathbb{N}_1$ and reduces the measure of non-users. We characterize the platform's optimal threshold and pricing decision in the following proposition.

Proposition 4 *The assistive technology provider sets the adoption threshold θ^* to the value that maximizes*

$$\max_{\theta \in [0,1]} \left(\mathbf{p}_0 \mathcal{T}(\theta) \mathbf{1} - \theta\right) \mu\left(\mathbb{U}_1(\theta) \cup \mathbb{U}_0(\theta)\right), \tag{20}$$

where, for each θ, the scalar $\mathbf{p}_0 \mathcal{T}(\theta) \mathbf{1}$ is uniquely determined by the implicit equation

$$\mathbf{p}_0 \mathcal{T}(\theta) \mathbf{1} = \mathbf{p}_0\, E\left(\mathcal{S} | \mathcal{S} \in \mathbb{U}_1(\theta) \cup \mathbb{N}_1(\theta)\right) \mathcal{M}\, \mathbf{1}, \tag{21}$$

with the sets $\mathbb{U}_1(\theta)$ *and* $\mathbb{N}_1(\theta)$*, specified in Proposition 3, themselves depend on* $\mathbf{p}_0\mathcal{T}(\theta)\mathbf{1}$*.*

The corresponding usage fee is $f^* = \mathbf{p}_0\mathcal{T}(\theta^*)\mathbf{1} - \theta^*$*.*

Similar to the case of Autonomous Technologies, the provider chooses $\theta^* > p_0$ when the technology is of reasonably high quality. Figure 2 illustrates the resulting partition when the provider selects the optimal fee and adoption threshold in response to an exogenously specified model ($\eta_m = 0.8$, $\delta_m = 0.5$). In the less common scenario where the technology is not of sufficiently high quality relative to the initial distribution, the provider sets $\theta^* = p_0$. Here, players either passively use the technology ($\mathbb{U}_0$) or make active decisions without it ($\mathbb{N}_1$), so the three-way partition collapses into a two-way one. Since active decision-making with technology ($\mathbb{U}_1$) no longer occurs, there is no distinction between Assistive and Autonomous Technology. They are effectively identical.

Because the technology $\mathcal{T}$ is endogenous to the provider's decision, we focus on the model $\mathcal{M}$, which is exogenous to the provider's choice. Variations in the model influence all three boundaries separating $\mathbb{U}_0$, $\mathbb{U}_1$, and $\mathbb{N}_1$. In particular, as the model quality η_m increases, the boundary between $\mathbb{U}_0$ and $\mathbb{U}_1$, $\eta_t = \mathbf{p}_0\mathcal{T}(\theta^*)\mathbf{1}$, rotates counterclockwise around $(s_x, s_v) = (1, 0)$, driven by the fact that

$$\mathbf{p}_0\mathcal{T}(\theta^*)\mathbf{1} = (1 - \delta_m)\eta_m + \delta_m\,\mathbf{p}_0 E\left(\mathcal{S} \mid \mathcal{S} \in \mathbb{U}_1(\theta^*) \cup \mathbb{N}_1(\theta^*)\right)\mathbf{1}\,, \tag{22}$$

increases with η_m. As a result, some users who were previously active $\mathbb{U}_1$ transition to passive usage $\mathbb{U}_0$.

The movement of the other two boundaries depends on the technology provider's choice θ^*. Corollary 2 characterizes how the optimal threshold θ^* depends on η_m and highlights the resulting implications for player choices.

Corollary 2 *As the model's invariant probability* η_m *rises, the optimal threshold* θ^* *increases. This improvement in model quality prompts active non-users* $\mathbb{N}_1$ *to transition to passive users* $\mathbb{U}_0$*. In addition, some technology users who previously made active decisions* $\mathbb{U}_1$ *shift to passive usage* $\mathbb{U}_0$*.*

Similar to the case of autonomous technology, an increase in η_m raises the optimal threshold θ^*, reflecting an expansion of the user base as model quality improves.

Equation (22) also highlights that the technology itself depends on input from active decision makers. Higher average skills among these decision makers enhance the technology's usefulness for all participants. The model's execution-innovation skill difference, δ_m, governs the relative importance of the model's inherent quality versus the contribution of active decision-making. This relative weighting also affects the technology provider's choice of adoption threshold. Corollary 3 characterizes how this decision varies with the model's execution-innovation skill difference δ_m.

Corollary 3 *When the average skill from active decision makers* $\mathbf{p}_0 E\left(\mathcal{S} \mid \mathcal{S} \in \mathbb{U}_1(\theta^*) \cup \mathbb{N}_1(\theta^*)\right) \mathbf{1}$ *equals the model invariant probability* η_m*, the optimal threshold* θ^* *increases with* δ_m*.*

This implies that, as the model places greater weight on the input from active decision makers, some active non-users $\mathbb{N}_1$ *transition to passive users* $\mathbb{U}_0$ *while others become active users* $\mathbb{U}_1$*.*

Corollary 3 holds $\mathbf{p}_0 E\left(\mathcal{S} \mid \mathcal{S} \in \mathbb{U}_1(\theta^*) \cup \mathbb{N}_1(\theta^*)\right) \mathbf{1} = \eta_m$ to isolate the effect of this relative weight. As δ_m increases, the influence of active decision makers grows, leading the technology provider to raise θ, thereby reducing the set of active decision makers and increasing the average skill level among those who remain active. Figure 3 illustrates this effect. The hatched region highlights active non-users who adopt the technology, transitioning from $\mathbb{N}_1$ to either $\mathbb{U}_0$ or $\mathbb{U}_1$. By keeping the average skill close to η_m, the figure isolates the impact of increasing δ_m on player behavior. As a result, the distribution of choices shifts: fewer players remain active, but those who do contribute more strongly to the technology, reflecting a concentration of influence among the most skilled participants.

4 Technology Usage and Active Choice

Technological progress reshapes who remains actively involved in decision making. The introduction of technology crowds out active decision making primarily among individuals with the lowest decision quality. As illustrated in Figures 1 and 2, players with weak skills in both execution and innovation and located closer to the origin optimally choose to become passive users of technology $\mathbb{U}_0$.

As technology improves, this effect intensifies. Improvements in assistive technology enhance its effectiveness as a substitute for human decisions, thereby shrinking the set of active decision makers, as formalized in Corollary 2. These comparative statics describe how technological quality affects individual choice, taking the structure of technology as given.

Although this paper treats the assistive versus autonomous form of technology as exogenous, improvements in quality may in practice influence how providers design and structure their systems. In particular, technologies are often deployed in an assistive mode when their quality is moderate. At

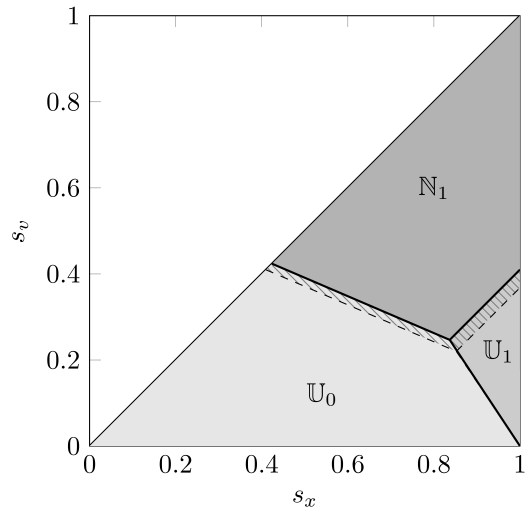

Figure 3: Player Choice Shifts with Stronger Technology Response to Active Input

The figure depicts how player choices shift when the technology assigns greater weight to input from active decision makers. The hatched region highlights active non-users who adopt the technology, transitioning from $\mathbb{N}_1$ to either $\mathbb{U}_0$ or $\mathbb{U}_1$. The parameters are set to $\eta_m = 0.6$, with δ_m chosen so that the equilibrium $\mathbf{p}_0 E\left(\mathcal{S} \mid \mathcal{S} \in \mathbb{U}_1(\theta^*) \cup \mathbb{N}_1(\theta^*)\right) \mathbf{1}$ remains close to η_m both before and after the change in δ_m.

intermediate performance levels, systems generate recommendations that reduce errors or improve efficiency while leaving critical decisions to humans. By contrast, autonomous technologies must operate reliably without human oversight, requiring substantially higher quality in sensing, reasoning, and control. Empirical and theoretical studies across domains such as autonomous vehicles, robotics, and enterprise AI illustrate this distinction: lower-capability systems function effectively as assistive tools, whereas true autonomy emerges only once performance crosses a threshold that ensures safe and reliable outcomes.

Counterintuitively, assistive technologies can crowd out active human decision-making more than autonomous technologies. With autonomous systems, initial human input remains valuable when it improves the state over the status quo. With autonomous systems, initial human input remains valuable whenever it improves the state relative to the status quo. Specifically, if $\mathbf{p}_0\mathcal{S}\mathbf{1} > p_0$, or equivalently, if the invariant probability satisfies $\eta_s > p_0$, active users achieve higher payoffs than passive users,

$$G(\mathcal{S}|\mathcal{T}) = \mathbf{p}_0\mathcal{S}\mathcal{T}\mathbf{1} > \mathbf{p}_0\mathcal{I}\mathcal{T}\mathbf{1} = G(\mathcal{I}|\mathcal{T})\,. \tag{23}$$

By contrast, assistive technologies generate the initial decision themselves, raising the threshold for human intervention to be beneficial and thereby discouraging active human involvement. According to Proposition 3, the human player's invariant probability η_s must exceed the elevated threshold $\mathbf{p}_0\mathcal{T}\mathbf{1}$ for their input to enhance production output.

This distinction between assistive and autonomous technologies has sharp implications for the extent of active human involvement, which we formalize in the following proposition.

Proposition 5 *Given the same initial state distribution* $\mathbf{p}_0 = (p_0, 1-p_0)$*, autonomous technologies always induce more active decision-making than assistive technologies, regardless of differences in model quality or technology specification,*

$$\mu_{Autonomous}(\mathbb{U}_1 \cup \mathbb{N}_1) \geq \mu_{Assistive}(\mathbb{U}_1 \cup \mathbb{N}_1)\,. \tag{24}$$

Autonomous technologies consistently promote more active decision-making than assistive technologies, independent of model quality or technology specification. This occurs because, under autonomous technologies, the set $\mathbb{U}_1 \cup \mathbb{N}_1$ is fixed by the triangular region defined by the boundary $p_0 s_x + (1-p_0)s_v = p_0$, whereas under assistive technologies, some players within this region may optimally choose to remain passive. Figure 4 compares player choices in Figure 1 and Figure 2. The vertically hatched trapezoid represents players who are passive users under assistive technology but become active users under autonomous technology.

Overall, the analysis highlights a surprising implication: as technology advances from moderate-quality, assistive systems to high-quality, autonomous systems, human engagement need not decline. Instead, reliable autonomous systems can create conditions that encourage active participation, as humans' contributions remain meaningful and beneficial. This suggests that technological progress, when coupled with effective system design, can strengthen rather than undermine human decision-making, highlighting how human skill and artificial intelligence can complement each other.

Figure 4 further illustrates how the shift from assistive to autonomous technologies reshapes the composition of technology users. Before the transition, the boundary between $\mathbb{N}_1$ and $\mathbb{U}_1$ has a positive slope; after the transition, it has a negative slope. As a result, players in the diagonally hatched trapezoid move from being active decision-makers who do not use the technology ($\mathbb{N}_1$) to active technology users ($\mathbb{U}_1$), while players in the dotted triangle shift from active technology users to active non-users. This reallocation of players across regimes is further characterized in Proposition 6.

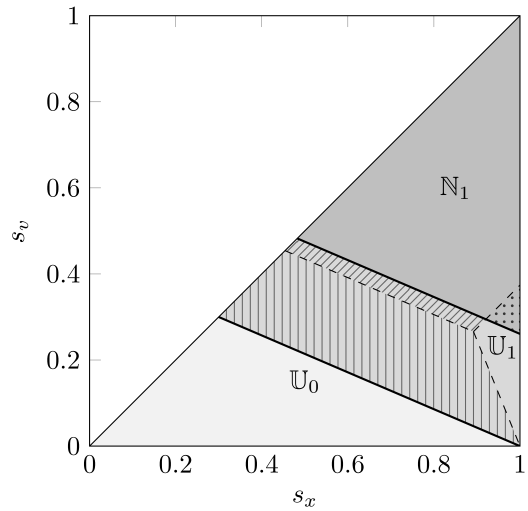

Figure 4: Evolution of player choices from assistive to autonomous technologies

The figure illustrates the transition of player choices between assistive and autonomous technologies. The vertically hatched trapezoid denotes passive users who become active ($\mathbb{U}_0 \to \mathbb{U}_1$), the diagonally hatched trapezoid represents the expansion of technology use by active decision-makers ($\mathbb{N}_1 \to \mathbb{U}_1$), and the dotted triangle indicates a contraction of usage ($\mathbb{U}_1 \to \mathbb{N}_1$). The parameters are consistent with those used in Figures 1 and 2, with the autonomous technology example characterized by a higher invariant probability η_t.

Proposition 6 *Moving from assistive to autonomous technology reallocates agents across regimes: players with high execution skill but only moderate innovation skill shift from active technology users* $\mathbb{U}_1$ *to active non-users* $\mathbb{N}_1$*, while a subset of players with moderate levels of both skills may move from* $\mathbb{N}_1$ *to* $\mathbb{U}_1$.

Proposition 6 highlights a reallocation in which players with high execution skills shift from active technology users to non-users, while those with relatively lower execution skills move in the opposite direction. This pattern arises because assistive and autonomous technologies require qualitatively different human skill sets. The distinction is ultimately driven by a stage-specific complementarity in this two-layer decision structure, whereby strong early-stage innovation enhances the returns to high execution skill in later stages, and strong execution similarly raises the value of early-stage innovation.

Autonomous technologies, which make the final decision, place a premium on execution quality and therefore complement humans with stronger innovation skills, who can provide high-quality initial inputs. When the probability of a viable human directive $\mathbf{p}_0\mathcal{S}\mathbf{1}$ is high, strong later-stage execution by the technology further amplifies the overall output, allowing early-stage innovation to have greater impact. Assistive technologies, by contrast, derive their value primarily from supporting innovative processes, and thus complement humans with stronger execution skills, who carry out the final implementation. The following expression illustrates that s_x is complementary to the technology, whereas s_v acts as a substitute,

$$\mathbf{p}_0 \mathcal{T} \mathcal{S} \mathbf{1} = \mathbf{p}_0 \mathcal{T} \mathbf{1} s_x + (1 - \mathbf{p}_0 \mathcal{T} \mathbf{1}) s_v \, . \tag{25}$$

When the technology's recommendations $\mathbf{p}_0 \mathcal{T} \mathbf{1}$ are sufficiently reliable, human innovation plays a diminished role.

Consequently, the composition of technology users differs sharply across the two modes. Among technology adopters, users of autonomous systems tend to exhibit higher innovation skills, while users of assistive systems tend to exhibit higher execution skills.

5 Conclusion

While the introduction of technologies like automatic transmissions and calculators has historically led to the deterioration of basic human executional skills, the age of artificial intelligence introduces a more complex dynamic where human input remains critical for model performance. This study demonstrates that the impact on human ability depends largely on whether a system is assistive or autonomous: while both forms crowd out active decision-making among those with low skill levels, assistive systems can paradoxically discourage human involvement more than autonomous ones by raising the threshold for beneficial intervention. Furthermore, the two modes complement distinct dimensions of expertise, with autonomous systems favoring those with high innovation skills to provide directives and assistive systems favoring those with high execution skills to implement suggestions. Ultimately, the transition toward high-quality autonomous technology does not necessitate the obsolescence of human skill; rather, it suggests that strategic system design can foster a sustainable complementarity where human expertise and artificial intelligence reinforce one another.

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Appendix

Proof of Lemma 1

We show that the action $\mathbb{N}_0$ is strictly dominated for any player.

Suppose, to the contrary, that under the optimal usage fee f^*, there exists a player who chooses $\mathbb{N}_0$. Then this player must strictly prefer $\mathbb{N}_0$ to passive use of the technology, $\mathbb{U}_0$, which requires

$$f^* > \mathbf{p}_0\mathcal{T}\mathbf{1} - \mathbf{p}_0\mathbf{1}. \tag{A1}$$

We show that condition (A1) implies that active use of the technology, $\mathbb{U}_1$, is strictly dominated for all players. First, consider players for whom $\mathbf{p}_0\mathcal{S}\mathbf{1} > \mathbf{p}_0\mathbf{1}$. Under (A1),

$$\mathbf{p}_0\mathcal{S}\mathcal{T}\mathbf{1} - f^* < \mathbf{p}_0\mathcal{S}\mathcal{T}\mathbf{1} - \big(\mathbf{p}_0\mathcal{T}\mathbf{1} - \mathbf{p}_0\mathbf{1}\big) = \mathbf{p}_0\mathcal{S}\mathbf{1},$$

so action $\mathbb{U}_1$ yields strictly lower payoff than $\mathbb{N}_1$ and is therefore strictly dominated.

Next, consider players for whom $\mathbf{p}_0\mathcal{S}\mathbf{1} \leq \mathbf{p}_0\mathbf{1}$. In this case,

$$\mathbf{p}_0\mathcal{S}\mathcal{T}\mathbf{1} - f^* \leq \mathbf{p}_0\mathcal{T}\mathbf{1} - f^* < \mathbf{p}_0\mathbf{1},$$

where the last inequality follows from (A1). Hence, $\mathbb{U}_1$ is strictly dominated by $\mathbb{N}_0$.

Thus, under (A1), no player optimally chooses $\mathbb{U}_1$, implying that no player uses the technology at all. Consequently, the technology provider earns zero revenue, implying that the fee f^* cannot be optimal. This contradicts the assumption that some player chooses $\mathbb{N}_0$. Therefore,

$$f^* \leq \mathbf{p}_0\mathcal{T}\mathbf{1} - \mathbf{p}_0\mathbf{1} = \mathbf{p}_0\mathcal{T}\mathbf{1} - p_0,$$

and $\mathbb{N}_0$ is never chosen in equilibrium, $\mathbb{N}_0^* = \emptyset$. ■

Proof of Proposition 1

Subtracting (7) from (8) yields

$$G(\mathcal{S} \mid \mathcal{T}) - G(\mathcal{I} \mid \mathcal{T}) = \delta_t\big(\mathbf{p}_0\mathcal{S}\mathbf{1} - p_0\big). \tag{A2}$$

This difference is nonnegative if and only if $\mathbf{p}_0\mathcal{S}\mathbf{1} \geq p_0$.

Next, from equation (9), we obtain

$$G(\mathcal{S} \mid \mathcal{T}) - f - Y(\mathcal{S}) = (1-\delta_t)\big(\eta_t - \mathbf{p}_0\mathcal{S}\mathbf{1}\big) - f. \tag{A3}$$

This expression is nonnegative if and only if $\mathbf{p}_0\mathcal{S}\mathbf{1} \leq \eta_t - f/(1-\delta_t)$.

Under the assumption $f \leq \mathbf{p}_0\mathcal{T}\mathbf{1} - p_0$, the latter threshold for $\mathbf{p}_0\mathcal{S}\mathbf{1}$ exceeds the former threshold,

$$\eta_t - \frac{f}{1-\delta_t} \geq \eta_t - \frac{\mathbf{p}_0\mathcal{T}\mathbf{1} - p_0}{1-\delta_t} = \eta_t - \frac{(1-\delta_t)\eta_t + \delta_t p_0 - p_0}{1-\delta_t} = p_0\,. \tag{A4}$$

where the equality follows from equation (8).

Taken together, these conditions characterize the player's optimal action, as summarized in (11).

■

Proof of Proposition 2

Because $\theta \geq p_0$, the set $p_0 s_x + (1-p_0)s_v > \theta$ forms a triangle in the (s_x, s_v) plane with vertices (θ, θ), $\big(1, (\theta - p_0)/(1-p_0)\big)$, and $(1,1)$. Its area is $\frac{(1-\theta)^2}{2(1-p_0)}$, and since players are uniformly distributed over a triangle of area $\frac{1}{2}$, the measure of this set is $\frac{(1-\theta)^2}{1-p_0}$. Consequently,

$$(\eta_t - \theta)\,\mu\big(\mathcal{S} \mid p_0 s_x + (1-p_0)s_v \leq \theta\big) = (\eta_t - \theta)\left(1 - \frac{(1-\theta)^2}{1-p_0}\right)\,. \tag{A5}$$

This expression is unimodal in θ over the interval $\theta \in [0,1]$. Differentiating with respect to θ yields the first-order condition

$$\theta = \frac{2 + \eta_t - \sqrt{(1-\eta_t)^2 + 3(1-p_0)}}{3} \tag{A6}$$

Thus, the provider's optimal adoption threshold is given by (13). The corresponding fee is determined by

$$\eta_t - \frac{f^*}{1-\delta_t} = \theta^*\,. \tag{A7}$$

■

Proof of Corollary 1

When $\theta^* > p_0$, differentiating (13) with respect to η_t gives

$$\frac{d\theta^*}{d\eta_t} = \frac{1}{3}\left(1 + \frac{1-\eta_t}{\sqrt{(1-\eta_t)^2 + 3(1-p_0)}}\right). \tag{A8}$$

It follows that θ^* is non-decreasing in η_t. Consequently, as the technology's invariant probability η_t increases, some active non-users $\mathbb{N}_1$ transition into active users $\mathbb{U}_1$. ■

Proof of Lemma 2

Suppose, toward a contradiction, that in equilibrium some player chooses action $\mathbb{N}_0$. Then this player must strictly prefer $\mathbb{N}_0$ to $\mathbb{U}_0$, which implies

$$f^* > \mathbf{p}_0\mathcal{T}\mathbf{1} - \mathbf{p}_0\mathbf{1}.$$

Under such a fee, all players strictly prefer $\mathbb{N}_0$ to $\mathbb{U}_0$.

Moreover, for any skill matrix $\mathcal{S}$,

$$\mathbf{p}_0\mathcal{T}\mathcal{S}\mathbf{1} - \mathbf{p}_0\mathcal{S}\mathbf{1} \;=\; \eta_s(\mathbf{p}_0\mathcal{T}\mathbf{1} - \mathbf{p}_0\mathbf{1}) \;\leq\; \mathbf{p}_0\mathcal{T}\mathbf{1} - \mathbf{p}_0\mathbf{1} \;<\; f^*,$$

where the equality follows from the fact that $\mathcal{S}$ shifts beliefs toward its invariant distribution. Hence, action $\mathbb{U}_1$ is strictly dominated by $\mathbb{N}_1$ for all players.

It follows that no player finds it optimal to adopt the technology under fee f^*, contradicting the assumption that $\mathbb{N}_0$ is chosen in equilibrium. Therefore, the provider never sets a fee above $\mathbf{p}_0\mathcal{T}\mathbf{1} - p_0$, and consequently $\mathbb{N}_0^* = \emptyset$. ■

Proof of Proposition 3

From equation (16), the player prefers $\mathbb{N}_1$ to $\mathbb{U}_1$ if and only if

$$\delta_s < \frac{f}{\mathbf{p}_0\mathcal{T}\mathbf{1} - p_0}. \tag{A9}$$

From equation (17), the player prefers $\mathbb{U}_1$ to $\mathbb{U}_0$ if and only if

$$\eta_s \geq \mathbf{p}_0 \mathcal{T} \mathbf{1}. \tag{A10}$$

Finally, the player prefers $\mathbb{N}_1$ to $\mathbb{U}_0$ if and only if

$$\mathbf{p}_0 \mathcal{S} \mathbf{1} = p_0 s_x + (1 - p_0) s_v > \mathbf{p}_0 \mathcal{T} \mathbf{1} - f. \tag{A11}$$

Taken together, these conditions characterize the player's optimal action, as summarized in (18).

■

Proof of Proposition 4

For a given θ, the associated usage fee satisfies $\theta = \mathbf{p}_0 \mathcal{T}(\theta) \mathbf{1} - f$, so that the provider's optimization problem is described by (20).

Let p^t represent $\mathbf{p}_0 \mathcal{T}(\theta) \mathbf{1}$. The set $\mathbb{U}_1(\theta) \cup \mathbb{N}_1(\theta)$ forms a non-convex quadrilateral with vertices

$$(\theta, \theta), \quad \left(\theta + \frac{p^t - \theta}{p^t - p_0}(1 - p_0),\ \theta - \frac{p^t - \theta}{p^t - p_0} p_0\right), \quad (1, 0), \quad (1, 1)\,.$$

Consequently, the average execution skill within $\mathbb{U}_1(\theta) \cup \mathbb{N}_1(\theta)$ is

$$E\left(s_x | \mathcal{S} \in \mathbb{U}_1(\theta) \cup \mathbb{N}_1(\theta)\right) = \frac{(1 - p^t)(p^t - p_0)\frac{2 + p^t}{3} + (p^t - \theta)^2\left(p^t + \frac{1 + p_0 - 2p^t}{3}\frac{p^t - \theta}{p^t - p_0}\right)}{(1 - p^t)(p^t - p_0) + (p^t - \theta)^2}, \tag{A12}$$

and the average innovation skill is

$$E\left(s_v | \mathcal{S} \in \mathbb{U}_1(\theta) \cup \mathbb{N}_1(\theta)\right) = \frac{(1 - p^t)(p^t - p_0)\frac{1 + p^t}{3} + (p^t - \theta)^2\left(p^t + \frac{p_0 - 2p^t}{3}\frac{p^t - \theta}{p^t - p_0}\right)}{(1 - p^t)(p^t - p_0) + (p^t - \theta)^2}. \tag{A13}$$

As a result, equation (21) can be expressed as

$$p^t = (1 - \delta_m)\eta_m + \delta_m\left[p_0 E\left(s_x | \mathcal{S} \in \mathbb{U}_1(\theta) \cup \mathbb{N}_1(\theta)\right) + (1 - p_0) E\left(s_v | \mathcal{S} \in \mathbb{U}_1(\theta) \cup \mathbb{N}_1(\theta)\right)\right], \tag{A14}$$

where the derivative of the right-hand side with respect to p^t is positive but less than 1. Therefore, for each θ, equation (21) admits a unique solution. ■

Proof of Corollary 2

The set $\mathbb{N}_1(\theta)$ forms a trapezoid with vertices

$$(p^t, p^t), \quad \left(\theta + \frac{p^t - \theta}{p^t - p_0}(1 - p_0),\ \theta - \frac{p^t - \theta}{p^t - p_0} p_0\right), \quad \left(1,\ \frac{\theta - p_0}{p^t - p_0}\right), \quad (1, 1)\,.$$

Consequently, the measure of assistive technology users is given by

$$\mu\big(\mathbb{U}_1(\theta) \cup \mathbb{U}_0(\theta)\big) = 1 - \frac{p^t - \theta}{p^t - p_0}\left[2(1 - p^t)\frac{\theta - p_0}{p^t - p_0} + (1 - p_0)\frac{p^t - \theta}{p^t - p_0}\right]. \tag{A15}$$

The first-order condition for the provider's objective (20) is

$$[p^t(\theta^*; \eta_m) - \theta]\mu_\theta(\theta^*; \eta_m) - [1 - p^t_\theta(\theta^*; \eta_m)]\mu(\theta^*; \eta_m) = 0 \tag{A16}$$

Applying the implicit function theorem,

$$\frac{d\theta^*}{d\eta_m} = \frac{p^t_{\eta_m}\mu_\theta + (p^t - \theta)\mu_{\theta\eta_m} + p_{\theta\eta_m}\mu - (1 - p^t_\theta)\mu_{\eta_m}}{2(1 - p^t_\theta)\mu_\theta - (p^t - \theta)\mu_{\theta\theta} - p_{\theta\theta}\mu}\,. \tag{A17}$$

From equation (A14), we have $p^t_{\eta_m} > 1 - \delta_m$. Since the term $p^t_{\eta_m}\mu_\theta$ is positive and dominates the remaining components, it follows that θ^* is increasing in η_m. Consequently, $p^t(\theta^*; \eta_m)$ is also increasing in η_m. By the players' choice rule in Proposition 3, these shifts in the thresholds induce active non-users $\mathbb{N}_1$ to transition to passive users $\mathbb{U}_0$, while some technology users who previously made active decisions $\mathbb{U}_1$ also switch to passive usage $\mathbb{U}_0$.

Proof of Corollary 3

An analogous argument applies to δ_m. The first-order condition for the provider's objective (20) can be written as

$$[p^t(\theta^*; \delta_m) - \theta]\mu_\theta(\theta^*; \delta_m) - [1 - p^t_\theta(\theta^*; \delta_m)]\mu(\theta^*; \delta_m) = 0 \tag{A18}$$

Applying the implicit function theorem,

$$\frac{d\theta^*}{d\delta_m} = \frac{p^t_{\delta_m}\mu_\theta + (p^t - \theta)\mu_{\theta\delta_m} + p_{\theta\delta_m}\mu - (1 - p^t_\theta)\mu_{\delta_m}}{2(1 - p^t_\theta)\mu_\theta - (p^t - \theta)\mu_{\theta\theta} - p_{\theta\theta}\mu}\,. \tag{A19}$$

From equation (A14) and condition $\mathbf{p}_0 E\left(\mathcal{S} \mid \mathcal{S} \in \mathbb{U}_1(\theta^*) \cup \mathbb{N}_1(\theta^*)\right)\mathbf{1} = \eta_m$, it follows that $p^t_{\delta_m}(\theta^*) = 0$. Consequently, the positive term $p^t_{\theta\delta_m}\mu$ dominates the remaining components, and therefore θ^* is increasing in δ_m. Since $p^t(\theta^*; \delta_m)$ responds to changes in θ^* less than one-for-one, the threshold (19) separating $\mathbb{N}_1$ and $\mathbb{U}_1$,

$$\frac{\mathbf{p}_0\mathcal{T}(\theta^*)\mathbf{1} - \theta^*}{\mathbf{p}_0\mathcal{T}(\theta^*)\mathbf{1} - p_0} = 1 - \frac{\theta^* - p_0}{p^t(\theta^*; \delta_m) - p_0}\,, \tag{A20}$$

is therefore increasing in δ_m. As a result, some active non-users $\mathbb{N}_1$ transition to passive users $\mathbb{U}_0$, while others become active users $\mathbb{U}_1$ as the model places greater weight on the input of active decision makers. ■

Proof of Proposition 5

Under autonomous technologies, the set $\mathbb{U}_1 \cup \mathbb{N}_1$ is determined by the triangular region defined by the boundary $p_0 s_x + (1 - p_0)s_v = p_0$. This boundary passes through (p_0, p_0) and $(1, 0)$, with slope $-\frac{p_0}{1-p_0}$.

Under assistive technologies, some players within this region may optimally choose to remain passive. From (18), the boundary between $\mathbb{U}_1$ and $\mathbb{U}_0$ is $\eta_s = \mathbf{p}_0\mathcal{T}\mathbf{1}$, which passes through $(1, 0)$ and has the steeper slope $-\frac{\mathbf{p}_0\mathcal{T}\mathbf{1}}{1-\mathbf{p}_0\mathcal{T}\mathbf{1}}$. The boundary between $\mathbb{N}_1$ and $\mathbb{U}_0$ is $p_0 s_x + (1 - p_0)s_v = \mathbf{p}_0\mathcal{T}\mathbf{1} - f$. By Lemma 2, we have $\mathbf{p}_0\mathcal{T}\mathbf{1} - f \geq p_0$. ■

Proof of Proposition 6

In the shift from assistive to autonomous technology, players who move from active technology users $\mathbb{U}_1$ to active non-users $\mathbb{N}_1$ are characterized by

$$\begin{aligned} s_x - s_v &> \frac{f_{\text{Assistive}}}{\mathbf{p}_0\mathcal{T}_{\text{Assistive}}\mathbf{1} - p_0}\,, \\ p_0 s_x + (1 - p_0)s_v &> \eta_{t,\text{Autonomous}} - \frac{f_{\text{Autonomous}}}{1 - \delta_{t,\text{Autonomous}}}\,. \end{aligned} \tag{A21}$$

and all have relatively high execution skills:

$$s_x > p_0 \cdot \frac{f_{\text{Assistive}}}{\mathbf{p}_0 \mathcal{T}_{\text{Assistive}} \mathbf{1} - p_0} + \eta_{t,\text{Autonomous}} - \frac{f_{\text{Autonomous}}}{1 - \delta_{t,\text{Autonomous}}} .$$

Conversely, players who shift from active non-users $\mathbb{N}_1$ to active users $\mathbb{U}_1$ satisfy

$$\begin{aligned} s_x - s_v &< \frac{f_{\text{Assistive}}}{\mathbf{p}_0 \mathcal{T}_{\text{Assistive}} \mathbf{1} - p_0} , \\ p_0 s_x + (1 - p_0) s_v &< \eta_{t,\text{Autonomous}} - \frac{f_{\text{Autonomous}}}{1 - \delta_{t,\text{Autonomous}}} . \end{aligned} \tag{A22}$$

and have comparatively lower execution skills, bounded by

$$s_x < p_0 \cdot \frac{f_{\text{Assistive}}}{\mathbf{p}_0 \mathcal{T}_{\text{Assistive}} \mathbf{1} - p_0} + \eta_{t,\text{Autonomous}} - \frac{f_{\text{Autonomous}}}{1 - \delta_{t,\text{Autonomous}}} .$$

■.