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Disagreement in Collateral Valuation

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Abstract

We present a model of secured lending in which borrowers and lenders agree to disagree about collateral values. Lenders' beliefs distort equilibrium prices of collateralized assets, and the extent to which lenders' beliefs distort prices is mediated by borrower riskiness. Specifically, prices are more reflective of lenders' beliefs when borrowers are riskier and more reflective of borrowers' beliefs when borrowers are safer. Disagreement in a dynamic setting can generate positive return autocorrelation that strengthens with borrower riskiness. We use data on U.S. residential mortgages to test the model's main predictions, for which we find strong empirical support.

Keywords: Disagreement, collateral, momentum, appraisal, real estate
JEL: G11, G12, G51, R30

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1 Introduction

Disagreement about asset valuation between agents has proved a powerful tool in explaining empirical puzzles such as abnormal trading volume, excess volatility, and return predictability.[1] Much of this work focuses on disagreement between agents who "agree to disagree" about the value of an asset and who buy or sell the asset according to their beliefs. In practice, the buyer of an asset often finances her purchase with a loan that is secured by the asset itself. When a borrower (i.e., buyer) defaults on a secured loan, the lender may seize the collateral from the borrower, sell the collateral, and apply the proceeds to the unpaid balance of the loan. It is therefore important for the lender to estimate the value of collateral at the time of loan issuance. However, borrowers and lenders may disagree about the value of collateral. In this paper, we explore how disagreement between borrowers and lenders distorts asset prices and affects returns.

We first develop a static model in which a borrower finances the purchase of an asset with debt and pledges the asset as collateral. In equilibrium, asset prices are more reflective of lenders' beliefs when borrowers are riskier and more reflective of borrowers' beliefs when borrowers are safer. We then extend the model to a dynamic setting. Here, we find that disagreement between borrowers and lenders can generate positive return autocorrelation that strengthens with the hazard rate of default. We end the paper by empirically testing the main predictions from each version of our model using data on U.S. residential mortgages. Consistent with these predictions, we find that when default risk is higher, (1) asset prices are closer to lenders' estimated values of collateral and (2) return momentum is stronger. Importantly, both our theoretical and empirical findings are robust to several extensions.

To understand how disagreement affects the price of a collateralized asset, we begin with a static model. In the model, a borrower wishes to purchase an indivisible asset from a seller and finances the purchase of the asset by borrowing from a lender. To obtain financing from the lender, the borrower pledges the asset as collateral. If the borrower remains solvent, she consumes the asset and makes a repayment to the lender. Conversely, if the borrower defaults, she obtains nothing, and the lender seizes the asset. In the baseline version of the static model, the repayment and the price of the collateralized asset are determined via multilateral bargaining.[2] Importantly, the borrower and the lender hold heterogenous beliefs about the value of collateral.

[1] See Banerjee and Kremer (2010) for a review.

[2] In an extension to the baseline model in which we consider sequential rounds of bilateral bargaining, we show that the assumption that all agents bargain simultaneously is not important for the results.

In contrast to existing models of disagreement, in which wealth shares or risk-aversion determine whose beliefs are more reflected in prices (e.g., Atmaz and Basak, 2018), our model highlights the role of borrower riskiness in mediating whose beliefs are more reflected. Specifically, we find that when the borrower is optimistic relative to the lender, the asset price decreases with borrower riskiness. Conversely, when the lender is optimistic relative to the borrower, the asset price increases with borrower's riskiness. Taken together, our results suggest that increasing borrower riskiness pulls the price towards the lender's value. In other words, we find a negative relation between borrower riskiness and the difference between the price and the lender's value. This finding is robust to changing the order of bargaining, the inclusion of repossession costs, endogenizing leverage, endogenizing default probabilities, and allowing the lender to sell the loan on a secondary market.

To develop some intuition for this result, it is helpful to decompose the lender's value from a loan into two parts. The first is the interest and principal repaid over the life of the loan, which the lender obtains while the borrower is solvent. The more optimistic the borrower is about the asset's value, the more she is willing to repay to the lender. The second is the value of collateral, which the lender obtains when the borrower defaults. The more optimistic the lender is about the asset's value, the more valuable is the collateral.

An increase in borrower riskiness affects the two parts of the loan's value differently. On the one hand, it decreases the expected present value of interest and principal payments. On the other hand, it increases the likelihood that the lender repossesses the asset. The dominant effect is determined by whether the borrower or the lender is more optimistic about the asset's value. If the lender is more optimistic than the borrower, the increase in value from possible repossession is larger than the decrease in value from a smaller repayment. Therefore, the lender's total value from the loan increases with borrower riskiness. Since the total value increases, the asset's price increases as well, which keeps the lender indifferent between lending and not. However, the opposite is true if the borrower is more optimistic than the lender: The effect of the smaller repayment dominates, so the lender's value – and hence the asset price – decreases with borrower riskiness.

To explore the return implications of disagreement between borrowers and lenders in secured lending, we extend the model to a dynamic setting. The asset's value, which is publicly observable, evolves with an unobserved growth rate. Whereas the static model takes agents' beliefs as given, we now model how agents arrive at different beliefs. Specifically, borrowers and lenders update their beliefs about the unobserved growth rate and agree to disagree about how much weight to place on new information. For example, borrowers and lenders place a weight of 25% and 15%, respectively, on new information while agreeing to disagree about which weight is actually correct (neither borrowers' nor lenders' weights need be Bayesian). The borrower makes the lender a fixed repayment until the exogenous arrival of default, at which time the lender repossesses the asset and sells it for the current market price.

We show that disagreement between borrowers and lenders can result in return autocorrelation. This result highlights the importance of heterogeneous beliefs. For example, even if one type of agent was at an informational disadvantage, a model of secured lending under rational expectations would not be able to generate return autocorrelation. In our setting, we find that returns are positively autocorrelated when lenders update more slowly than borrowers and borrowers update like Bayesians. This return autocorrelation is stronger when borrowers are riskier, which is when default and lender repossession are more likely. Intuitively, prices are more reflective of lenders' beliefs when borrowers are riskier, and since lenders' beliefs incorporate new information more slowly, prices incorporate new information more slowly as well. Although we focus on the case in which lenders update more slowly than borrowers, the model generates alternative return predictions for when borrowers and lenders update their beliefs in other ways.

To demonstrate the model's empirical relevance, we use data on U.S. residential mortgages. This market is an ideal setting to study disagreement between borrowers and lenders for two reasons. First, we can observe both the equilibrium price (i.e., sale price of the home) and the lender's estimated value of the collateral (i.e., appraised value).[3] Second, borrowers and lenders come to different conclusions, which we interpret as disagreement, about the value of collateral even though they have access to similar information. For example, sale prices differ from appraised values 64% of the time in our sample of over 10 million transactions.

Our first testable prediction from the model is that the difference between the appraised value and sale price decreases with borrower riskiness. Since we do not have ex-ante estimates of each loan's probability of default, we proxy for default risk (i.e., borrower riskiness) with three different variables: initial home equity, initial loan-to-value (LTV) ratio, and FICO score at time of origination. A meta-analysis reveals that home equity and FICO are negatively associated with default risk, and initial loan-to-value ratio is positively associated with default risk (Jones and Sirmans, 2015). We find that in both ordinary least squares and

[3]If the lender sells the loan to an outside investor, the appraised value can be interpreted as the investor's estimate of value rather than the lender's. We show that the same results obtain in an extension with this interpretation.

Poisson regressions, the coefficients on each of the three proxies, as well as the coefficient on their first principal component, are of the expected sign and highly significant. This result is robust to considering appraisal bias, accounting for lender repossession costs, and focusing on loans that are more likely to be sold on the secondary market.

The second testable prediction from the model is with regards to return autocorrelation. We focus on the scenario in which borrowers (who in practice are represented by well-informed real estate agents) update their information set in accordance with Bayes' rule, but lenders update more slowly than prescribed by Bayes' rule. This relative lag on the part of lenders is consistent with the slow-moving nature of appraisals (Clayton et al., 2001), which are often legally required by lenders (Eriksen et al., 2020). In this scenario, the model predicts positive return autocorrelation (i.e., return momentum) in the time series, which has been widely documented in the real estate market (Beracha and Skiba, 2011; Ghysels et al., 2013). A much more novel prediction produced by the model is that return momentum is stronger when default is more likely. Using the same three proxies for default risk (i.e., initial home equity, LTV ratio, credit score) and their first principal component, we find that return momentum is indeed stronger when default risk is higher.

2 Related Literature

In this section, we review some of the key papers in the literature most closely related to our paper and highlight our contributions.

2.1 Related studies on disagreement

Broadly speaking, the literature has focused on two reasons for disagreement. In the first, agents have different information sets and therefore come to different conclusions about asset values, prices, and returns. These are models of rational expectations under asymmetric information.[4] In the second, agents hold dogmatic beliefs regarding some aspect of the economy (e.g., asset values, signal precision) and agree to disagree with other agents.[5] For example, an agent may be overconfident and place too much weight on her own information and not enough on other agent's information. An attractive feature of this latter approach

[4] See Grossman and Stiglitz (1980); Kyle (1985) for examples.

[5] See Harrison and Kreps (1978); Harris and Raviv (1993); Kandel and Pearson (1995); Scheinkman and Xiong (2003); Cao and Ou-Yang (2008); Banerjee et al. (2009); Banerjee and Kremer (2010); Banerjee (2011); Atmaz and Basak (2018); Kyle et al. (2023) for examples.

is its ability to explain empirical puzzles, such as persistent return momentum (e.g., Kyle et al., 2023). These papers almost always model disagreement between traders and focus on explaining puzzles from the equity markets (i.e., return predictability, volatility, and trading volume).[6] In contrast, we model disagreement between borrowers and lenders to generate price distortions and return predictability, and we focus on secured lending.

Our application of disagreement to secured lending is closest to that in Simsek (2013), who also considers a static setting in which agents disagree about the value of collateral. In his model, pessimists lend to optimists who wish to purchase an asset that will serve as collateral. Default arises endogenously when collateral value falls below the borrower's repayment. He finds that what agents disagree about (e.g., the probability of good states versus bad states and the recover values therein), matters more for asset prices than the level of disagreement. Our focus is markedly different. Taking borrowers' and lenders' beliefs as given, we study the role of borrower riskiness (rather than asset riskiness) in mediating disagreement.

2.2 Related studies on collateral valuation

Our paper is also related to the literature on collateral valuation. Recent work has shown that higher collateral values are associated with lower credit spreads and higher loan amounts (Benmelech and Bergman, 2009; Cerqueiro et al., 2016; Luck and Santos, 2022). Our model implicitly shows that higher estimates of collateral value are associated with higher loan amounts. Stroebel (2016) shows that lenders with relatively superior information about the value of collateral earn higher returns on their secured loans. Jiang and Zhang (2023) find that mortgages collateralized by houses whose estimated values are more disperse, which the authors argue may be driven by information asymmetry, receive higher interest rates and are smaller in size. In contrast, agents in our model have the same information sets but choose to incorporate new information into their estimates of asset values at different rates. Our contribution to this strand of literature is to show that lenders' estimates of collateral values can also affect asset prices, not just loan terms.

[6] Additionally, Broer (2018) looks at disagreement in the context of securitization; Burnside et al. (2016) examines the role of disagreement in creating booms and busts in the housing market; and Xiong and Yan (2010) explores disagreement in bond markets.

2.3 Related studies on return momentum

Lastly, we contribute to the literature on return momentum (i.e., positive return autocorrelation), which has been documented across asset classes and geographies (Moskowitz et al., 2012; Asness et al., 2013). More specifically, our paper fits into the literature on return momentum in residential real estate, which was first documented by Case and Shiller (1989, 1990).[7] Explanations put forth for this phenomenon include extrapolative expectations (Case and Shiller, 1987; Glaeser and Nathanson, 2017), information frictions (Capozza et al., 2004; Anenberg, 2016), search costs (Head et al., 2014, 2016), and strategic complementarity (Guren, 2018). We contribute to this literature in two ways. First, we provide a novel mechanism (i.e., disagreement) for momentum. In our model, return momentum can be generated when lenders incorporate new information about collateral values more slowly than do buyers. This difference in beliefs is similar to the empirical evidence in Genesove and Hanse (2023), who argue that return momentum in the residential housing market may be driven by sellers updating their values much more slowly than buyers. Second, we show that momentum is strongest when borrowers are riskier. To the best of our knowledge, we are the first to document this stylized fact.

Martel and Van Wesep (2016) examine the effect of appraisal-based price constraints on prices, returns, and liquidity. In their model, buyers cannot pay significantly more than the average price in recent transactions, which causes sluggish adjustment to changes in fundamental value. They show that in a rising market, sellers strategically delay sale, which exacerbates the already sluggish adjustment process. In contrast to this paper, estimates of value based on historical data arise endogenously in our model because lenders require an estimate of collateral value. Moreover, we explore the role of borrower risk in appraisals' affect on prices.

3 Static Model

In this section, we present a static model to illustrate how disagreement between a borrower and a lender distorts asset prices. The main result is that the lender's belief about an asset's future growth rate affects the price the borrower pays for an asset that will serve as collateral. We go on to show that the borrower riskiness mediates disagreement between the borrower and the lender. Specifically, a safer borrower will have her beliefs reflected more in

[7] See Ghysels et al. (2013) for a review.

the price of an asset than a riskier borrower. We show that this result is robust to a number of extensions.

3.1 Base Model

There are two dates, $t = 0, 1$, and three agents, which are indexed by $j \in \{\mathcal{B}, \mathcal{L}, \mathcal{S}\}$: borrower/buyer $\mathcal{B}$ (she), lender $\mathcal{L}$ (he), and seller $\mathcal{S}$. Agents are risk-neutral. In this section, we normalize the risk-free rate to zero. There is an indivisible asset for which the borrower has unit demand. The date $t = 1$ value of owning the asset is

$$v_1 = v_0 + x, \tag{1}$$

where v_0 is a constant and x is a normally distributed growth rate. The main assumption of the model is that agents agree to disagree about the exact distribution of x. Specifically, agents agree that x is normally distributed with precision $\tau_0 > 0$ but agree to disagree about the mean of x. That is, agent j believes the mean is $\widehat{x}_j$, which need not equal $\widehat{x}_i$ for some other agent i. Let

$$a_j = v_0 + x_j, \tag{2}$$

where a_j represents agent j's estimate of the asset's value. At this point, we take as given that agents disagree (we provide a rationale for disagreement in Section 4).

The borrower wishes to purchase the asset from the seller and seeks a loan from the lender to finance the purchase. To secure the loan, the borrower pledges the asset as collateral. The borrower may default on her loan, in which case the lender repossesses the asset. Let d be a Bernoulli random variable representing default: $d = 1$ (default) with probability $\lambda \in (0, 1)$, and $d = 0$ (solvency) with probability $1 - \lambda$. Furthermore, let c be the borrower's repayment in solvency, p be the price paid by the borrower to the seller for the asset, and $\ell \in (0, 1)$ be the percent of the purchase price financed by the lender (the remaining percent $1 - \ell$ is financed by the borrower). We take ℓ to be exogenous and solve for the equilibrium repayment c and asset price p. In an extension, we endogenize ℓ by allowing the borrower to choose it.

Suppose the three agents agree on repayment c and asset price p. The surpluses of the borrower, the lender, and the seller are

$$S_{\mathcal{B}}(c,p) = (1-d)(v_1 - c) - (1-\ell)p \tag{3}$$

$$S_{\mathcal{L}}(c,p) = dv_1 + (1-d)c - \ell p \tag{4}$$

$$S_{\mathcal{S}}(c,p) = p - v_1. \tag{5}$$

The borrower obtains the asset value v_1 less the repayment c if she is solvent and nothing if she defaults. She has an initial outlay of $(1-\ell)p$. The lender obtains the asset value v_1 if the borrower defaults and the repayment c if she remains solvent. The lender has an initial outlay of ℓp. The seller obtains the price p but gives up the asset, which has value v_1.

We have thus far assumed that all agents value the asset at v_1, even if they disagree about its mean. In practice, the seller, the borrower, and the lender may have different values for the asset, even if they agree about the distribution of the asset's value. For example, a homeowner who just took a job in another city has a lower value for her current house than a potential buyer, regardless of her views on the local real estate market. Similarly, a lender values a foreclosed house less than a potential homeowner (we explore this possibility in a subsequent section). For the moment, we set aside these important issues and focus on how disagreement affects prices.

To determine whether or not there are gains from trade, we need to evaluate the agents' expected surpluses under their respective subjective probability measures:

$$\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c,p)] = (1-\lambda)(a_{\mathcal{B}} - c) - (1-\ell)p \tag{6}$$

$$\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c,p)] = \lambda a_{\mathcal{L}} + (1-\lambda)c - \ell p \tag{7}$$

$$\mathbb{E}^{\mathcal{S}}[S_{\mathcal{S}}(c,p)] = p - a_{\mathcal{S}}. \tag{8}$$

The total expected surplus is therefore

$$\sum_j \mathbb{E}^j[S_j(c,p)] = (1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}} - a_{\mathcal{S}}. \tag{9}$$

The repayment c and asset price p are determined via multilateral bargaining.[8] Specifically, let η_j be j's bargaining weight where $\sum_j \eta_j = 1$. These weights may be interpreted literally as bargaining weights or more abstractly as parameters that capture agents' relative

[8] We assume that the borrower, the lender, and the seller bargain over the repayment and price simultaneously. In an extension, we explore sequential bargaining.

market power in a competitive market. The key assumption is that agent j obtains a fraction η_j of the total surplus in equation (9). The following proposition gives the equilibrium repayment and price.

Proposition 1. *The equilibrium repayment and price are*

$$c^*(\lambda) = (1-\lambda)^{-1}((1-\lambda)\ell\eta_{\mathcal{S}}a_{\mathcal{B}} + \lambda(\ell\eta_{\mathcal{S}} - 1)a_{\mathcal{L}} + \ell(1-\eta_{\mathcal{S}})a_{\mathcal{S}}) \tag{10}$$

$$p^*(\lambda) = \eta_{\mathcal{S}}((1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}) + (1-\eta_{\mathcal{S}})a_{\mathcal{S}}. \tag{11}$$

From equation (11), it is clear that the equilibrium price not only reflects the beliefs of the borrower (i.e., the buyer) and the seller but also the beliefs of the lender. Furthermore, the lender's beliefs are more reflected in the price when the borrower is riskier. It follows from equation (11) that when the borrower is optimistic relative to the lender ($a_{\mathcal{B}} > a_{\mathcal{L}}$), the price is decreasing in the borrower's riskiness ($p^{*\prime}(\lambda) < 0$). Conversely, when the borrower is pessimistic relative to the lender ($a_{\mathcal{B}} < a_{\mathcal{L}}$), the price is increasing in the borrower's riskiness ($p^{*\prime}(\lambda) > 0$).

To study the role of disagreement between the borrower and the lender, we often focus on the case in which neither agent earns surplus ($\eta_{\mathcal{B}} = \eta_{\mathcal{L}} = 0$).[9] In this case, the borrower's beliefs and lender's beliefs are reflected in the asset price, but the seller's beliefs are not. Although we readily acknowledge that all agents may obtain surplus in practice, focusing on this case allows us to highlight the economic forces behind Proposition 1.

To develop the intuition behind Proposition 1, consider the indifference conditions for the borrower and the lender when neither agent obtains surplus:

$$\text{Borrower: } 0 = (1-\lambda)(a_{\mathcal{B}} - c(\lambda)) - (1-\ell)p(\lambda) \tag{12}$$

$$\text{Lender: } 0 = \lambda a_{\mathcal{L}} + (1-\lambda)c(\lambda) - \ell p(\lambda). \tag{13}$$

From the borrower's indifference condition, we see that to keep the borrower indifferent, the unconditional expected repayment $(1-\lambda)c(\lambda)$ must equal the unconditional asset value $(1-\lambda)a_{\mathcal{B}}$ less the amount of the purchase price she finances herself $(1-\ell)p(\lambda)$. Substituting $(1-\lambda)c(\lambda)$ into the lender's indifference condition, we obtain

$$0 = \overbrace{\lambda a_{\mathcal{L}}}^{\text{(A)}} + \overbrace{(1-\lambda)a_{\mathcal{B}} - (1-\ell)p(\lambda)}^{\text{(B)}} - \overbrace{\ell p(\lambda)}^{\text{(C)}}. \tag{14}$$

[9]Equivalently, one might imagine assets markets are competitive, loan markets are competitive, and that the asset is in sufficiently short supply.

From equation (14), we see that increasing borrower riskiness *increases* the unconditional collateral value (A) at a rate of $a_{\mathcal{L}}$ and *decreases* the unconditional repayment (B) at a rate of $a_{\mathcal{B}} + (1-\ell)p'(\lambda)$. To keep the lender indifferent between lending and not, the loan amount (C) must change to offset the net change of (A) and (B):

$$\ell p'(\lambda) = a_{\mathcal{L}} - (a_{\mathcal{B}} + (1-\ell)p'(\lambda)), \tag{15}$$

from which it follows that $p'(\lambda) > 0$ when $a_{\mathcal{L}} > a_{\mathcal{B}}$ and $p'(\lambda) > 0$ when $a_{\mathcal{L}} < a_{\mathcal{B}}$.

Before concluding this section, we state an important empirical prediction of the model. In many applications, one may observe the lender's value of the asset and the price paid by the borrower for the asset, but not the borrower's value of the asset (in fact, one of the main points of our paper is that the price paid for the asset is distorted by the lender's beliefs and does not fully reflect the borrower's beliefs).

Corollary 1. *If neither the borrower nor the lender earn surplus* ($\eta_{\mathcal{B}} = \eta_{\mathcal{L}} = 0$)*, then*

$$p^*(\lambda) = (1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}} \tag{16}$$

and hence

$$p^{*\prime}(\lambda) = a_{\mathcal{L}} - a_{\mathcal{B}} = (1-\lambda)^{-1}(a_{\mathcal{L}} - p^*(\lambda)). \tag{17}$$

As in Proposition 1, the price is decreasing in the borrower's riskiness ($p^{*\prime}(\lambda) < 0$) when the borrower is relatively optimistic ($a_{\mathcal{B}} > a_{\mathcal{L}}$) and increasing in the borrower's riskiness ($p^{*\prime}(\lambda) > 0$) when the borrower is relatively pessimistic ($a_{\mathcal{B}} < a_{\mathcal{L}}$). However, Corollary 1 enables us to establish that the equilibrium price is greater than the lender's value ($p^*(\lambda) > a_{\mathcal{L}}$) when the borrower is relatively optimistic, and the equilibrium price is less than the lender's value ($p^*(\lambda) < a_{\mathcal{L}}$) when the borrower is relatively pessimistic. Taken together, these results suggest that increasing borrower riskiness pulls the price towards the lender's value.

3.2 Extensions

The model considered in the previous section is admittedly stylized. We therefore present a number of extensions to demonstrate that the the basic intuition of Proposition 1 is robust to alternative specifications of the model.

3.2.1 Sequential Bargaining

In practice, the repayment and the price are determined through multiple rounds of bilateral bargaining. For example, a small business owner may first determine the loan terms he will be able to obtain before approaching the seller for a piece of equipment or vehicle. To that end, we now consider an extension of the model in which the borrower and lender bargain over a repayment, and the borrower can commit to the repayment in a subsequent round of bargaining with the seller. The goal of this extension is to illustrate that the multilateral nature of price formation in the previous section does not materially affect the equilibrium outcome.

There are now three dates, $t = 0, 1, 2$. On date $t = 0$, the borrower and the lender bargain over a repayment, c. On date $t = 1$, the borrower and the seller bargain over the price p. On date $t = 2$, payoffs are realized. Since each stage of bargaining is bilateral, we appeal to Nash (1950) directly for the bargaining solution.

Proposition 2 (Sequential Bargaining)**.** *The equilibrium repayment and price are*

$$c^*(\lambda) = \frac{(1-\lambda)(\ell\eta_{\mathcal{S}} + (1-\ell)\eta_{\mathcal{L}})a_{\mathcal{B}} - \lambda(1-\ell)(\eta_{\mathcal{B}} + \eta_{\mathcal{S}})a_{\mathcal{L}} + (1-\ell)(\ell\eta_{\mathcal{B}} + (1-\ell)\eta_{\mathcal{L}})a_{\mathcal{S}}}{(1-\lambda)(\eta_{\mathcal{S}} + (1-\ell)(1-\eta_{\mathcal{S}}))} \tag{18}$$

$$p^*(\lambda) = \frac{\eta_{\mathcal{S}}((1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}) + (1-\ell)(1-\eta_{\mathcal{S}})a_{\mathcal{S}}}{\eta_{\mathcal{S}} + (1-\ell)(1-\eta_{\mathcal{S}})}. \tag{19}$$

Three facts follow immediately. First, the price under sequential bargaining places less weight on the seller's belief than under simultaneous bargaining (simply compare the coefficient on $a_{\mathcal{S}}$ in equation (11) to the corresponding coefficient in equation (19)). The borrower and lender now enjoy a first-mover advantage relative to the seller, so the equilibrium price reflects their views more. Second, as the surpluses of the borrower and the lender go to zero, the prices under sequential bargaining and simultaneous bargaining converge to $p(\lambda) = (1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}$. Third, $p^{*\prime}(\lambda) > 0$ if $a_{\mathcal{L}} > a_{\mathcal{B}}$ and $p^{*\prime}(\lambda) < 0$ if $a_{\mathcal{L}} < a_{\mathcal{B}}$, just as in Proposition 1.

3.2.2 Repossession Costs

In many applications, collateral repossession is costly; lenders may incur substantial holding costs (in the case of equipment repossession) or have to sell at a foreclosure sale discount (Conklin et al., 2023). In this subsection, we extend the model to include repossession costs.

We show that increasing borrower riskiness pulls the price of the asset towards the lender's effective value (his value net of repossession costs).

Suppose that when the borrower defaults, the lender is able to recover only a fraction $\xi \in (0, 1)$ of the collateral value. The subjective, expected surpluses of the borrower and the seller are as before (see equations (6) and (8)), but the lender's expected surplus is now

$$\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c, p)] = \lambda \xi a_{\mathcal{L}} + (1 - \lambda)c - \ell p. \tag{20}$$

Solving for the equilibrium repayment and price amounts to substituting $\xi a_{\mathcal{L}}$ for $a_{\mathcal{L}}$ in Proposition 1.

Proposition 3 (Repossession Costs)**.** *The equilibrium repayment and price are*

$$c^*(\lambda) = (1 - \lambda)^{-1}((1 - \lambda)\ell\eta_{\mathcal{S}} a_{\mathcal{B}} + \lambda(\ell\eta_{\mathcal{S}} - 1)\xi a_{\mathcal{L}} + \ell(1 - \eta_{\mathcal{S}})a_{\mathcal{S}}) \tag{21}$$

$$p^*(\lambda) = \eta_{\mathcal{S}}((1 - \lambda)a_{\mathcal{B}} + \lambda \xi a_{\mathcal{L}}) + (1 - \eta_{\mathcal{S}})a_{\mathcal{S}}. \tag{22}$$

Now $p^{*\prime}(\lambda) > 0$ if $\xi a_{\mathcal{L}} > a_{\mathcal{B}}$, and $p^{*\prime}(\lambda) < 0$ if $\xi a_{\mathcal{L}} < a_{\mathcal{B}}$. The analogue of Corollary 1 follows:

Corollary 2. *If neither the borrower nor the lender earn surplus* ($\eta_{\mathcal{B}} = \eta_{\mathcal{L}} = 0$)*, then*

$$p^*(\lambda) = (1 - \lambda)a_{\mathcal{B}} + \lambda \xi a_{\mathcal{L}} \tag{23}$$

and hence

$$p^{*\prime}(\lambda) = \xi a_{\mathcal{L}} - a_{\mathcal{B}} = (1 - \lambda)^{-1}(\xi a_{\mathcal{L}} - p^*(\lambda)). \tag{24}$$

As in Corollary 1, the price is decreasing in borrower riskiness when the price is substantially greater than the lender's value ($p^*(\lambda) >> a_{\mathcal{L}}$) and increasing in borrower riskiness when the price is substantially less than the lender's value ($p^*(\lambda) << a_{\mathcal{L}}$). Mathematically, the point at which the sign of $p^{*\prime}(\lambda)$ flips now depends on the magnitude of the repossession costs; when repossession costs are higher, the point at which the sign of $p^{*\prime}(\lambda)$ flips is lower. These results suggest that increasing borrower riskiness pulls the price towards the lender's value less repossession costs. In our empirical section, we provide evidence supporting Corollary 2 assuming different estimates of repossession costs.

3.2.3 Endogenous Leverage

In the base model, the fraction of the asset price financed by the lender is taken to be exogenous. In practice, borrowers choose how much they borrow (perhaps subject to a constraint on loan-to-value). In this subsection, we take the repayment c as fixed and let agents bargain over the fraction of the purchase price to be financed ℓ as well as the asset price p.

Proposition 4 (Endogenous Leverage)**.** *The equilibrium leverage and price are*

$$\ell^*(\lambda) = \frac{(1-\lambda)(c-\eta_{\mathcal{L}}a_{\mathcal{B}}) + \lambda(1-\eta_{\mathcal{L}})a_{\mathcal{L}} + \eta_{\mathcal{L}}a_{\mathcal{S}}}{\eta_{\mathcal{S}}((1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}) + (1-\eta_{\mathcal{S}})a_{\mathcal{S}}} \tag{25}$$

$$p^*(\lambda) = \eta_{\mathcal{S}}((1-\lambda)a_{\mathcal{B}} + \lambda a_{\mathcal{L}}) + (1-\eta_{\mathcal{S}})a_{\mathcal{S}}. \tag{26}$$

The asset price in equation (26) of Proposition 4 is identical to the asset price in equation (11) of Proposition 1. The reason is simple: total surplus (equation (9) in both cases is the same. Neither the total surplus nor the seller's surplus depend explicitly on how the borrower finances her purchase (i.e., the fraction of the price to be financed ℓ or the repayment c). We therefore conclude that $p^{*\prime}(\lambda) > 0$ if $a_{\mathcal{L}} > a_{\mathcal{B}}$ and $p^{*\prime}(\lambda) < 0$ if $a_{\mathcal{L}} < a_{\mathcal{B}}$, just as in Proposition 1.

3.2.4 Endogenous Default Probabilities

We now consider an extension of the base model in which the probability of default is correlated with the value of the collateral. In practice, borrowers are most likely to default when the asset value is low, either for non-strategic reasons (asset values are low precisely when borrowers are illiquid) or strategic reasons (the borrower is underwater and chooses to walk away from both the loan and the asset).[10] In this extension, the borrower and the lender agree about borrower riskiness and the probabilities of good and bad states of the world (e.g., expansions and recessions), but they disagree about the value of collateral in these different states of the world.[11]

To keep the extension simple, we modify our original distributional assumptions. Suppose there are two states indexed by $s \in \{H, L\}$. $s = H$ with probability $\pi \in (0,1)$ and $s = L$ with probability $1-\pi$. If $s = H$, the borrower does not default. If $s = L$, the borrower defaults with probability $\lambda \in (0,1)$ and remains solvent with probability $1-\lambda$. Agent j believes that the asset is worth a_j^s in state s. In general, agents believe that the asset is less

[10] In housing, a relatively small fraction of homeowners default explicitly because of a decline in home values (i.e., strategic default). Estimates range from 6.4% (Foote et al., 2008) to 35.1% (Guiso et al., 2013).

[11] See Simsek (2013) for a thorough analysis of disagreement about the distribution of collateral values. Our primary departure is the consideration of borrower riskiness (λ), which determines how the views of the borrower and the lender are incorporated into the asset price.

valuable in state L than in state H: $a_j^L < a_j^H$. This simple specification captures the idea that the collateral value should be low precisely when default is most likely.

We can now write the subjective, expected surpluses of the borrower and the lender:

$$\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c,p)] = \pi(a_{\mathcal{B}}^H - c) + (1-\pi)(1-\lambda)(a_{\mathcal{B}}^L - c) - (1-\ell)p \tag{27}$$

$$\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c,p)] = \pi c + (1-\pi)(\lambda a_{\mathcal{L}}^L + (1-\lambda)c) - \ell p. \tag{28}$$

The following proposition gives the equilibrium repayment and price with endogenous default probabilities.

Proposition 5 (Endogenous Default)**.** *If neither the borrower nor the lender earn surplus* $(\eta_{\mathcal{B}} = \eta_{\mathcal{L}} = 0)$*, the equilibrium repayment and price are*

$$c^*(\lambda) = (1-(1-\pi)\lambda)^{-1}(\ell(\pi a_{\mathcal{B}}^H + (1-\pi)(1-\lambda)a_{\mathcal{B}}^L) - (1-\ell)(1-\pi)\lambda a_{\mathcal{L}}^L) \tag{29}$$

$$p^*(\lambda) = \pi a_{\mathcal{B}}^H + (1-\pi)((1-\lambda)a_{\mathcal{B}}^L + \lambda a_{\mathcal{L}}^L). \tag{30}$$

Moreover, $p^{*\prime}(\lambda) > 0$ *if the lender is optimistic, in the sense that* $a_{\mathcal{L}}^L > a_{\mathcal{B}}^L$*, and* $p^{*\prime}(\lambda) < 0$ *if the borrower is pessimistic, in the sense that* $a_{\mathcal{L}}^L < a_{\mathcal{B}}^L$*.*

Although the exact notions of "optimistic" and "pessimistic" are different from those of Proposition 1, the basic finding that prices increase with borrower riskiness when the lender is optimistic and decrease when the borrower is optimistic survives.

3.2.5 Secondary Market

We now consider the possibility that the lender can sell the loan. We therefore add an investor $\mathcal{I}$ to our model (e.g., a government-sponsored enterprise or an investor in an asset-backed security). In this case, the lender may agree with the borrower and the seller about the asset value. Alternatively, the lender may use his informational advantage to sell riskier loans (Agarwal et al., 2012) or engage in lax borrower screening because the lender is going to sell the loan to an outside investor regardless of the asset's value (Keys et al., 2010). In either situation, the ultimate investor holds a differing view from the borrower and the seller.

Consider a lender who originates home loans. He will not make a particular loan unless the loan can be sold to an outside investor. However, the investor will not buy the loan unless the originator uses a particular appraised value of the home, even if that appraised value differs markedly from the originator's value. In this way, an investor's beliefs can distort the asset price just as much as a lender's beliefs can.

We reiterate that although we consider a bargaining game between four agents, we could reinterpret the bargaining weights as agents' respective market power in a competitive market. Let p_A be the price of the asset, p_L be the price of the loan, and $\varphi \in (0,1)$ be the fraction of the loan retained by the lender. Let $\eta_{\mathcal{I}}$ be the investor's bargaining weight, and suppose that all agents' weights sum to unity. The surpluses are

$$S_{\mathcal{B}}(c, p_A, p_L) = (1-d)(v_1 - c) - (1-\ell)p_A \tag{31}$$

$$S_{\mathcal{L}}(c, p_A, p_L) = p_L + \varphi(dv_1 + (1-d)c) - \ell p_A \tag{32}$$

$$S_{\mathcal{S}}(c, p_A, p_L) = p_A - v_1 \tag{33}$$

$$S_{\mathcal{I}}(c, p_A, p_L) = (1-\varphi)(dv_1 + (1-d)c) - p_L. \tag{34}$$

Therefore, the subjective, expected surpluses are

$$\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c, p_A, p_L)] = (1-\lambda)(a_{\mathcal{B}} - c) - (1-\ell)p_A \tag{35}$$

$$\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c, p_A, p_L)] = p_L + \varphi(\lambda a_{\mathcal{L}} + (1-\lambda)c) - \ell p_A \tag{36}$$

$$\mathbb{E}^{\mathcal{S}}[S_{\mathcal{S}}(c, p_A, p_L)] = p_A - a_{\mathcal{S}} \tag{37}$$

$$\mathbb{E}^{\mathcal{I}}[S_{\mathcal{I}}(c, p_A, p_L)] = (1-\varphi)(\lambda a_{\mathcal{I}} + (1-\lambda)c) - p_L. \tag{38}$$

The total expected surplus is therefore

$$\sum_j \mathbb{E}^j[S_j(c, p)] = (1-\lambda)a_{\mathcal{B}} + \lambda(\varphi a_{\mathcal{L}} + (1-\varphi)a_{\mathcal{I}}) - a_{\mathcal{S}}. \tag{39}$$

We characterize the equilibrium in the following proposition.

Proposition 6 (Secondary Market)**.** *If neither the borrower, the lender, nor the investor earn surplus and the borrower and the lender agree on the estimated value of the asset, then the equilibrium repayment, price of the asset, and price of the loan are*

$$c^*(\lambda, \varphi) = (1-\lambda)^{-1}(((1-\lambda) - \ell(1-(1-\varphi)\lambda))a_{\mathcal{B}} - \ell(1-\varphi)\lambda a_{\mathcal{I}}) \tag{40}$$

$$p_A^*(\lambda, \varphi) = (1-\varphi)\lambda a_{\mathcal{I}} + (1-(1-\varphi)\lambda)a_{\mathcal{B}} \tag{41}$$

$$p_L^*(\lambda, \varphi) = (1-\varphi)(((1-\lambda) - \ell(1-(1-\varphi)\lambda))a_{\mathcal{B}} + (\lambda - \ell(1-\varphi)\lambda)a_{\mathcal{I}}). \tag{42}$$

Corollary 3. *If neither the borrower, the lender, nor the investor earn surplus and the* *borrower and the lender agree on the estimated value of the asset, then*

$$\frac{\partial p_A^*}{\partial \lambda} = (1-\varphi)(a_{\mathcal{I}} - a_{\mathcal{B}}) = (1-(1-\varphi)\lambda)^{-1}(1-\varphi)(a_{\mathcal{I}} - p_A^*(\lambda,\varphi)). \tag{43}$$

Corollary 3 states that even if the lender has the option to sell the loan on a secondary market, there is still an effect of borrower riskiness on the asset price as mediated by disagreement between the borrower and the ultimate investor. That is, the price is decreasing in borrower riskiness when the price is greater than the investor's value ($p_A^*(\lambda,\varphi) > a_{\mathcal{I}}$) and increasing in borrower riskiness when the price is less than the investor's value ($p_A^*(\lambda,\varphi) < a_{\mathcal{I}}$).

4 Dynamic Model

To understand the consequences of disagreement for return dynamics, we extend the static model developed in Section 3 to continuous time. Importantly, we now model the source of disagreement. We assume that borrowers and lenders disagree because they place different weights on new information when updating their beliefs about collateral values. The dynamic model introduces other realistic features, such as estimates of value based on historical data, uncertainty about the arrival time of default, and recovery values based on market prices for the asset. The model combines the price structure of Glaeser and Nathanson (2017) with the non-Bayesian information processing of Berrada (2009).

4.1 Setup

Time is continuous and indexed by $t \in (-\infty, \infty)$. There are two types of agents: borrowers ($\mathcal{B}$) and lenders ($\mathcal{L}$). As in a number of iterations of the static model, neither borrowers nor lenders earn surplus, so we omit the seller. Agents are risk-neutral and discount utility flows at a rate $r > 0$. There is an indivisible asset for which borrowers have unit demand. Suppose that the common stock value of the asset V_t evolves according to

$$dV_t = X_t dt + \sigma_V dB_t^V \tag{44}$$

$$dX_t = -\kappa X_t dt + \sqrt{2\kappa}\sigma_X dB_t^X, \tag{45}$$

where B_t^V and B_t^X are standard and uncorrelated one-dimensional Brownian motions, and κ, σ_V, and σ_X are positive, known constants (let $\tau_V = 1/\sigma_V^2$ and $\tau_X = 1/\sigma_X^2$ be the corresponding precisions). V_t is observed by all agents, and X_t is observed by none. Let τ_0 be the positive root of

$$0 = \tau_X \tau_V + 2\kappa(\tau_X - \tau_0)\tau_0. \tag{46}$$

To obtain a steady-state equilibrium, we assume that the unconditional prior belief is that X_t is normally distributed with mean zero and precision τ_0. Let $\widehat{X}_t^j$ denote agent j's subjective expectation of X_t at time t. For $j \in \{\mathcal{B}, \mathcal{L}\}$, we assume that $\widehat{X}_t^j$ evolves according to

$$d\widehat{X}_t^j = -\kappa \widehat{X}_t^j dt + w_j(dV_t - \widehat{X}_t^j dt), \tag{47}$$

where w_j is constant. Equation (47) nests the Kalman-Bucy filter, which has coefficient $w_0 \equiv \tau_V/\tau_0$. Following the disagreement literature, we assume that $\mathcal{B}$ knows that $\mathcal{L}$'s growth rate estimate is $\widehat{X}_t^{\mathcal{L}}$, $\mathcal{L}$ knows that $\mathcal{B}$'s growth rate estimate is $\widehat{X}_t^{\mathcal{B}}$, and $\mathcal{B}$ and $\mathcal{L}$ agree to disagree (e.g., Banerjee and Kremer, 2010). The primary friction in the model is the possibility that $w_{\mathcal{B}}$ and $w_{\mathcal{L}}$ differ from each other and differ from the Bayesian weight w_0. This assumption is similar to that made by Berrada (2009), who studies securities trading.

Consider a borrower who buys the asset at time t. The lender finances a fraction $\ell \in (0, 1)$ of the loan. The time of default $T \geq t$ arrives exogenously with rate $\lambda > 0$.[12] Let C_t be the perpetuity value of repayment (over a time interval of length ds, the borrower makes a repayment of $rC_t ds$ to the lender), P_t be the price of the collateral asset at time t, $U_s^{\mathcal{B}} = U^{\mathcal{B}}(s, C_t, P_t)$ be the borrower's value, and $U_s^{\mathcal{L}} = U^{\mathcal{L}}(s, C_t, P_t)$ be the lender's value. $U_s^{\mathcal{B}}$ and $U_s^{\mathcal{L}}$ evolve according to the following Bellman equations:

$$rU_s^{\mathcal{B}} ds = (rV_s - rC_t)ds - \lambda U_s^{\mathcal{B}} ds + \mathbb{E}_s^{\mathcal{B}}[dU_s^{\mathcal{B}}] \tag{48}$$

$$rU_s^{\mathcal{L}} ds = rC_t ds + \lambda(P_s - U_s^{\mathcal{L}})ds + \mathbb{E}_s^{\mathcal{L}}[dU_s^{\mathcal{L}}]. \tag{49}$$

The borrower's flow utility $rU_s^{\mathcal{B}} ds$ equals the flow utility she obtains from the asset $rV_s ds$ less the fixed repayment $rC_t ds$ she makes to the lender. With probability λds, she defaults and loses everything. Her continuation value is $\mathbb{E}_s^{\mathcal{B}}[dU_s^{\mathcal{B}}]$. The lender earns fixed payment $rC_t ds$, and with probability λds, the borrower defaults, the lender seizes the collateral, and the lender sells it for P_t. His continuation value is $\mathbb{E}_s^{\mathcal{L}}[dU_s^{\mathcal{L}}]$.

[12] In practice, borrowers are likely to default when the asset value is low, either strategically or non-strategically. If we were to endogenize default (as in Section 3.2.4), the equilibrium price of the asset would be lower, but the main result (i.e., return autocorrelation), which we detail in Section 4.2.3, would be preserved.

Integrating equations (48) and (49) yields

$$U^{\mathcal{B}}(t, C_t, P_t) = \mathbb{E}_t^{\mathcal{B}}\left[\int_t^T r(V_s - C_t)e^{-r(s-t)}ds\right]. \tag{50}$$

$$U^{\mathcal{L}}(t, C_t, P_t) = \mathbb{E}_t^{\mathcal{L}}\left[\int_t^T rC_t e^{-r(s-t)}ds + P_T e^{-r(T-t)}\right]. \tag{51}$$

To better study the role of disagreement between borrowers and lenders, we focus our analysis on the case in which neither type of agent earns surplus:

$$0 = U^{\mathcal{B}}(t, C_t, P_t) - (1-\ell)P_t \tag{52}$$

$$0 = U^{\mathcal{L}}(t, C_t, P_t) - \ell P_t. \tag{53}$$

Equations (52) and (53) are the continuous-time analogues of equations (12) and (13). Adding equations (52) and (53), it follows that

$$P_t = \mathbb{E}_t\left[\int_t^T r\mathbb{E}_t^{\mathcal{B}}[V_s|T]e^{-r(s-t)}ds + \mathbb{E}_t^{\mathcal{L}}[P_T|T]e^{-r(T-t)}\right]. \tag{54}$$

Equation (54) illustrates how the differing beliefs of borrowers and lenders are incorporated into prices. Up until default, the borrower expects to obtain a flow utility of $r\mathbb{E}_t^{\mathcal{B}}[V_s|T]$. In the event of default, the lender seizes the collateral and sells it for an expected market price of $\mathbb{E}_t^{\mathcal{L}}[P_T|T]$. In the next subsection, we show precisely how the agents' beliefs about the growth rate are reflected in the price of the collateral asset.

We have thus far been silent on why transactions occur, why buyers and sellers transact when they do, and how many transactions occur during a particular period of time. Suffice it to say, buyers and sellers are assumed to transact at a particular time for exogenous reasons (e.g., a firm acquires a vehicle or a piece of equipment to finish a project). Transactions are assumed to be sufficiently frequent so that agents can, in principle, extract the asset value from the history of asset prices. Although lack of liquidity is undoubtedly important in housing markets (Sagi, 2021), our goal is simply to illustrate how disagreement affects returns of collateralized assets, absent search frictions.

4.2 Equilibrium

Following standard practice, we restrict attention to equilibria in which the price is linear in the state variables V_t, $\widehat{X}_t^{\mathcal{B}}$, and $\widehat{X}_t^{\mathcal{B}}$.

Proposition 7. *The unique linear equilibrium repayment C_t^* and price P_t^* are*

$$C_t^* = b_V(\lambda)V_t + b_{\mathcal{B}}(\lambda)\widehat{X}_t^{\mathcal{B}} + b_{\mathcal{L}}(\lambda)\widehat{X}_t^{\mathcal{L}} \text{ and} \tag{55}$$

$$P_t^* = V_t + c_{\mathcal{B}}(\lambda)\widehat{X}_t^{\mathcal{B}} + c_{\mathcal{L}}(\lambda)\widehat{X}_t^{\mathcal{L}}, \tag{56}$$

where

$$b_V(\lambda) = 1 - r^{-1}(r+\lambda)(1-\ell) \tag{57}$$

$$b_{\mathcal{B}}(\lambda) = (r+\kappa+\lambda)^{-1} - r^{-1}(r+\lambda)(1-\ell)c_{\mathcal{B}}(\lambda) \tag{58}$$

$$b_{\mathcal{L}}(\lambda) = -r^{-1}(r+\lambda)(1-\ell)c_{\mathcal{L}}(\lambda) \tag{59}$$

$$c_{\mathcal{B}}(\lambda) = r((w_{\mathcal{B}}+\kappa)+r)^{-1}(r+\lambda)^{-1}(r+\kappa+\lambda)^{-1}((w_{\mathcal{B}}+\kappa)+r+\lambda) \tag{60}$$

$$c_{\mathcal{L}}(\lambda) = \lambda(r+\kappa)^{-1}(r+\lambda)^{-1}\left(1 + w_{\mathcal{B}}r((w_{\mathcal{B}}+\kappa)+r)^{-1}(r+\kappa+\lambda)^{-1}\right). \tag{61}$$

Moreover, $c_{\mathcal{B}}(\lambda) + c_{\mathcal{L}}(\lambda) = 1/(r+\kappa)$, $c'_{\mathcal{B}}(\lambda) < 0$, *and* $c'_{\mathcal{L}}(\lambda) > 0$.

Proposition 7 generalizes Proposition 1 to the dynamic setting. In particular, it retains the feature that lenders' beliefs are more reflected in equilibrium prices when borrowers are riskier (i.e., $c'_{\mathcal{B}}(\lambda) < 0$ and $c'_{\mathcal{L}}(\lambda) > 0$).

4.2.1 Price and Lenders' Value

In the model, the lender's estimate of value is

$$A_t^{\mathcal{L}} = \mathbb{E}_t^{\mathcal{L}}\left[\int_t^{\infty} rV_s e^{-r(s-t)}ds\right] = V_t + (r+\kappa)^{-1}\widehat{X}_t^{\mathcal{L}}. \tag{62}$$

From Proposition 7, it follows that

$$P_t - A_t = c_{\mathcal{B}}(\lambda)(\widehat{X}_t^B - \widehat{X}_t^L). \tag{63}$$

Equation (63) is the continuous-time analogue of Corollary 1. If lenders are more optimistic than borrowers ($\widehat{X}_t^L > \widehat{X}_t^B$), then the price is less than the lender's value ($P_t < A_t$). Since $c_{\mathcal{B}}$ is decreasing in λ, $P_t - A_t$ is increasing in λ (P_t increases towards A_t from below). If instead borrowers are more optimistic than lenders ($\widehat{X}_t^B > \widehat{X}_t^L$), then the price is greater than the lender's value ($P_t > A_t$). Again, since $c_{\mathcal{B}}$ is decreasing in λ, $P_t - A_t$ is decreasing in λ (P_t decreases towards A_t from above). Taken together, these results suggest that increasing borrower riskiness pulls the price towards the lender's value.

4.2.2 Return Predictability

If agents follow Bayes' rule, they place a weight of w_0 on new information (specifically, $w_j = w_0$ in Equation (47)). However, we consider the possibility that agents place different weights on new information than each other and than that prescribed by Bayes' rule. In this way, disagreement about growth rates stems from disagreement about how much weight to place on new information. In this subsection, we explain why this type of disagreement generates return predictability. This discussion will lay the groundwork for our discussion of return autocorrelation in the next subsection.

For each agent $i \in \{\mathcal{B}, \mathcal{L}\}$, let

$$Z_t^i = X_t - \widehat{X}_t^i. \tag{64}$$

Z_t^i is the difference between the true growth rate X_t and agent i's estimate of the growth rate $\widehat{X}_t^i$. Z_t^i evolves according to

$$dZ_t^i = -(\kappa + w_i) Z_t dt - w_i \sigma_V dB_t^V + \sqrt{2\kappa}\sigma_X dB_t^X. \tag{65}$$

Let

$$\alpha_i(\lambda) = (r + \kappa + w_i) c_i(\lambda). \tag{66}$$

From equation (56) in Proposition 7 and the dynamics for the borrowers' and lenders' beliefs in equation (47), it follows that

$$\begin{aligned} dP_t + r(V_t - P_t)dt &= dV_t + c_{\mathcal{B}}(\lambda) d\widehat{X}_t^{\mathcal{B}} + c_{\mathcal{L}}(\lambda) d\widehat{X}_t^{\mathcal{L}} - r(c_{\mathcal{B}}(\lambda)\widehat{X}_t^{\mathcal{B}} + c_{\mathcal{L}}(\lambda)\widehat{X}_t^{\mathcal{L}})dt & (67)\\ &= \alpha_{\mathcal{B}}(\lambda)(dV_t - \widehat{X}_t^{\mathcal{B}} dt) + \alpha_{\mathcal{L}}(\lambda)(dV_t - \widehat{X}_t^{\mathcal{L}} dt) & (68)\\ &= \alpha_{\mathcal{B}}(\lambda)(Z_t^{\mathcal{B}} dt + \sigma_V dB_t^V) + \alpha_{\mathcal{L}}(\lambda)(Z_t^{\mathcal{L}} dt + \sigma_V dB_t^V). & (69) \end{aligned}$$

Note that $dP_t + r(V_t - P_t)dt$ represents the instantaneous total return. During a time interval of length dt, the owner of the asset enjoys a capital gain of dP_t plus utility from the asset $rV_t dt$, minus the financing (or opportunity) cost $rP_t dt$. Under the objective probability measure,

$$\mathbb{E}[dP_t + r(V_t - P_t)dt] = \alpha_{\mathcal{B}}(\lambda)(\mathbb{E}[X_t] - \widehat{X}_t^{\mathcal{B}}) + \alpha_{\mathcal{L}}(\lambda)(\mathbb{E}[X_t] - \widehat{X}_t^{\mathcal{L}}). \tag{70}$$

Pessimism (in the sense that $\mathbb{E}[X_t] > \widehat{X}_t^j$), from either the borrower or the lender, predicts positive future returns, and optimism ($\mathbb{E}[X_t] < \widehat{X}_t^j$) predicts negative future returns. Since $\alpha_{\mathcal{B}}$ is decreasing in λ and $\alpha_{\mathcal{L}}$ is increasing in λ, we additionally have that when borrowers are relatively safer (riskier), borrowers' sentiment (i.e., optimism or pessimism) is more (less) predictive of returns than lenders' sentiment.

4.2.3 Return Autocorrelation

We now turn to the central question of the dynamic model: How does disagreement about how much weight to place on new information affect return autocorrelation? To maintain tractability, we define the holding period return as in Kyle et al. (2023). The return to an investor who finances the purchase of the asset at the risk-free rate at time t, "reinvests" utility flows during the holding period at the risk-free, and sells the asset at time $t + \theta$ is

$$R_{t,t+\theta} \equiv P_{t+\theta} - P_t e^{r\theta} + e^{r\theta} \int_t^{t+\theta} rV_s e^{-r(s-t)} ds. \tag{71}$$

Note that $\lim_{\theta \to 0} R_{t,t+\theta} = dP_t + rV_t dt - rP_t dt$. Under no-arbitrage, $\lim_{\theta \to 0} R_{t,t+\theta}$ should be identically zero.

The unconditional return autocorrelation is

$$\rho_{t-\theta,t+\theta}(\lambda, w_{\mathcal{L}}, w_{\mathcal{B}}) = \frac{\text{Cov}(R_{t,t+\theta}, R_{t-\theta,t})}{\text{Var}(R_{t-\theta,t})}. \tag{72}$$

As we show in the proposition that follows, return autocorrelation is time-invariant. We therefore omit the time dependence in the discussion that follows. We are interested in the sign of the autocorrelation $\rho(\lambda; w_{\mathcal{B}}, w_{\mathcal{L}})$ and the sign of its derivative with respect to borrower riskiness $\partial_\lambda \rho(\lambda; w_{\mathcal{B}}, w_{\mathcal{L}})$.

Before deriving any analytical results, we first consider a numerical simulation. Figure 1 shows the sign of the autocorrelation and the sign of its derivative with respect to λ in the $w_{\mathcal{B}}$-$w_{\mathcal{L}}$ plane for a particular set of parameters. The plane is demarcated into four regions, which correspond to the four combinations of signs. When the sum (or average) of borrowers' and lenders' weights ($w_{\mathcal{B}} + w_{\mathcal{L}}$) is large (i.e., regions (2) and (3)), there is reversal (negative return autocorrelation). Here, agents place "too much" weight on new information. Conversely, when $w_{\mathcal{B}} + w_{\mathcal{L}}$ is small (i.e., regions (1) and (4)), there is momentum (positive return autocorrelation). Here, agents place "too little" weight on new information. When the difference between borrowers' weights and lenders' weights ($w_{\mathcal{B}} - w_{\mathcal{L}}$) is large (i.e., regions (1) and (2)), momentum (or reversal) increases with the hazard rate of default, but when $w_{\mathcal{B}} - w_{\mathcal{L}}$ is small (i.e., regions (3) and (4)), it decreases with the hazard rate.

To develop intuition and prepare for the empirical work that follows, we focus on region

(1). In this region, there is momentum (positive return autocorrelation) because borrowers are updating approximately like a Bayesian, but lenders are always updating more slowly than a Bayesian. From an econometrician's point of view, prices adjust too slowly and returns exhibit momentum. When the hazard rate of default is small, prices do not reflect lenders' slow moving beliefs because they are unlikely to repossess the collateral anytime soon (see Proposition 7). When the hazard rate is high, prices are more reflective of lenders' beliefs' relative to buyers' beliefs, and returns exhibit strong momentum.

To conclude this section, we formally state our result regarding return autocorrelation.

Proposition 8 (Return Autocorrelation)**.** *The unconditional return autocorrelation is time-invariant. If lenders update more slowly than borrowers ($w_{\mathcal{L}} < w_{\mathcal{B}}$) and borrowers update like Bayesians ($w_{\mathcal{B}} = w_0$), returns are positively autocorrelated. Moreover, if θ is sufficiently small, the autocorrelation is strictly increasing in the hazard rate of default.*

The condition that θ be sufficiently small does not appear to be necessary but greatly simplifies the proof.

4.2.4 Capital Gain Rate Autocorrelation

In many applications, utility flows or lease rates are difficult to observe, so returns are expressed exclusively in terms of capital gains. We therefore conclude this section with a discussion of the applicability of the previous results to such applications. Let $\widetilde{R}_{t,t+\theta}$ be the change in the log-price:

$$\widetilde{R}_{t,t+\theta} \equiv \log(P_{t+\theta}) - \log(P_t). \tag{73}$$

The unconditional autocorrelation of the log price changes is

$$\widetilde{\rho}_{t-\theta,t+\theta}(\lambda, w_{\mathcal{L}}, w_{\mathcal{B}}) = \frac{\mathrm{Cov}(\widetilde{R}_{t,t+\theta}, \widetilde{R}_{t-\theta,t})}{\mathrm{Var}(\widetilde{R}_{t-\theta,t})}. \tag{74}$$

In Figure 2, we plot the autocorrelation $\widetilde{\rho}$ for different values of λ under the assumptions that $w_{\mathcal{B}} = w_0$ and $w_{\mathcal{L}} < w_{\mathcal{B}}$. We draw two observations from Figure 2. First, the autocorrelation is increasing in borrower riskiness, just as in Proposition 8. Increasing borrower riskiness increases the extent to which the price reflects lenders' beliefs, which update more slowly than prescribed by Bayes' rule. Second, the autocorrelation is non-zero when borrowers do not default ($\lambda = 0$) and the price fully reflects borrowers' Bayesian beliefs. This result follows is because, even under no-arbitrage, capital gain rates are predictable.

5 Residential Real Estate

In this section, we use U.S. residential real estate as a laboratory to test the model's predictions.

5.1 Institutional Details

In real estate, outside appraisers are used for at least two reasons. First, an outside appraiser helps the lender obtain an independent opinion of the market value of the property that a borrower pledges as collateral for a home loan (Eriksen et al., 2019, 2020). These valuations are important because they are used by lenders, investors, and other appraisers. Second, a home appraisal is often required by law. The Real Estate Appraisal Reform Act of 1988 requires that an appraisal be conducted by an independent and qualified appraiser for all federally-related mortgage loans. A federally-related mortgage loan includes any loan that is secured by a first lien or subordinate lien on residential real property and falls into one of the following categories: a loan made by a lender that is regulated by or whose deposits are insured by any agency of the federal government; a loan made by or insured by an agency of the federal government; a loan made in connection with a housing or urban development program administered by an agency of the federal government; a loan made and intended to be sold by the originating lender to FNMA, Government National Mortgage Association (GNMA), or FHLMC; or a loan that is the subject of a home equity conversion mortgage or reverse mortgage issued by a lender or creditor subject to the regulation.[13] These mortgages accounted for at least 75% of all active single-family mortgages as of early 2021 (Pendleton, 2021).

Although the requirements to become a licensed appraiser vary across states, most states require a combination of coursework and apprenticeships (Eriksen et al., 2020). Appraisers must follow the Universal Standards of Professional Appraisal Practice, which was adopted by Congress in 1989, in reaching an estimated value of the property. There are a number of valuation methods available to the appraiser, but the comparable sales method of valuation is the near universally-adopted approach used by appraisers to value residential property (Eriksen et al., 2020). This method can be described in three steps (Eriksen et al., 2019):

1. Find transactions of comparable properties that best match the subject property in physical attributes, geographic proximity, and temporal proximity.

[13] See RESPA and 12 USC §2602 for more details.

2. Adjust for differences in attributes between each comparable transaction and the subject property to estimate an adjusted value for each transaction.

3. Apply weights to each comparable transaction to arrive at an appraised value of the subject property.

Upon completion of the above steps, the appraiser provides the lender an appraised value of the subject property.

In summary, lenders who use outside appraisals in the residential real estate market likely update their estimated collateral values using a different methodology than do borrowers. Since the resulting estimates of collateral values likely differ, we argue that residential real estate is an appropriate setting to test the model's predictions.

5.2 Prices and Lenders' Values

In this subsection, we empirically test the hypothesis that greater borrower riskiness is associated with a smaller difference between the lender's estimate of value and the asset price. Unlike data on secured lending in most settings, data on mortgages for residential real estate are unique in that they allow us to observe both the equilibrium price (i.e., sale price) and the lender's estimate of value (i.e., appraised value). Although we are unable to observe an objective estimate of borrower riskiness, we are able to observe variables that are highly correlated with borrowers' default risk. A meta-analysis of the determinants of residential mortgage default shows that home equity and FICO score are consistently negatively associated with default risk, and LTV ratio is consistently positively associated with default risk (Jones and Sirmans, 2015). We use these three variables as proxies for default risk.

5.2.1 Empirical Specification

To see the relation between borrower riskiness and the difference between the lender's value and the price of the asset, we consider two cases. Suppose the appraised value is greater than the sale price: $a_{\mathcal{L}} > p^*(\lambda)$. According to Corollary 1, the lender is optimistic relative to the borrower (i.e., $a_{\mathcal{L}} > a_{\mathcal{B}}$). Rearranging and taking the natural logarithm of Equation

(17), we obtain

$$\log(a_{\mathcal{L}} - p^*(\lambda)) = \log(1-\lambda) + \log(p^{*\prime}(\lambda)) \tag{75}$$

$$= \log(1-\lambda) + \log(a_{\mathcal{L}} - a_{\mathcal{B}}) \tag{76}$$

$$= \beta_0 + \beta_1 \log(1/(1-\lambda)) + \varepsilon, \tag{77}$$

where $\beta_0 = \mathbb{E}[\log(a_{\mathcal{L}} - a_{\mathcal{B}})]$, $\beta_1 = -1$, and $\varepsilon = \log(a_{\mathcal{L}} - a_{\mathcal{B}}) - \beta_0$ represents an error term. Therefore, this equation predicts a negative relation between borrower riskiness and the difference between the appraised value and the sale price.

Conversely, suppose $p^*(\lambda) > a_{\mathcal{L}}$. According to Corollary 1, the borrower is optimistic relative to the lender (i.e., $a_{\mathcal{B}} > a_{\mathcal{L}}$). Rearranging and taking the natural logarithm of Equation (17), we obtain

$$\log(p^*(\lambda) - a_{\mathcal{L}}) = \log(1-\lambda) + \log(-p^{*\prime}(\lambda)) \tag{78}$$

$$= \log(1-\lambda) + \log(a_{\mathcal{B}} - a_{\mathcal{L}}) \tag{79}$$

$$= \beta_0 + \beta_1 \log(1/(1-\lambda)) + \varepsilon, \tag{80}$$

where $\beta_0 = \mathbb{E}[\log(a_{\mathcal{B}} - a_{\mathcal{L}})]$, $\beta_1 = -1$, and $\varepsilon = \log(a_{\mathcal{B}} - a_{\mathcal{L}}) - \beta_0$ represents an error term. Therefore, this equation also predicts a negative relation between borrower riskiness and the difference between the appraised value and the sale price.

The identifying assumption, from the perspective of the model, is that the difference between the borrower's belief and the lender's belief is uncorrelated with the borrower's riskiness (i.e., home equity, LTV, or FICO score at origination). Therefore, we test Corollary 1 by simply regressing the difference between appraised value and sale price ($\log(A-P)$ or $\log(P-A)$, where A and P are the appraised value and sale price) on each of our three proxies for default risk.[14]

A number of real-world frictions complicate the basic prediction of Corollary 1. Specifically, we consider the role of appraisal bias, repossession costs (see Corollary 2), and a secondary markets for loans (see Corollary 3).

[14] We note that since 'value' in the LTV ratio is the lesser of appraised value and sale price, the left-hand side and right-hand side of our regressions that include the LTV ratio include some of the same information. As a result, the relation between the LTV ratio and the difference between appraised value and sale price could be, but is not necessarily, mechanical in some instances. Therefore, any results that include the LTV ratio as a regressor should be interpreted with caution.

5.2.2 Data: Corelogic

For each of the tests in this subsection, we use data from CoreLogic Loan-Level Market Analytics.[15] Since the home price indexes we use in later tests are for single-family homes whose mortgages were originated for purchase, we focus our loan-level analysis on the same types of mortgages. We exclude observations for which any of the following are missing: appraised value, sale price, original loan balance, original loan-to-value, zip code. Since loan-to-value ratios use the lesser of appraised value and sale price, we use original loan balances, appraised values, and sale prices to check the integrity of the original loan-to-value ratios provided by CoreLogic. If the loan-to-value ratio we estimate using the variables individually are not within one percent of the original loan-to-value ratio provided by CoreLogic, we drop the observation. We assume that loans whose loan-to-value ratios are above 100% are collateralized in part with assets we cannot observe, so we drop those observations. We also drop observations that, in our estimation, likely have wrong values for appraised value or sale price. That includes observations for which appraised value is less than or equal to 20% of the sale price or greater than or equal to 500% of the sale price. Lastly, we drop singleton observations.[16]

We present summary statistics of our variables in Table 1. Panel A focuses on observations for which appraised value is larger than sale price. The first two rows of Panel A show that the distributions of both appraised values and sale prices are right-skewed. The dependent variable for these regressions, $\log(A - P)$, is relatively normally distributed. Average LTV is higher than the traditional 80% LTV threshold, and both *Home Equity* and *FICO* are slightly left-skewed but close to normally distributed.

Panel B of Table 1 focuses on observations for which sale price is larger than appraised value. The distributions of both appraised value and sale price are again right-skewed. Interestingly, the distributions of both these variables are larger than they are in Panel A. Intuitively, instances in which the sales price is larger than the appraised value are more likely to occur with higher-priced homes because the buyers of those homes are more likely to be wealthy, and thus, be less constrained by low appraisals (via the LTV channel). We see further evidence of this lack of financial constraints in Panel B with relatively lower LTV ratios.

To better understand the distribution of the difference between appraised values and sale

[15] CoreLogic collects detailed data on both conforming and non-conforming mortgages (appraised value, sale price, loan-to-value, FICO score, etc.) at origination from the 25 largest mortgages servicers in the U.S. (Lewis, 2023).

[16] Singleton observations can overstate statistical significance and lead to incorrect inferences (Correia, 2015).

prices, we present four histograms of the difference in Figure 3. Panel A presents the raw difference between appraised values and sale prices (bin width of $1,000). Nearly 50% of all raw differences between the appraised value and sale price are between $0 and $1,000. This concentration is being driven by observations in which the appraised value and sale price are identical, which occurs in 36% of all observations. Just over 60% of all appraised values are greater than the sale price, and 4% of the appraised values are below the sale price. These percentages are similar to those in previous work (e.g., Chinloy et al., 1997, Eriksen et al., 2020, Calem et al., 2021). We also present a trimmed version of the distribution of the raw differences in Panel B. The data are trimmed at raw differences of $0 and $25,000, relative differences of 0% and 15%, and comprise 89% of the overall sample. Of these differences, over 50% are between $0 and $1,000.

Panel C presents the relative difference between appraised values and sale prices (bin width of 0.5%). Nearly 40% of all differences are between 0% and 0.5%, and of course, the overwhelming majority of these differences are 0%. Panel D shows that in the trimmed data, there is a similarly steep decline in the distribution of differences from the concentration around zero.

There are several reasons why there are many more observations with appraised value greater than sale price. One reason for this asymmetry is that when the appraised value is lower than the sale price, the buyer can use the low appraisal to successfully renegotiate a lower sale price, which will lead to more observations in which the appraised value equals the sale price (Calem et al., 2021). Another reason for this distribution asymmetry is when the appraised value is lower than the sale price, renegotiation fails, and no mortgage is originated (Nakamura, 2010; Fout and Yao, 2016). These observations do not show up in our data. The last reason for this distribution asymmetry is the upward bias in appraised values.[17] This bias has been estimated to be about 5% to 6% (Agarwal et al., 2015; Eriksen et al., 2020). Based on Figure 2 of Eriksen et al. (2020), it appears that the bias predominantly affects observations in which the appraised value is at most 5% higher than the sale price.

The reasons for this distribution asymmetry can therefore affect our results if the reasons are in some way correlated with our proxies for default risk. We discuss and address this potential issue below.

[17] See Cho and Megbolugbe (1996), Chinloy et al. (1997), Nakamura (2010), Agarwal et al. (2015), Ding and Nakamura (2016), Fout and Yao (2016), Eriksen et al. (2019, 2020), Calem et al. (2021), and Mayer and Nothaft (2022) for examples.

5.2.3 Appraisal, Price, and Default Risk

Table 2 presents results of our empirical tests of Corollary 1. We take the negative of *Home Equity* and *FICO*, so that the predicted sign of the coefficient is negative for all proxies for default risk. Since the relation between the difference in appraised value and sale price and default risk likely varies by geographic area and time, we include zip code by year by month fixed effects in all of our specifications. Standard errors are likely to be correlated in both the cross section and the time series, so we cluster standard errors by zip code, year, and month.

Panel A of Table 2 focuses on observations in which the appraised value is at least as large as the price, so the dependent variable is either $log(A - P)$ (for OLS) or $A - P$ (for Poisson). The first four columns present results from our OLS regressions. As predicted by Corollary 1, the first three columns show that the coefficient estimates on all three proxies for default risk are negative and highly significant. For example, the coefficient on *LTV* is -0.134, which implies that a one standard deviation increase in the LTV ratio is associated with a 13.4% decline in the difference between appraised value and sale price.

Our three proxies for default risk are highly correlated with each other, so including them all in the same regression leads to multicollinearity issues. To include information from all three proxies in the same specification and avoid these issues, we use the first principal component (i.e., *PC1*) from a principal component analysis of the three proxies.[18] The fourth column shows that the coefficient on *PC1* is -0.169, which implies that a one standard deviation increase in the first principal component is associated with a 16.9% decrease in the difference between appraised value and sale price. Since *PC1* is 91%, 89%, and 64% correlated with *Home Equity*, *LTV*, and *FICO*, respectively, the results are consistent with Corollary 1.

Using OLS allows us to better map the dependent variable in Corollary 1 to our empirical test, but since the appraised value often equals the sale price, we must drop a significant number of observations when taking the natural log of the difference between the two. To avoid dropping these observations, we also estimate our specification using Poisson regressions, which are well suited to accommodate outcomes with a value of zero (Cohn et al., 2022).

In the last four columns in Panel A of Table 2, we present results from our Poisson regressions. In these specifications, we include all observations from the first four columns and all observations in which the appraised value equals the sale price. Importantly, the

[18] The first principal component contains nearly 70% of the explained variance.

last four columns of Panel A show that coefficients obtained from estimating our Poisson regressions are also negative and highly significant, which is consistent with our results obtained from our OLS regressions and Corollary 1.

Panel B focuses on observations in which sale price is at least as large as the appraised value. Since sale prices are seldomly higher than appraised values, the number of observations in these regressions is much smaller. Nonetheless, the results are largely consistent with those in Panel A. Specifically, all coefficient estimates from both OLS regressions and Poisson regressions are negative, and most are highly significant.

Overall, the results in Table 2 provide evidence consistent with Corollary 1, which implies that the difference between appraised value and sale price is smaller when default risk is higher.

5.2.4 Appraisal, Price, and Default Risk: Appraisal Bias

As mentioned above, a significant portion of our observations might be affected by appraisal bias or renegotiation, and these affected observations might lead to a mechanical relation between our dependent variable and independent variables. To alleviate this concern, we separately estimate our specification for subsamples that include less affected observations and more affected observations. Less affected observations are defined to be those for which the appraised value is at least 5% higher than the sale price, and more affected observations are those for which the appraised value is at most 5% higher than the sale price. We present the results in Table 3.

Panel A presents results for less affected observations. Since these observations are less likely to be affected by appraisal bias, the dependent variable is unchanged relative to that in Table 2 (i.e., $log(A-P)$ for OLS, $A-P$ for Poisson). Consistent with Corollary 1, coefficient estimates on all proxies for default risk and their first principal component are negative and highly significant.

In Panel B of Table 3, we focus on observations that are likely more affected by appraisal bias. As mentioned above, this bias has been estimated to be about 5% to 6%. Therefore, the dependent variable in these regressions is $log(P - A/1.05)$ in our OLS regressions and $P - A/1.05$ in our Poisson regressions.[19] Importantly, Panel B shows that after accounting for appraisal bias in the difference between sale price and appraised value, the estimates

[19]We recognize that appraisal bias is likely more than 5% for observations whose sales price is further from the unobservable, unbiased appraised value and likely less than 5% for observations whose sales price is closer to the unobservable, unbiased appraised value. However, without more details on the amount of bias of different types of observations, we use the average bias of 5% as a rough estimate.

on all proxies for default risk and their first principal component are negative and highly significant.

In summary, Table 3 shows that our results are robust to adjusting appraised values for the well-documented appraisal bias.

5.2.5 Appraisal, Price, and Default Risk: Repossession Costs

In Corollary 2, we consider the possibility that the lender is unable to recover the full value of the asset in the event of default. Specifically, Equation (24) in Corollary 2 states that borrower riskiness is negatively associated with the difference between the equilibrium price and the lender's effective value, which we define as the lender's value net of repossession costs. In housing, these costs are largely driven by a foreclosure sale discount. There are four broad reasons for this discount (Conklin et al., 2023). First, there are observable and unobservable prior differences between distressed and non-distressed properties (Frame, 2010). Second, there are differences in the condition of the house that is caused by distressed homeowners' reduction in maintenance (Lambie-Hanson, 2015). Third, distressed sellers, which are often financial institutions (i.e., lenders) have greater urgency, and the briefer time the house is on the market is correlated with lower price (Clauretie and Daneshvary, 2009). Fourth, there is a stigma associated with distressed sales, which is often driven by the asymmetric information endemic to real estate transactions (Stroebel, 2016; Lopez, 2021).

Although there is wide agreement that there is a material foreclosure sale discount, there is no consensus on the size of the discount (Conklin et al., 2023). Some estimates been shown to be as high as 25% or 30% (e.g., Campbell et al., 2011), but others have found discounts to be as low as 5% or 10% (e.g., Conklin et al., 2023, Clauretie and Daneshvary, 2009). Given that 5% is perhaps the most recent discount put forth in the literature, we use that figure as our estimate of repossession costs. However, as we show in the Internet Appendix, our results are not dependent on this 5% estimate. Note that regardless of the exact value of repossession costs, Corollaries 1 and 2 make the same prediction when the appraised value and sale price are sufficiently far from each other.

Table 4 presents results from our tests of Corollary 2, which is essentially a robustness test of Corollary 1. Panel A focuses on observations in which sale price is equal to or less than 95% of appraised value. The dependent variable is $log(0.95 \times A\text{-}P)$ for our OLS regressions and $0.95 \times A\text{-}P$ for our Poisson regressions. All coefficient estimates across all proxies for default risk and both types of specifications are negative and highly significant.

Panel B of Table 4 considers observations in which sale price is equal to or greater than

95% of appraised value and includes the overwhelming number of observations in the sample. The dependent variable here is *log(P-0.95×A)* for our OLS regressions and *P-0.95×A* for our Poisson regressions. Consistent with both Panel A and Corollary 2, coefficient estimates on each proxy for default risk and on their first principal component are negative and highly significant.

Overall, the results in Table 4 are consistent with Corollary 2. Furthermore, as mentioned above, results presented in the Internet Appendix (Table IA1) show that our results are not dependent on repossession costs of 5%.

5.2.6 Appraisal, Price, and Default Risk: Secondary Market

In Corollary 3, we focus on the situation in which the lender can sell the loan. In this setting, the lender agrees with the borrower and seller about the value of the asset, but the investor holds a different view on the value of the asset. The investor is able to impose her beliefs on the value of the asset, because if those beliefs are not reflected, the investor will not purchase the loan. Corollary 3 therefore implies that borrower riskiness is negatively associated with the difference between appraised value and sale price among loans that are sold to outside investors.

To empirically test this implication, we would ideally have information on which loan is sold to an outside investor and which loan is kept on the lender's balance sheet. Although we unfortunately do not have that information, we can proxy for this delineation by using the GSE Eligible Flag provided by CoreLogic. This flag turns on when the loan conforms to the GSE standard eligible requirements criteria. The GSEs do not buy every conforming loan, but according to the Federal Housing Finance Agency (FHFA), GSE share of all conforming mortgages has been between 50% and 65% over the past fifteen years.[20] Therefore, we estimate our baseline specification on the subsample of conforming loans. We present the results in Table 5.

Since almost 90% of all mortgages in our main sample are conforming mortgages, the results in Table 5 are very similar to those in Table 3. Specifically, Panel A, which focuses on observations in which appraised value is at least as large as sale price, shows that the coefficients on all proxies for default risk and their first principal component are negative and highly significant. The coefficient estimates in Panel B, which focuses on all other observations, are not all statistically significant, but they are almost all the correct sign.

[20]See https://www.fhfa.gov/Media/Blog/Pages/What-Types-of-Mortgages-Do-Fannie-Mae-and-Freddie-Mac-Acqui aspx for more details.

Overall, Table 5 shows that our baseline prediction holds for mortgages that are likely to be sold to outside investors.

5.3 Return Autocorrelation

Thus far, we have empirically evaluated three corollaries from our static model. In this subsection, we test Proposition 8, which states that if lenders update their beliefs more slowly than borrowers and borrowers update like Bayesians, returns are positively autocorrelated, and this autocorrelation increases in borrower riskiness.

5.3.1 Slow Appraisals

As discussed above, the near universally-adopted approach used by appraisers to value residential property is the comparable sales valuation method. However, this method is likely a major contributor for why appraisers engage in appraisal smoothing, or appraisal lag (Clayton et al., 2001).[21] Appraisal lag leads to appraised values that tend to lag the real estate price cycle (Matysiak and Wang, 1995). Appraisers tend to underestimate value in rising markets and overestimate value in falling markets (Diaz III and Wolverton, 1998). In other words, appraisers, and by extension lenders, update their estimates of asset values more slowly than buyers and sellers.[22]

This phenomenon can be seen in Figure 4, which plots two national FHFA Home Price Indexes (HPIs). The FHFA HPIs are weighted, repeat-sales indexes estimated from repeat mortgage transactions on single-family properties whose mortgages have been purchased or securitized by Fannie Mae or Freddie Mac. The all-transactions index is estimated using appraised values and sale prices, and the purchase-only index is estimated using only sale prices.[23] Since the all-transactions index includes estimates from both appraisals and sales, and not just appraisals, we can interpret any differences between the two indexes as a lower bound on the actual difference between a hypothetical appraisal-only index and the purchase-only index.

[21] See Geltner et al. (2003) and the references therein for a review of the literature on appraisal smoothing.

[22] It is important to note that appraisal lag and appraisal bias are not incompatible with each other. Appraised values may be systematically higher than sale prices but changes in appraised values may still lag changes in sale prices. Since indexes, such as those in Figure 4, are estimated as a function of the starting value of the same series, only comparisons of changes, and not levels, can be made between indexes.

[23] Both indexes, which are at the national level, have been indexed to 100 beginning the first quarter of 1991, for that is the first available data for the purchase-only index. See https://www.fhfa.gov/DataTools/Downloads/Pages/House-Price-Index-Datasets.aspx#mpo for more details.

With that interpretation in mind, Figure 4 shows that after home prices peaked in early 2007, the all-transactions index stayed above the purchase-only index for several years (i.e., overestimated value in falling markets). Once the purchase-only index started to rise again at the beginning of 2012, it quickly surpassed the all-transactions index and stayed above it through 2021 (i.e., underestimated value in rising markets).

In summary, lenders who use appraised values update their estimates of asset values more slowly than buyers and sellers. According to Proposition 8, these more slowly updating beliefs should lead to positive return autocorrelation, and this return autocorrelation should increase with the hazard rate of default.

5.3.2 Return Autocorrelation and Borrower Riskiness

For our test of Proposition 8, we obtain data on our proxies for default risk (i.e., initial home equity, LTV, and credit score) from Freddie Mac.[24] Freddie Mac is one of two sources of mortgages for the FHFA HPI, which we use to estimate housing returns. In this test, we focus on the quarterly purchase-only indexes of the 100 largest metropolitan statistical areas.

We present summary statistics of the data in Table 6. Estimates of *Log return*, which is the annual change in the log value of the FHFA HPI at the CBSA level (Guren, 2018), are mostly positive and slightly right-skewed. For each of our proxies for default risk, we estimate weighted quarterly averages within each CBSA (i.e., Census Bureau-defined area) with value used as the weight. The distributions are similar to those in Table 1, but since we average loan-level observations at the CBSA×quarter level before estimating the distribution, the ranges and standard deviations are smaller.

Table 7 presents results of our test of Proposition 8. Rather than simply interact our proxies with lagged return, as stated in Proposition 8, we interact lagged return with dummy variables that turn on when our proxies are in a given part of the distribution. Specifically, we interact lagged return with *Low*, *Mid*, and *High*, which are dummy variables indicating whether the sorting variable of interest is in the lowest, middle, or highest tercile within a given CBSA. Using dummies allows us to account for any non-linearities in the relation between returns and the interaction of our proxies and lagged return. Since standard errors are likely to be correlated in both the cross section and the time series, we cluster standard errors by CBSA, year, and quarter.

[24] We use quarterly loan-level data on loans for purchase of single-family homes. These data span 1999 through 2021. See https://www.freddiemac.com/research/datasets/sf-loanlevel-dataset for more details.

The first column of Table 7 shows that annual returns are autocorrelated, which is consistent with previous work. In the next three columns, we create dummies using our proxies for default risk and interact them with lagged return. Recall that default risk is higher when home equity is lower, LTV is higher, and credit score is lower. Therefore, the results in these three columns show that return autocorrelation increases nearly monotonically as default risk (as captured by our proxies) increases.

In the last column, we sort by *PC1*, which is the first principal component of the three variables. We find that return autocorrelation increases as *PC1* decreases. Since *PC1* is positively correlated with both *Home Equity* and *Credit Score* and negatively correlated with *LTV*, the results in the last column also show that return autocorrelation increases as default risk increases.[25]

In summary, the results in Table 7 provide evidence consistent with Proposition 8, which states that return autocorrelation is higher when default is more likely.

6 Conclusion

In this paper, we present a model of secured lending in which borrowers and lenders disagree about the value of collateral. The prices of collateralized assets reflect the beliefs of both borrowers and lenders. Specifically, prices are more reflective of lender's beliefs when borrowers are riskier and more reflective of borrower's beliefs when borrowers are safer. In a dynamic version of the model, we consider the possibility that borrowers and lenders place different (and potentially non-Bayesian) weights on new information. We establish conditions under which returns exhibit momentum or reversal. Most relevant for collateralized lending, we explore the role of default risk as a mediating factor. Specifically, we investigate conditions under which default risk exacerbates or attenuates momentum/reversal.

To demonstrate the model's empirical relevance, we investigate the model's implications for the U.S. residential real estate market. Using a large sample of U.S. home loans, we show that sale prices are closer to appraised values when default risk is higher. Using housing returns in the 100 largest U.S. metro areas, we show that momentum is strongest when loans are riskier. Taken together, our evidence is consistent with a specification of the model in which borrowers update more-or-less like Bayesians, and lenders update less aggressively than prescribed by Bayes' rule. Although we focus on residential real estate, the model

[25]Table IA2 in the Internet Appendix shows that our results are nearly identical when using simple, rather than weighted, averages to calculate our quarter-level proxies for default risk.

applies to any setting in which an asset collateralizes its own financing, such as accounts receivable, buildings, equipment, inventory, land, and vehicles.

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Figure 1: Return Autocorrelation

This figure shows how return autocorrelation ($\rho(\lambda, w_{\mathcal{B}}, w_{\mathcal{L}})$) in our model changes as borrowers' weight on new information ($w_{\mathcal{B}}$) and lenders' weight on new information ($w_{\mathcal{L}}$) change. The parameters used in the simulation to generate this figure are $r = \kappa = 0.1$, $\tau_X = \tau_V = 1$, $\lambda = 0.05$, and $\theta = 0.2$. The hatched line in Region 1 is the beliefs space we consider in Proposition 8.

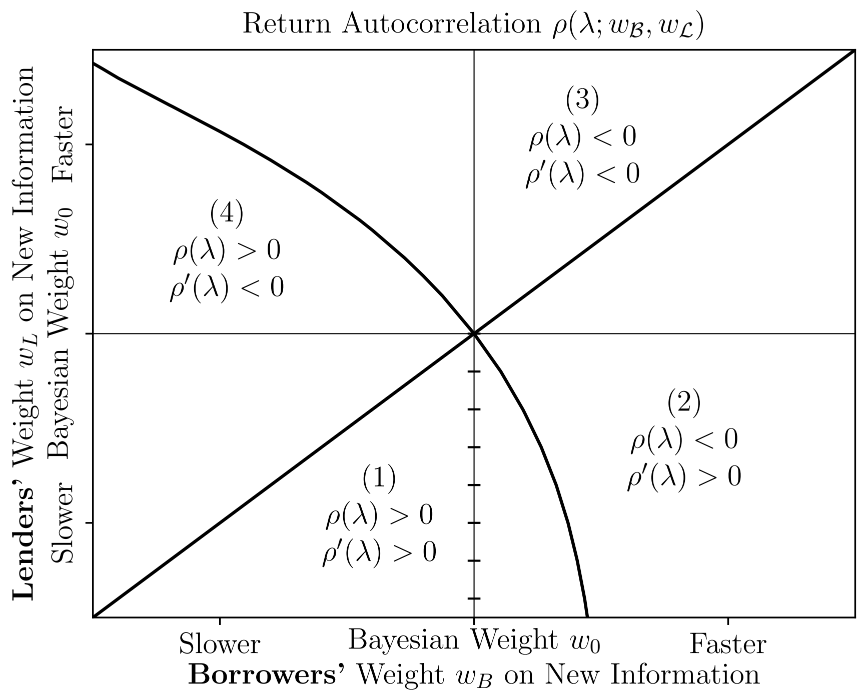

Figure 2: Capital Gain Rate Autocorrelation

This figure presents the log capital gain rate autocorrelation for different values of the hazard rate of default λ. The parameters used in the simulation to generate this figure are $r = \kappa = 0.1$, $\tau_X = \tau_V = 1$, $\theta = 0.2$, $w_{\mathcal{B}} = w_0$, and $w_{\mathcal{L}} = w_0/2$. For each λ, we simulate 10 million return series over the time interval $[0, 20]$ to compute the autocorrelation.

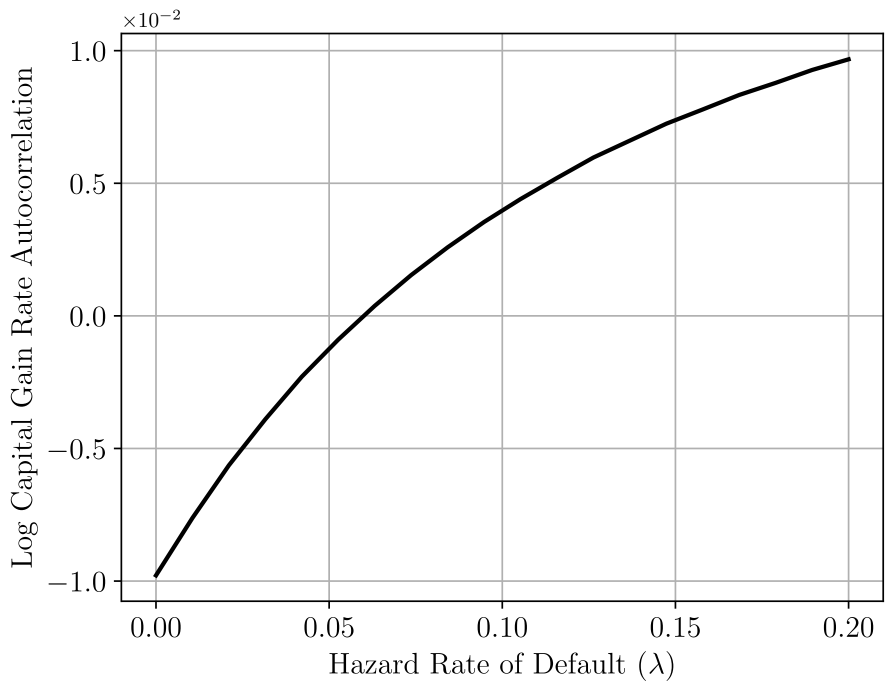

Figure 3: Appraised Value and Sale Price

This figure presents differences between appraised values and sale prices in our loan-level data from CoreLogic. Panel A presents the raw difference (in thousands of dollars). Panel B presents the raw difference (in thousands of dollars) after trimming the data. Panel C presents the relative difference. Panel D presents the relative difference after trimming the data. In Panels A and C, both *Appraised Value* and *Sale Price* are winsorized at the 0.5% and 99.5% levels. In Panels B and D, data are trimmed at raw differences of \$0 and \$25,000 and relative differences of 0% and 15%.

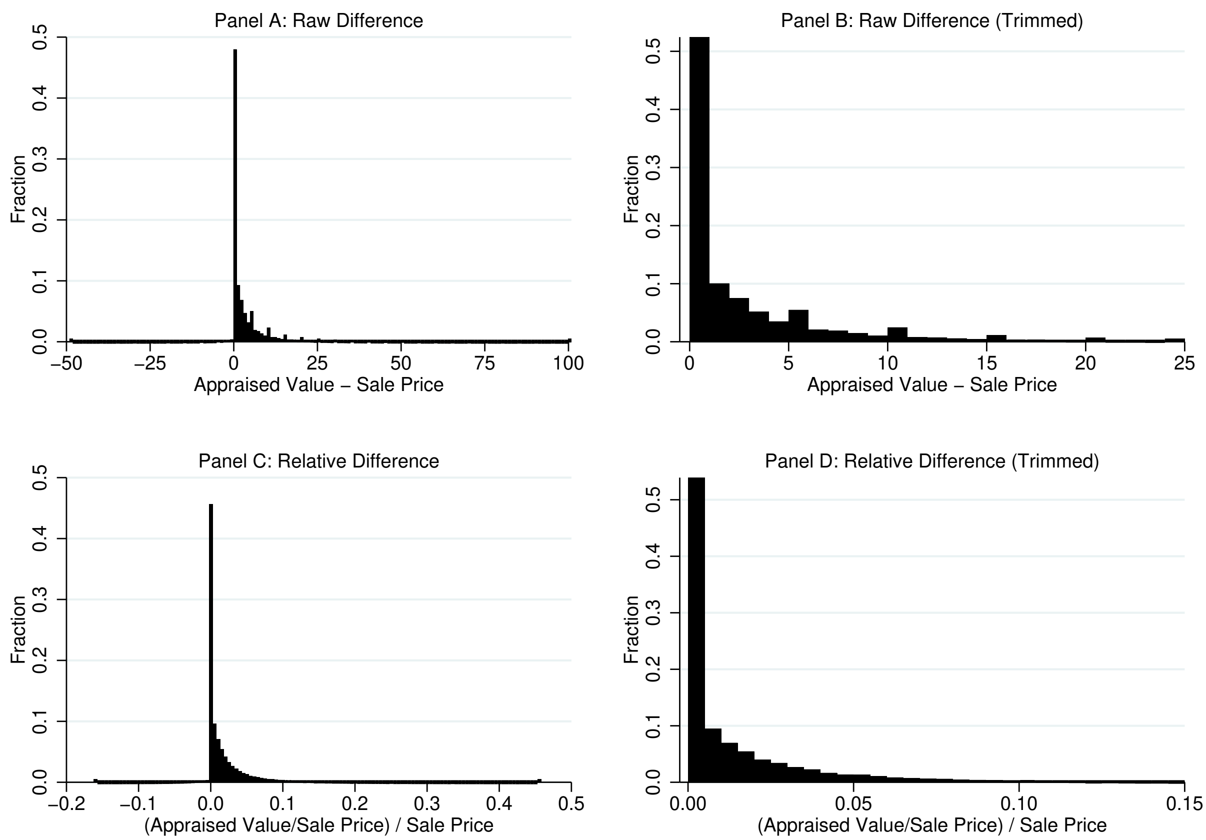

Figure 4: Slow Appraisals

This figure presents two price indexes from the Federal Housing Finance Agency. The All-Transactions Index is estimated using appraised values and sale prices. The Purchase-Only Index is estimated using sales prices. Both indexes are at the national level (i.e., United States), indexed to 100 beginning 1991, and are at the quarterly level.

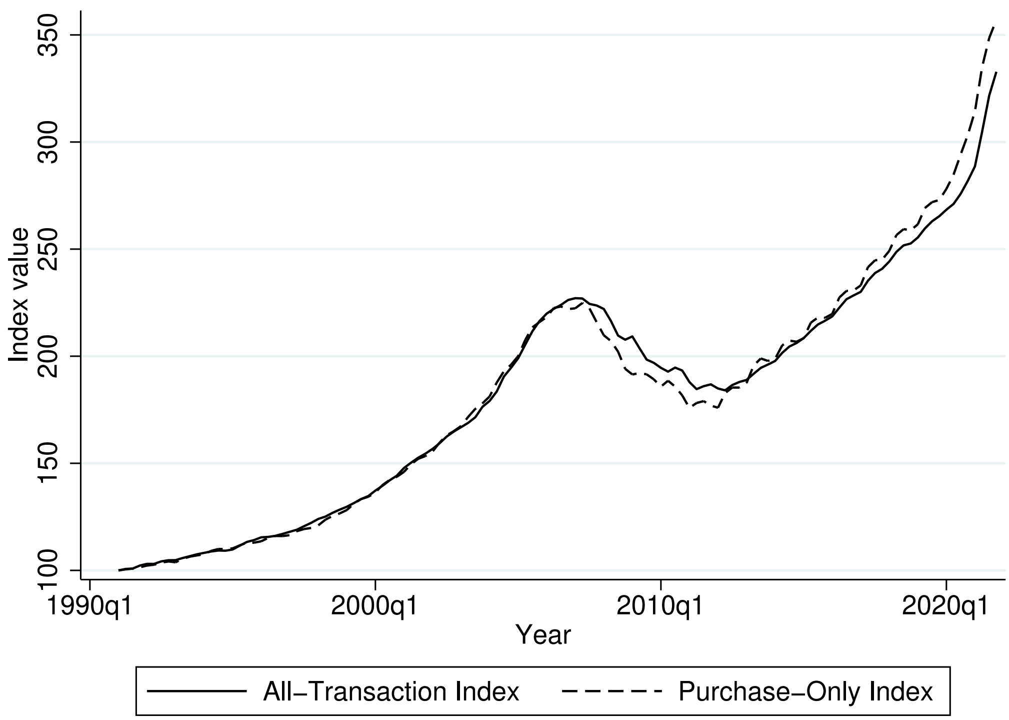

Table 1: Summary Statistics: CoreLogic

This table presents summary statistics of our loan-level data from CoreLogic. Panel A presents summary statistics for variables when appraised value is larger than sale price. Panel B presents summary statistics for variables when sale price is larger than appraised value. *Appraised Value* is the reported fair market value of the property (presented in thousands). *Sale Price* is the sale price of the property (presented in thousands). $\log(A-P)$ is the natural logarithm of the *Appraised Value* less *Sale Price*. $\log(P-A)$ is the natural logarithm of the opposite. *Home Equity* is the natural logarithm of sale price minus loan amount at the time of loan origination (presented in thousands). *LTV* is the original mortgage amount divided by the lesser of *Appraised Value* or *Sale Price*. *FICO* is the borrower's FICO score at the time of loan origination. All variables but *Appraised Value* and *Sale Price* are winsorized at the 0.5% and 99.5% levels.

	Mean	SD	10th	25th	Median	75th	90th	Obs.
Panel A: Appraised value larger than price								
Appraised value (A)	230.8	211.7	82.0	117.0	171.0	275.0	430.0	11,030,416
Sale price (P)	222.8	203.9	78.5	112.9	165.6	267.0	419.0	11,030,416
log(A-P)	7.79	1.65	5.52	6.91	8.01	8.85	9.77	11,030,416
Home equity	9.67	1.73	7.27	8.60	10.04	10.90	11.57	11,030,416
LTV	83.8%	14.3%	67.2%	80.0%	80.0%	95.0%	98.6%	11,030,416
FICO	711	66.6	619	666	720	766	790	7,274,646
Panel B: Price larger than appraised value								
	Mean	SD	10th	25th	Median	75th	90th	Obs.
Appraised value (A)	283.4	278.1	92.5	137.5	215.0	345.0	522.4	309,186
Sale price (P)	318.8	309.4	106.5	155.0	243.2	389.0	590.2	309,186
log(P-A)	9.04	2.00	6.40	7.82	9.21	10.55	11.52	309,186
Home equity	10.99	1.25	9.29	10.38	11.84	12.39	12.71	309,186
LTV	77.5%	16.4%	54.8%	71.2%	80.0%	90.0%	96.5%	309,186
FICO	721	63.8	631	680	732	774	795	126,969

Table 2: Appraisal, Price, and Default Risk

This table presents results from our tests of Corollary 1. In our ordinary least squares (OLS) regressions, the dependent variable is $\log(A - P)$ in Panel A and $\log(P - A)$ in Panel B. In our Poisson regressions, the dependent variable is A-P in Panel A and P-A in Panel B. A is *Appraised Value* (i.e., lender's value), and P is *Sale Price* (i.e., price). Our proxies for default risk are *Home Equity*, *LTV*, and *FICO*. We take the negative of *Home Equity* and *FICO* so that the sign of the predicted coefficient is the same across all variables. *PC1* is the first principal component of the three proxies. See Table 1 for variable definitions. All regression variables are winsorized at the 0.5% and 99.5% levels. *LTV*, *FICO*, and *PC1* are standardized to zero mean and unit standard deviation. Standard errors below coefficient estimates are adjusted for clustering at the zip code, year, and month levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

Panel A: Appraised value at least as large as price

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Home equity	-0.122***				-0.079***			
	(0.006)				(0.005)			
LTV		-0.134***				-0.058***		
		(0.005)				(0.008)		
FICO			-0.076***				-0.060***	
			(0.007)				(0.008)	
PC1				-0.169***				-0.120***
				(0.007)				(0.007)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	17.8%	16.5%	15.0%	15.9%				
Pseudo R^2					37.2%	34.9%	35.7%	36.3%
Observations	11,030,416	11,030,416	7,274,646	7,274,646	20,095,354	20,095,354	12,318,206	12,318,206

Panel B: Price at least as large as appraised value

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Home equity	-0.988***				-1.967***			
	(0.065)				(0.098)			
LTV		-0.069				-0.055		
		(0.051)				(0.048)		
FICO			-0.083*				-0.016	
			(0.038)				(0.081)	
PC1				-0.521***				-0.771***
				(0.022)				(0.051)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	56.8%	39.4%	34.3%	39.0%				
Pseudo R^2					72.2%	45.1%	45.0%	50.3%
Observations	309,186	309,186	126,969	126,969	3,524,647	3,524,647	1,532,085	1,532,085

Table 3: Appraisal, Price, and Default Risk: Appraisal bias

This table presents results from our tests of Corollary 1 after adjusting for appraisal bias. Panel A includes observations that are less affected by appraisal bias, which we define as observations in which the appraised value is equal to or greater than 5% greater than the sale price. Panel B includes observations that are more affected by appraisal bias, which we define as observations in which the appraised value is equal to or less than 5% greater than the sale price. In our ordinary least squares (OLS) regressions, the dependent variable is $\log(A - P)$ in Panel A and *log(P-A/1.05)* in Panel B. In our Poisson regressions, the dependent variable is *A-P* in Panel A and *P-A/1.05* in Panel B. *A* is *Appraised Value* (i.e., lender's value), and *P* is *Sale Price* (i.e., price). Our proxies for default risk are *Home Equity*, *LTV*, and *FICO*. We take the negative of *Home Equity* and *FICO* so that the sign of the predicted coefficient is the same across all variables. *PC1* is the first principal component of the three proxies. See Table 1 for variable definitions. All regression variables are winsorized at the 0.5% and 99.5% levels. *LTV*, *FICO*, and *PC1* are standardized to zero mean and unit standard deviation. Standard errors below coefficient estimates are adjusted for clustering at the zip code, year, and month levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

Panel A: Less affected by appraisal bias

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Home equity	-0.147***				-0.174***			
	(0.007)				(0.005)			
LTV		-0.128***				-0.135***		
		(0.004)				(0.005)		
FICO			-0.060***				-0.067***	
			(0.004)				(0.006)	
PC1				-0.170***				-0.186***
				(0.003)				(0.005)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	51.1%	48.5%	45.6%	47.8%				
Pseudo R^2					70.2%	68.8%	67.3%	68.7%
Observations	1,603,591	1,603,591	1,032,505	1,032,505	1,603,591	1,603,591	1,032,505	1,032,505

Panel B: More affected by appraisal bias

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Home equity	-0.192***				-0.317***			
	(0.014)				(0.027)			
LTV		-0.137***				-0.129***		
		(0.004)				(0.006)		
FICO			-0.063***				-0.064***	
			(0.004)				(0.006)	
PC1				-0.209***				-0.246***
				(0.006)				(0.006)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	54.3%	48.6%	47.7%	50.7%				
Pseudo R^2					72.0%	62.5%	65.1%	65.1%
Observations	19,118,683	19,118,683	11,364,711	11,364,711	19,153,826	19,153,826	11,389,122	11,389,122

Table 4: Appraisal, Price, and Default Risk: Repossession Costs

This table presents results from our tests of Corollary 2, which states that the difference between the lender's effective estimate of collateral value (i.e., estimate of collateral value less repossession costs) and equilibrium price is negatively associated with borrower riskiness. We assume repossession costs are 5% of the appraised value. In our ordinary least squares (OLS) regressions, the dependent variable is *log(0.95×A-P)* in Panel A and *log(P-0.95×A)* in Panel B. In our Poisson regressions, the dependent variable is *0.95×A-P* in Panel A and *P-0.95×A* in Panel B. *A* is *Appraised Value* (i.e., lender's value), and *P* is *Sale Price* (i.e., price). Our proxies for default risk are *Home Equity*, *LTV*, and *FICO*. We take the negative of *Home Equity* and *FICO* so that the sign of the predicted coefficient is the same across all variables. *PC1* is the first principal component of the three proxies. See Table 1 for variable definitions. All regression variables are winsorized at the 0.5% and 99.5% levels. *LTV*, *FICO*, and *PC1* are standardized to zero mean and unit standard deviation. Standard errors below coefficient estimates are adjusted for clustering at the zip code, year, and month levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

Panel A: Sale price equal to or less than 95% of appraised value

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Home equity	-0.131***				-0.141***			
	(0.004)				(0.006)			
LTV		-0.146***				-0.144***		
		(0.006)				(0.005)		
FICO			-0.075***				-0.072***	
			(0.008)				(0.009)	
PC1				-0.164***				-0.166***
				(0.006)				(0.008)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	25.2%	24.8%	22.5%	23.1%				
Pseudo R^2					62.0%	61.8%	60.4%	61.0%
Observations	1,460,622	1,460,622	940,219	940,219	1,489,309	1,489,309	958,997	958,997

Panel B: Sale price equal to or greater than 95% of appraised value

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Home equity	-0.191***				-0.313***			
	(0.014)				(0.026)			
LTV		-0.137***				-0.128***		
		(0.004)				(0.006)		
FICO			-0.063***				-0.064***	
			(0.004)				(0.006)	
PC1				-0.208***				-0.244***
				(0.006)				(0.005)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	54.8%	49.0%	48.1%	51.2%				
Pseudo R^2					72.4%	62.9%	65.5%	69.0%
Observations	19,255,219	19,255,219	11,459,270	11,459,270	19,292,239	19,292,239	11,485,196	11,485,196

Table 5: Appraisal, Price, and Default Risk: Secondary Market

This table presents results from our tests of Corollary 3, which states that the difference between the lender's estimate of collateral value and equilibrium price is negatively associated with borrower riskiness among mortgages that are likely to be sold to an outside investor. In our ordinary least squares (OLS) regressions, the dependent variable is $\log(A - P)$ in Panel A and $\log(P - A)$ in Panel B. In our Poisson regressions, the dependent variable is A-P in Panel A and P-A in Panel B. A is *Appraised Value* (i.e., lender's value), and P is *Sale Price* (i.e., price). Our proxies for default risk are *Home Equity*, *LTV*, and *FICO*. We take the negative of *Home Equity* and *FICO* so that the sign of the predicted coefficient is the same across all variables. *PC1* is the first principal component of the three proxies. See Table 1 for variable definitions. All regression variables are winsorized at the 0.5% and 99.5% levels. *LTV*, *FICO*, and *PC1* are standardized to zero mean and unit standard deviation. Standard errors below coefficient estimates are adjusted for clustering at the zip code, year, and month levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

Panel A: Appraised value at least as large as price

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Home equity	-0.108***				-0.050***			
	(0.006)				(0.006)			
LTV		-0.131***				-0.048***		
		(0.006)				(0.008)		
FICO			-0.077***				-0.062***	
			(0.007)				(0.009)	
PC1				-0.156***				-0.097***
				(0.007)				(0.009)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	15.3%	15.1%	14.1%	14.6%				
Pseudo R^2					33.8%	33.7%	35.9%	36.0%
Observations	10,206,248	10,206,248	6,747,257	6,747,257	18,232,404	18,232,404	11,217,204	11,217,204

Panel B: Price at least as large as appraised value

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Home equity	-0.978***				-2.057***			
	(0.070)				(0.121)			
LTV		-0.050				-0.015		
		(0.056)				(0.054)		
FICO			-0.074				0.002	
			(0.042)				(0.088)	
PC1				-0.503***				-0.770***
				(0.022)				(0.057)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	58.5%	41.1%	36.3%	40.8%				
Pseudo R^2					75.3%	46.6%	46.3%	51.8%
Observations	267,148	267,148	108,721	108,721	2,972,029	2,972,029	1,279,052	1,279,052

Table 6: Summary Statistics: Freddie Mac and FHFA HPI

This table presents summary statistics of returns derived from FHFA HPI and our loan data from Freddie Mac. *Log return* is the annual change in log value of the non-seasonally adjusted FHFA HPI. *Home Equity* is the weighted average of quarterly home equity values (i.e., natural logarithm of home equity estimated from LTV and the loan amount at the time of loan origination) within a CBSA. *LTV* is the weighted average of quarterly loan-to-value ratios within a CBSA. *Credit Score* is the weighted average of quarterly credit scores within a CBSA. All averages are calculated with value used as the weight. Each data set focuses on single-family homes that were purchased, not refinanced. Our sample spans 2000 through 2021 and includes 8,772 quarterly observations from the 100 largest CBSAs.

	Mean	SD	10th	25th	Median	75th	90th
Log return	4.40%	8.02%	-0.05%	1.33%	4.67%	8.34%	13.30%
Home Equity	11.30	0.53	10.72	10.92	11.21	11.58	12.04
LTV	76.6%	6.1%	70.1%	74.1%	77.3%	80.6%	82.9%
Credit Score	744	17.3	720	730	749	757	763

Table 7: Return Autocorrelation

This table presents results from our tests of Proposition 8, which we test by regressing *Log return* on the one-year lag of itself interacted with dummies for our different proxies for default risk. *Log Return* is the annual change in log value of the non-seasonally adjusted FHFA HPI. *Low*, *Mid*, and *High* are dummy variables indicating whether the proxy in that column is in the lowest, middle, or highest tercile within a given CBSA. Our proxies for default risk are *Home Equity*, *LTV*, and *Credit Score*. *PC1* is the first principal component of the three proxies. See Table 6 for variable definitions. Standard errors below coefficient estimates are adjusted for clustering at the CBSA, year, and quarter levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

	No Sort (1)	Home Equity (2)	LTV (3)	Credit Score (4)	PC1 (5)
ρ	0.68***				
	(0.08)				
$\rho \times$ Low		0.79***	0.56***	0.78***	0.90***
		(0.10)	(0.09)	(0.12)	(0.12)
$\rho \times$ Mid		0.73***	0.72***	0.65***	0.78***
		(0.09)	(0.09)	(0.07)	(0.09)
$\rho \times$ High		0.58**	0.93**	0.58**	0.52**
		(0.11)	(0.17)	(0.18)	(0.08)
Adjusted R^2	41.4%	42.3%	43.6%	42.1%	44.1%
Observations	8,772	8,772	8,772	8,772	8,772

A Proofs

A.1 Proof of Proposition 1

Require that equation (6) equal $\eta_{\mathcal{B}}$ times equation (9) and equation (7) equal $\eta_{\mathcal{L}}$ times equation (9). Solve for c and p. ■

A.2 Proof of Proposition 2

We proceed by backwards induction, starting on date $t = 1$. Suppose that the borrower had committed to a repayment c on date $t = 0$. The seller's and borrower's surpluses are

$$S_{\mathcal{S}}(c, p) = p - v_1 \tag{81}$$

$$S_{\mathcal{B}}(c, p) = (1 - d)(v_1 - c(p)) - (1 - \ell)p. \tag{82}$$

The seller's and borrower's expected surpluses

$$\mathbb{E}^{\mathcal{S}}[S_{\mathcal{S}}(c, p)] = p - a_{\mathcal{S}} \tag{83}$$

$$\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c, p)] = (1 - \lambda)(a_{\mathcal{B}} - c) - (1 - \ell)p. \tag{84}$$

As a function of the repayment c, the price $p(c)$ that maximizes the Nash product is

$$p(c) = \frac{\eta_{\mathcal{B}}}{\eta_{\mathcal{B}} + \eta_{\mathcal{S}}} \cdot a_{\mathcal{S}} + \frac{\eta_{\mathcal{S}}}{\eta_{\mathcal{B}} + \eta_{\mathcal{S}}} \cdot \frac{(1 - \lambda)(a_{\mathcal{B}} - c)}{(1 - \ell)}. \tag{85}$$

Now on date $t = 0$, the borrower's and lender's surpluses are

$$S_{\mathcal{B}}(c, p(c)) = (1 - d)(v_1 - c) - (1 - \ell)p(c) \tag{86}$$

$$S_{\mathcal{L}}(c, p(c)) = dv_1 + (1 - d)c - \ell p(c) \tag{87}$$

Therefore, the borrower's and lender's subjective, expected surpluses are

$$\mathbb{E}^{\mathcal{B}}[S_{\mathcal{B}}(c, p(c))] = (1 - \lambda)(a_{\mathcal{B}} - c) - (1 - \ell)p(c) \tag{88}$$

$$\mathbb{E}^{\mathcal{L}}[S_{\mathcal{L}}(c, p(c))] = \lambda a_{\mathcal{L}} + (1 - \lambda)c - \ell p(c). \tag{89}$$

The equilibrium c maximizes the Nash product. ■

A.3 Proof of Proposition 3

Just as in the proof of Proposition 1, but now substitute $\xi a_{\mathcal{L}}$ for $a_{\mathcal{L}}$. ■

A.4 Proof of Proposition 4

Just as in the proof of Proposition 1, but now solve for ℓ and p (rather than c and p). ■

A.5 Proof of Proposition 5

Set equations (27) and (28) equal to zero and solve for c and p. ■

A.6 Proof of Proposition 6

Set equations (38), (36), and (35) equal to zero and solve for c, p_L, and p_A. ■

A.7 Proof of Proposition 7

Conjecture a linear equilibrium of the form

$$P_t = c_V V_t + c_{\mathcal{B}} \widehat{X}_t^{\mathcal{B}} + c_{\mathcal{L}} \widehat{X}_t^{\mathcal{L}}, \tag{90}$$

where c_V, $c_{\mathcal{B}}$, and $c_{\mathcal{L}}$ are known constants. Integrating equation (45) from t to $s > t$, we obtain

$$X_s = X_t e^{-\kappa(s-t)} + \sqrt{2\kappa}\sigma_X \int_t^s e^{-\kappa(s-\tau)} dB_\tau^X. \tag{91}$$

Therefore,

$$\mathbb{E}_t^j[X_s] = \widehat{X}_t^j e^{-\kappa(s-t)}. \tag{92}$$

Integrating equation (47) from t to T, we obtain

$$\widehat{X}_T^j = e^{-(\kappa+w_j)(T-t)} \widehat{X}_t^j + w_j \int_t^T e^{-(\kappa+w_j)(T-s)} dV_s. \tag{93}$$

Therefore, i's expectation of j's belief is

$$\mathbb{E}_t^i[\widehat{X}_T^j|T] = e^{-(\kappa+w_j)(T-t)} \widehat{X}_t^j + w_j \int_t^T e^{-(\kappa+w_j)(T-s)} \widehat{X}_t^i e^{-\kappa(s-t)} ds \tag{94}$$

$$= e^{-(\kappa+w_j)(T-t)} \widehat{X}_t^j + \left(e^{-\kappa(T-t)} - e^{-(\kappa+w_j)(T-t)}\right) \widehat{X}_t^i, \tag{95}$$

having used the fact that

$$\mathbb{E}_t^i[dV_s] = \mathbb{E}_t^i[X_s ds + \sigma_V dB_s^V] = \widehat{X}_t^i e^{-\kappa(s-t)} ds. \tag{96}$$

The value at time T is given by

$$V_T = V_t + \int_t^T X_s ds + \sigma_V \left(B_T^V - B_t^V\right). \tag{97}$$

Hence,

$$\mathbb{E}_t^j[V_T] = V_t + \int_t^T \mathbb{E}_t^j[X_s] ds = V_t + \int_t^T \widehat{X}_t^j e^{-\kappa(s-t)} ds = V_t + \kappa^{-1}\left(1 - e^{-\kappa(T-t)}\right) \widehat{X}_t^j. \tag{98}$$

For any constant $a > 0$,

$$\mathbb{E}_t\left[e^{-a(T-t)}\right] = \int_t^\infty \lambda e^{-(a+\lambda)(T-t)} dT = \frac{\lambda}{a+\lambda}. \tag{99}$$

First, the buyer's expected value from owning the asset is

$$\mathcal{V}_t = \mathbb{E}_t\left[\int_t^T r\mathbb{E}_t^{\mathcal{B}}[V_s|T] e^{-r(s-t)} ds\right] \tag{100}$$

$$= \mathbb{E}_t\left[\int_t^T r\left(V_t + \kappa^{-1}\left(1 - e^{-\kappa(s-t)}\right)\widehat{X}_t^{\mathcal{B}}\right) e^{-r(s-t)} ds\right] \tag{101}$$

$$= r(r+\lambda)^{-1} V_t + r(r+\lambda)^{-1}(r+\kappa+\lambda)^{-1}\widehat{X}_t^{\mathcal{B}}. \tag{102}$$

Next, we compute the lender's expected recovery value. From equation (95),

$$\mathbb{E}_t^{\mathcal{L}}[V_T|T] = V_t + \kappa^{-1}\left(1 - e^{-\kappa(T-t)}\right)\widehat{X}_t^{\mathcal{L}} \tag{103}$$

$$\mathbb{E}_t^{\mathcal{L}}[\widehat{X}_T^{\mathcal{B}}|T] = e^{-(w_{\mathcal{B}}+\kappa)(T-t)}\widehat{X}_t^{\mathcal{B}} + \left(e^{-\kappa(T-t)} - e^{-(w_{\mathcal{B}}+\kappa)(T-t)}\right)\widehat{X}_t^{\mathcal{L}} \tag{104}$$

$$\mathbb{E}_t^{\mathcal{L}}[\widehat{X}_T^{\mathcal{L}}|T] = e^{-\kappa(T-t)}\widehat{X}_t^{\mathcal{L}}. \tag{105}$$

Therefore,

$$\mathbb{E}_t\left[\mathbb{E}_t^{\mathcal{L}}[V_T|T]e^{-r(T-t)}\right] = \lambda(r+\lambda)^{-1}V_t + \lambda(r+\lambda)^{-1}(r+\kappa+\lambda)^{-1}\widehat{X}_t^{\mathcal{L}} \tag{106}$$

$$\mathbb{E}_t\left[\mathbb{E}_t^{\mathcal{L}}[\widehat{X}_T^{\mathcal{B}}|T]e^{-r(T-t)}\right] = \lambda(w_{\mathcal{B}}+\kappa+r+\lambda)^{-1}\widehat{X}_t^{\mathcal{B}} + \lambda w_{\mathcal{B}}(r+\kappa+\lambda)^{-1}(r+w_{\mathcal{B}}+\kappa+\lambda)^{-1}\widehat{X}_t^{\mathcal{L}} \tag{107}$$

$$\mathbb{E}_t\left[\mathbb{E}_t^{\mathcal{L}}[\widehat{X}_T^{\mathcal{L}}|T]e^{-r(T-t)}\right] = \lambda(r+\kappa+\lambda)^{-1}\widehat{X}_t^{\mathcal{L}} \tag{108}$$

and therefore, the expected recovery value is

$$\mathcal{R}_t = \mathbb{E}_t\left[\mathbb{E}_t^{\mathcal{L}}[P_T|T]e^{-r(T-t)}\right] \tag{109}$$

$$= \mathbb{E}_t\left[\left(c_V\mathbb{E}_t^{\mathcal{L}}[V_T|T] + c_{\mathcal{B}}\mathbb{E}_t^{\mathcal{L}}[\widehat{X}_T^{\mathcal{B}}|T] + c_{\mathcal{L}}\mathbb{E}_t^{\mathcal{L}}[\widehat{X}_T^{\mathcal{L}}|T]\right)e^{-r(T-t)}\right], \tag{110}$$

which can be computed using equations (106) through (108). Rewriting equation (54),

$$P_t = \mathcal{V}_t + \mathcal{R}_t \tag{111}$$

$$\begin{aligned} &= \left(r(r+\lambda)^{-1} + c_V\lambda(r+\lambda)^{-1}\right)V_t \\ &+ \left(r(r+\lambda)^{-1}(r+\kappa+\lambda)^{-1} + c_{\mathcal{B}}\lambda((w_{\mathcal{B}}+\kappa)+r+\lambda)^{-1}\right)\widehat{X}_t^{\mathcal{B}} \\ &+ \lambda(r+\kappa+\lambda)^{-1}\left(c_V(r+\lambda)^{-1} + c_{\mathcal{B}}w_{\mathcal{B}}(r+(w_{\mathcal{B}}+\kappa)+\lambda)^{-1} + c_{\mathcal{L}}\right)\widehat{X}_t^{\mathcal{L}}. \end{aligned} \tag{112}$$

Comparing terms, we have that $c_V(\lambda) = 1$, and $c_{\mathcal{B}}$ and $c_{\mathcal{L}}$ are given by the expressions in equations (60) and (61) respectively. It follows that

$$c'_{\mathcal{L}}(\lambda) = \frac{r\left(w_{\mathcal{B}}(\kappa+2(\lambda+r)) + (\kappa+\lambda+r)^2\right)}{(\lambda+r)^2(\kappa+\lambda+r)^2(\kappa+r+w_{\mathcal{B}})} > 0. \tag{113}$$

and

$$(r+\kappa)(c_{\mathcal{B}}(\lambda) + c_{\mathcal{L}}(\lambda)) = 1. \tag{114}$$

It remains to compute the equilibrium repayment. Rearranging the no-surplus condition for the borrower, we obtain

$$(1-\ell)P_t = \mathbb{E}_t\left[\int_t^T r\mathbb{E}_t^{\mathcal{B}}[V_s|T]e^{-r(s-t)}ds\right] + \mathbb{E}_t\left[\int_t^T rC_te^{-r(s-t)}\right] \tag{115}$$

$$= r(r+\lambda)^{-1}V_t + r(r+\lambda)^{-1}(r+\kappa+\lambda)^{-1}\widehat{X}_t^{\mathcal{B}} - r(r+\lambda)^{-1}C_t, \tag{116}$$

and hence

$$C_t = V_t + (r+\kappa+\lambda)^{-1}\widehat{X}_t^{\mathcal{B}} - r^{-1}(r+\lambda)(1-\ell)P_t \tag{117}$$

$$= b_V(\lambda)V_t + b_{\mathcal{B}}(\lambda)\widehat{X}_t^{\mathcal{B}} + b_{\mathcal{L}}(\lambda)\widehat{X}_t^{\mathcal{L}}, \tag{118}$$

where b_V, $b_{\mathcal{B}}$, and $b_{\mathcal{L}}$ are given by equations (57), (58), and (59) respectively. This concludes the proof. ■

A.8 Proof of Proposition 8

We begin by establishing some facts about several unconditional expectations that will appear later in the proof. For each $i \in \{\mathcal{B}, \mathcal{L}\}$, let $\eta_i = r + w_i$, $\zeta_{i,j} = (r+\eta_i)(r+\eta_j)$,

$$h_i(s,t,\tau) = e^{r(\tau-t)+(\kappa+w_i)(s-t)}, \text{ and} \tag{119}$$

$$H_{i,\tau_0,\tau_1}(s,\tau) = \int_{\tau_0}^{\tau_1} h_i(s,t,\tau)dt. \tag{120}$$

In particular,

$$H_{i,s,\tau}(s,\tau) = (r+\kappa+w_i)^{-1}(e^{r(\tau-s)} - e^{-(\kappa+w_i)(\tau-s)}) \tag{121}$$

$$H_{i,\tau-\theta,\tau}(s,\tau) = (r+\kappa+w_i)^{-1}(e^{(r+\kappa+w_i)\theta} - 1)e^{-(\kappa+w_i)(\tau-s)} \tag{122}$$

$$H_{i,\tau,\tau+\theta}(s,\tau) = (r+\kappa+w_i)^{-1}(1 - e^{-(r+\kappa+w_i)\theta})e^{-(\kappa+w_i)(\tau-s)}. \tag{123}$$

In what follows, we omit arguments for parsimony. For a standard 1-dimensional Brownian motion B_t, one can write

$$\int_{\tau-\theta}^{\tau}\int_{-\infty}^{t} h_i dB_s dt = \int_{-\infty}^{\tau-\theta} H_{i,\tau-\theta,\tau}dB_s + \int_{\tau-\theta}^{\tau} H_{i,s,\tau}dB_s \tag{124}$$

$$\int_{\tau}^{\tau+\theta}\int_{-\infty}^{t} h_i dB_s dt = \int_{-\infty}^{\tau-\theta} H_{i,\tau,\tau+\theta}dB_s + \int_{\tau-\theta}^{\tau} H_{i,\tau,\tau+\theta}dB_s + \int_{\tau}^{\tau+\theta} H_{i,s,\tau+\theta}dB_s. \tag{125}$$

Consider the following unconditional expectations:

$$\mathcal{I}^{(1)} \equiv \mathbb{E}\left[\left(\int_{\tau-\theta}^{\tau} e^{r(\tau-t)} dB_t\right)^2\right] \tag{126}$$

$$\mathcal{I}_i^{(2)} \equiv \mathbb{E}\left[\int_{\tau-\theta}^{\tau} e^{r(\tau-t)} dB_t \cdot \int_{\tau-\theta}^{\tau}\int_{-\infty}^{t} h_i dB_s dt\right] \tag{127}$$

$$\mathcal{I}_{i,j}^{(3)} \equiv \mathbb{E}\left[\int_{\tau-\theta}^{\tau}\int_{-\infty}^{t} h_i dB_s dt \cdot \int_{\tau-\theta}^{\tau}\int_{-\infty}^{t} h_j dB_s dt\right] \tag{128}$$

$$\mathcal{I}_i^{(4)} \equiv \mathbb{E}\left[\int_{\tau-\theta}^{\tau} e^{r(\tau-t)} dB_t \cdot \int_{\tau}^{\tau+\theta}\int_{-\infty}^{t} h_i dB_s dt\right] \tag{129}$$

$$\mathcal{I}_{i,j}^{(5)} \equiv \mathbb{E}\left[\int_{\tau-\theta}^{\tau}\int_{-\infty}^{t} h_i dB_s dt \cdot \int_{\tau}^{\tau+\theta}\int_{-\infty}^{t} h_j dB_s dt\right]. \tag{130}$$

By the Itô isometry (and equations (124) and (125)), we have

$$\mathcal{I}^{(1)} = \int_{\tau-\theta}^{\tau} e^{2r(\tau-s)} ds = \frac{e^{2r\theta}-1}{2r} \tag{131}$$

$$\mathcal{I}_i^{(2)} = \int_{\tau-\theta}^{\tau} e^{r(\tau-s)} H_{i,s,\tau} ds = \frac{1}{r+\eta_i}\left(\mathcal{I}^{(1)} + \frac{1-e^{(r-\eta_i)\theta}}{r-\eta_i}\right) \tag{132}$$

$$\mathcal{I}_{i,j}^{(3,a)} \equiv \int_{-\infty}^{\tau-\theta} H_{i,\tau-\theta,\tau} H_{j,\tau-\theta,\tau} ds = \frac{(e^{(r+\eta_i)\theta}-1)(e^{(r+\eta_j)\theta}-1)e^{-(\eta_i+\eta_j)\theta}}{(\eta_i+\eta_j)\zeta_{i,j}} \tag{133}$$

$$\mathcal{I}_{i,j}^{(3,b)} \equiv \int_{\tau-\theta}^{\tau} H_{i,s,\tau} H_{j,s,\tau} ds = \frac{e^{2r\theta}-1}{2r\zeta_{i,j}} - \frac{e^{(r-\eta_i)\theta}-1}{(r-\eta_i)\zeta_{i,j}} - \frac{e^{(r-\eta_j)\theta}-1}{(r-\eta_j)\zeta_{i,j}} + \frac{1-e^{-(\eta_i+\eta_j)\theta}}{(\eta_i+\eta_j)\zeta_{i,j}} \tag{134}$$

$$\mathcal{I}_{i,j}^{(3)} = \mathcal{I}_{i,j}^{(3,a)} + \mathcal{I}_{i,j}^{(3,b)} \tag{135}$$

and

$$\mathcal{I}_i^{(4)} = \int_{\tau-\theta}^{\tau} e^{r(\tau-s)} H_{i,\tau,\tau+\theta} ds = \frac{2(\cosh(r\theta)-\cosh(\eta_i\theta))e^{-\eta_i\theta}}{(r+\eta_i)(r-\eta_i)} \tag{136}$$

$$\mathcal{I}_{i,j}^{(5,a)} = \int_{-\infty}^{\tau-\theta} H_{i,\tau-\theta,\tau} H_{j,\tau,\tau+\theta} ds \tag{137}$$

$$\mathcal{I}_{i,j}^{(5,b)} = \int_{\tau-\theta}^{\tau} H_{i,s,\tau} H_{j,\tau,\tau+\theta} ds \tag{138}$$

$$\mathcal{I}_{i,j}^{(5)} = \mathcal{I}_{i,j}^{(5,a)} + \mathcal{I}_{i,j}^{(5,b)} = \frac{2(\cosh(r\theta)-\cosh(\eta_j\theta))e^{-\eta_j\theta}}{(\eta_i+\eta_j)(r+\eta_j)(r-\eta_j)}. \tag{139}$$

Note that

$$\frac{\mathcal{I}_i^{(2)} + \mathcal{I}_j^{(2)}}{\mathcal{I}_{i,j}^{(3)}} = \frac{\mathcal{I}_j^{(4)}}{\mathcal{I}_{i,j}^{(5)}} = \eta_i + \eta_j \tag{140}$$

and

$$\mathcal{I}^{(1)\prime}(\theta) = e^{2r\theta} \tag{141}$$

$$\mathcal{I}_i^{(2)\prime}(\theta) = \frac{1}{r+\eta_i}\left(e^{2r\theta} - e^{(r-\eta_i)\theta}\right) \tag{142}$$

$$\mathcal{I}_{i,j}^{(3)\prime}(\theta) = \frac{1}{\eta_i+\eta_j}\left(\mathcal{I}_i^{(2)\prime}(\theta) + \mathcal{I}_j^{(2)\prime}(\theta)\right). \tag{143}$$

In particular, $\mathcal{I}^{(1)\prime}(\theta) = 1$ and $\mathcal{I}_i^{(2)\prime}(\theta) = \mathcal{I}_{i,j}^{(3)\prime}(\theta) = 0$. Having established these facts, we now turn our attention to the error in borrowers' and lenders' estimates of the growth rate. Integrating equation (65) from $-\infty$ to t, we obtain

$$Z_t^i = \sqrt{2\kappa}\sigma_X M_t^{X,i} - w_i \sigma_V M_t^{V,i}, \tag{144}$$

where

$$M_t^{X,j} \equiv \int_{-\infty}^{t} e^{(\kappa+w_j)(s-t)} dB_s^X \tag{145}$$

$$M_t^{V,j} \equiv \int_{-\infty}^{t} e^{(\kappa+w_j)(s-t)} dB_s^V. \tag{146}$$

Importantly, $\mathbb{E}[M_{\tau_0}^{X,j} M_{\tau_1}^{V,j}] = 0$ for any times τ_0 and τ_1. Define the integrands

$$N_{\tau-\theta,\tau}^{Z^j} \equiv \alpha_j(\lambda) \int_{\tau-\theta}^{\tau} e^{r(\tau-t)} Z_t^j dt \tag{147}$$

$$N_{\tau-\theta,\tau}^{V} \equiv \overline{\alpha} \int_{\tau-\theta}^{\tau} e^{r(\tau-t)} dB_t^V, \tag{148}$$

where $\overline{\alpha} = (\eta_{\mathcal{B}} + \eta_{\mathcal{L}})\sigma_V$. It follows that

$$N_{\tau-\theta,\tau}^{Z^j} = \alpha_j(\lambda) \int_{\tau-\theta}^{\tau} e^{r(\tau-t)} \left(\sqrt{2\kappa}\sigma_X M_t^{X,j} - w_j \sigma_V M_t^{V,j}\right) dt. \tag{149}$$

Now consider the return from time τ to time $\tau+\theta$ as defined by equation (71):

$$R_{\tau,\tau+\theta} = \int_{\tau}^{\tau+\theta} e^{r((\tau+\theta)-t)}(dP_t + r(V_t - P_t)dt) \tag{150}$$

$$= \eta_{\mathcal{B}}(\lambda)\int_{\tau}^{\tau+\theta} e^{r((\tau+\theta)-t)}(Z_t^{\mathcal{B}}dt + \sigma_V dB_t^V) + \eta_{\mathcal{L}}(\lambda)\int_{\tau}^{\tau+\theta} e^{r((\tau+\theta)-t)}(Z_t^{\mathcal{L}}dt + \sigma_V dB_t^V) \tag{151}$$

$$= N_{\tau,\tau+\theta}^{Z^{\mathcal{B}}} + N_{\tau,\tau+\theta}^{Z^{\mathcal{L}}} + N_{\tau,\tau+\theta}^{V}. \tag{152}$$

It follows immediately that $R_{\tau-\theta,\tau} = N_{\tau-\theta,\tau}^{Z^{\mathcal{B}}} + N_{\tau-\theta,\tau}^{Z^{\mathcal{L}}} + N_{\tau-\theta,\tau}^{V}$. Let

$$\varphi_{i,j} = 2\kappa\tau_X^{-1} + w_i w_j \tau_V^{-1}. \tag{153}$$

Consider the following variances and covariances:

$$\mathbb{E}[(N_{\tau-\theta,\tau}^{V})^2] = (\eta_{\mathcal{B}}(\lambda) + \eta_{\mathcal{L}}(\lambda))^2\tau_V^{-1}\mathcal{I}^{(1)} \tag{154}$$

$$\mathbb{E}[N_{\tau-\theta,\tau}^{Z^i}N_{\tau-\theta,\tau}^{V}] = -w_i\alpha_i(\lambda)(\eta_{\mathcal{B}}(\lambda) + \eta_{\mathcal{L}}(\lambda))\tau_V^{-1}\mathcal{I}_i^{(2)} \tag{155}$$

$$\mathbb{E}[N_{\tau-\theta,\tau}^{Z^i}N_{\tau-\theta,\tau}^{Z^j}] = \varphi_{i,j}\alpha_i(\lambda)\alpha_j(\lambda)\mathcal{I}_{i,j}^{(3)} \tag{156}$$

$$\mathbb{E}[N_{\tau-\theta,\tau}^{V}N_{\tau,\tau+\theta}^{Z^i}] = -w_i\alpha_i(\lambda)(\eta_{\mathcal{B}}(\lambda) + \eta_{\mathcal{L}}(\lambda))\tau_V^{-1}e^{r\theta}\mathcal{I}_i^{(4)} \tag{157}$$

$$\mathbb{E}[N_{\tau-\theta,\tau}^{Z^i}N_{\tau,\tau+\theta}^{Z^j}] = \varphi_{i,j}\alpha_i(\lambda)\alpha_j(\lambda)e^{r\theta}\mathcal{I}_{i,j}^{(5)}. \tag{158}$$

The unconditional variance of $R_{\tau-\theta,\tau}$ is

$$\sigma_1^2(\lambda,\theta) \equiv \mathbb{E}[(R_{\tau-\theta,\tau})^2] \tag{159}$$

$$= \mathbb{E}[(N_{\tau-\theta,\tau}^{Z^{\mathcal{B}}})^2] + \mathbb{E}[(N_{\tau-\theta,\tau}^{Z^{\mathcal{L}}})^2] + \mathbb{E}[(N_{\tau-\theta,\tau}^{V})^2] + 2\mathbb{E}[N_{\tau-\theta,\tau}^{Z^{\mathcal{B}}}N_{\tau-\theta,\tau}^{Z^{\mathcal{L}}}] + 2\mathbb{E}[N_{\tau-\theta,\tau}^{V}N_{\tau-\theta,\tau}^{Z^{\mathcal{B}}}] + 2\mathbb{E}[N_{\tau-\theta,\tau}^{V}N_{\tau-\theta,\tau}^{Z^{\mathcal{L}}}] \tag{160}$$

$$= \varphi_{\mathcal{B},\mathcal{B}}\eta_{\mathcal{B}}(\lambda)^2\mathcal{I}_{\mathcal{B},\mathcal{B}}^{(3)}(\theta) + \varphi_{\mathcal{L},\mathcal{L}}\eta_{\mathcal{L}}(\lambda)^2\mathcal{I}_{\mathcal{L},\mathcal{L}}^{(3)}(\theta) + (\eta_{\mathcal{B}}(\lambda) + \eta_{\mathcal{L}}(\lambda))^2\tau_V^{-1}\mathcal{I}^{(1)}(\theta) + 2\varphi_{\mathcal{B},\mathcal{L}}\eta_{\mathcal{B}}(\lambda)\eta_{\mathcal{L}}(\lambda)\mathcal{I}_{\mathcal{B},\mathcal{L}}^{(3)}(\theta) - 2w_{\mathcal{B}}\eta_{\mathcal{B}}(\lambda)(\eta_{\mathcal{B}}(\lambda) + \eta_{\mathcal{L}}(\lambda))\tau_V^{-1}\mathcal{I}_{\mathcal{B}}^{(2)}(\theta) - 2w_{\mathcal{L}}\eta_{\mathcal{L}}(\lambda)(\eta_{\mathcal{B}}(\lambda) + \eta_{\mathcal{L}}(\lambda))\tau_V^{-1}\mathcal{I}_{\mathcal{L}}^{(2)}(\theta) \tag{161}$$

$$= \psi(w_{\mathcal{B}})\mathcal{I}_{\mathcal{B},\mathcal{B}}^{(3)}\eta_{\mathcal{B}}(\lambda)^2 + \psi(w_{\mathcal{L}})\mathcal{I}_{\mathcal{L},\mathcal{L}}^{(3)}\eta_{\mathcal{L}}(\lambda)^2 + \tau_V^{-1}\mathcal{I}^{(1)}(\eta_{\mathcal{B}}(\lambda) + \eta_{\mathcal{B}}(\lambda))^2 + 2(\psi(w_{\mathcal{B}})\mathcal{I}_{\mathcal{B},\mathcal{I}}^{(3)} + \tau_V^{-1}(w_{\mathcal{B}} - w_{\mathcal{L}})\mathcal{I}_{\mathcal{L}}^{(2)})\eta_{\mathcal{B}}(\lambda)\eta_{\mathcal{L}}(\lambda) \tag{162}$$

which does not depend on τ. Let

$$\beta_{i,j}(\lambda) = \alpha_i'(\lambda)\alpha_j(\lambda) + \alpha_i(\lambda)\alpha_j'(\lambda). \tag{163}$$

Differentiating with respect to λ:

$$\begin{aligned}\partial_\lambda \sigma_1^2(\lambda) = {} & \psi(w_\mathcal{B})\mathcal{I}^{(3)}_{\mathcal{B},\mathcal{B}}\beta_{\mathcal{B},\mathcal{B}}(\lambda) + \psi(w_\mathcal{L})\mathcal{I}^{(3)}_{\mathcal{L},\mathcal{L}}\beta_{\mathcal{L},\mathcal{L}}(\lambda) \\ & + \tau_V^{-1}\mathcal{I}^{(1)}(\beta_{\mathcal{B},\mathcal{B}}(\lambda) + 2\beta_{\mathcal{B},\mathcal{L}}(\lambda) + \beta_{\mathcal{L},\mathcal{L}}(\lambda)) \\ & + 2(\psi(w_\mathcal{B})\mathcal{I}^{(3)}_{\mathcal{B},\mathcal{L}} + \tau_V^{-1}(w_\mathcal{B} - w_\mathcal{L})\mathcal{I}^{(2)}_\mathcal{L})\beta_{\mathcal{B},\mathcal{L}}(\lambda). \end{aligned} \tag{164}$$

Define

$$\psi(w) = 2\kappa\tau_X^{-1} - w(w + 2\kappa)\tau_V^{-1}. \tag{165}$$

Recall that w_0 is the positive root of ψ. Let w_{-1} denote the negative root so that

$$\psi(w) = (w_0 - w)(w - w_{-1}). \tag{166}$$

Note that $\psi(w) > 0$ for $w \in [0, w_0)$, $\psi(w_0) = 0$, and $\psi(w) < 0$ for $w > w_0$. Let

$$D_i(\lambda) = \frac{\alpha_i(\lambda)(w_i\alpha_{-i}(\lambda) + w_{-i}\alpha_i(\lambda) + (2\kappa + w_i)(\alpha_i(\lambda) + \alpha_{-i}(\lambda)))\psi(w_i)}{\eta_i + \eta_{-i}}. \tag{167}$$

We have written D_i as a function of λ as we will need the derivative of D_i with respect to λ:

$$D_i'(\lambda) = \frac{(w_i\beta_{i,-i}(\lambda) + w_{-i}\beta_{i,i}(\lambda) + (2\kappa + w_i)(\beta_{i,i}(\lambda) + \beta_{i,-i}(\lambda)))\psi(w_i)}{\eta_i + \eta_{-i}}. \tag{168}$$

The unconditional auto-covariance is

$$\begin{aligned}\sigma_{1,2}(\lambda, \theta) & \equiv \mathbb{E}[R_{\tau-\theta,\tau}R_{\tau,\tau+\theta}] \qquad (169) \\ & = \sum_{j\in\{\mathcal{B},\mathcal{L}\}} \mathbb{E}[N^{Z^\mathcal{B}}_{\tau-\theta,\tau}N^{Z^j}_{\tau,\tau+\theta}] + \mathbb{E}[N^{Z^\mathcal{L}}_{\tau-\theta,\tau}N^{Z^j}_{\tau,\tau+\theta}] + \mathbb{E}[N^V_{\tau-\theta,\tau}N^{Z^j}_{\tau,\tau+\theta}] \qquad (170) \\ & = \varphi_{\mathcal{B},\mathcal{B}}\eta_\mathcal{B}(\lambda)^2 e^{r\theta}\mathcal{I}^{(5)}_{\mathcal{B},\mathcal{B}} + \varphi_{\mathcal{B},\mathcal{L}}\eta_\mathcal{B}(\lambda)\eta_\mathcal{L}(\lambda)e^{r\theta}(\mathcal{I}^{(5)}_{\mathcal{B},\mathcal{L}} + \mathcal{I}^{(5)}_{\mathcal{L},\mathcal{B}}) \\ & \quad + \varphi_{\mathcal{L},\mathcal{L}}\eta_\mathcal{L}(\lambda)^2 e^{r\theta}\mathcal{I}^{(5)}_{\mathcal{L},\mathcal{L}} - w_\mathcal{B}\eta_\mathcal{B}(\lambda)(\eta_\mathcal{B}(\lambda) + \eta_\mathcal{L}(\lambda))\tau_V^{-1}e^{r\theta}\mathcal{I}^{(4)}_\mathcal{B} \\ & \quad - w_\mathcal{L}\eta_\mathcal{L}(\lambda)(\eta_\mathcal{B}(\lambda) + \eta_\mathcal{L}(\lambda))\tau_V^{-1}e^{r\theta}\mathcal{I}^{(4)}_\mathcal{L} \qquad (171) \\ & = (D_\mathcal{B}(\lambda)\mathcal{I}^{(5)}_{\mathcal{B},\mathcal{B}}(\theta) + D_\mathcal{L}(\lambda)\mathcal{I}^{(5)}_{\mathcal{L},\mathcal{L}}(\theta))e^{r\theta}, \qquad (172) \end{aligned}$$

having used the fact that $\mathbb{E}[N^{Z^{\mathcal{B}}}_{\tau-\theta,\tau}N^{V}_{\tau,\tau+\theta}] = \mathbb{E}[N^{Z^{\mathcal{L}}}_{\tau-\theta,\tau}N^{V}_{\tau,\tau+\theta}] = \mathbb{E}[N^{V}_{\tau-\theta,\tau}N^{V}_{\tau,\tau+\theta}] = 0$. Therefore,

$$\partial_\lambda \sigma_{1,2}(\lambda) = (D'_{\mathcal{B}}(\lambda)\mathcal{I}^{(5)}_{\mathcal{B},\mathcal{B}} + D'_{\mathcal{L}}(\lambda)\mathcal{I}^{(5)}_{\mathcal{L},\mathcal{L}})e^{r\theta}. \tag{173}$$

Finally,

$$\rho(\lambda,\theta;w_{\mathcal{B}},w_{\mathcal{L}}) = \frac{\sigma_{1,2}(\lambda,\theta;w_{\mathcal{B}},w_{\mathcal{L}})}{\sigma_1(\lambda,\theta;w_{\mathcal{B}},w_{\mathcal{L}})\sigma_2(\lambda,w_{\mathcal{B}},w_{\mathcal{L}})} = \frac{\sigma_{1,2}(\lambda,\theta;w_{\mathcal{B}},w_{\mathcal{L}})}{\sigma_1^2(\lambda,\theta;w_{\mathcal{B}},w_{\mathcal{L}})}. \tag{174}$$

Now consider the case in which $w_{\mathcal{B}} = w_0$. Then $D_{\mathcal{B}}(\lambda;w_0,w_{\mathcal{L}}) = D'_{\mathcal{B}}(\lambda;w_0,w_{\mathcal{L}}) = 0$ and hence

$$\sigma_{1,2}(\lambda,\theta;w_0,w_{\mathcal{L}}) = D_{\mathcal{L}}(\lambda;w_0,w_{\mathcal{L}})\mathcal{I}^{(5)}_{\mathcal{L},\mathcal{L}}(\theta;w_0,w_{\mathcal{L}})e^{r\theta} \tag{175}$$

$$\partial_\lambda\sigma_{1,2}(\lambda,\theta;w_0,w_{\mathcal{L}}) = D'_{\mathcal{L}}(\lambda;w_0,w_{\mathcal{L}})\mathcal{I}^{(5)}_{\mathcal{L},\mathcal{L}}(\theta;w_0,w_{\mathcal{L}})e^{r\theta}. \tag{176}$$

We now wish to show that if $w_{\mathcal{L}} < w_0$ as assumed, then $\sigma_{1,2}(\lambda;w_0,w_{\mathcal{L}}) > 0$. Note first that since $w_{\mathcal{L}} < w_0$, $D_{\mathcal{L}}(\lambda;w_0,w_{\mathcal{L}}) > 0$. Next,

$$\mathcal{I}^{(5)}_{\mathcal{L},\mathcal{L}} = \frac{2(\cosh(r\theta) - \cosh(\eta_{\mathcal{L}}\theta))e^{-\eta_{\mathcal{L}}\theta}}{(\eta_{\mathcal{L}}+\eta_{\mathcal{L}})(r+\eta_{\mathcal{L}})(r-\eta_{\mathcal{L}})} > 0, \tag{177}$$

which follows from the fact that $(\cosh(r\theta) - \cosh(\eta_{\mathcal{L}}\theta))/(r\theta - \eta_{\mathcal{L}}\theta) > 0$. We conclude that $\sigma_{1,2}(\lambda;w_0,w_{\mathcal{L}}) > 0$. We have that $D_{\mathcal{L}}(\lambda) = \eta_{\mathcal{L}}(\lambda)\widetilde{D}_{\mathcal{L}}(\lambda)\psi(w_{\mathcal{L}})/(2\kappa + w_{\mathcal{B}} + w_{\mathcal{L}})$, where

$$\widetilde{D}_{\mathcal{L}}(\lambda) = w_{\mathcal{L}}\eta_{\mathcal{B}}(\lambda) + w_{\mathcal{B}}\eta_{\mathcal{L}}(\lambda) + (2\kappa + w_{\mathcal{L}})(\eta_{\mathcal{L}}(\lambda) + \eta_{\mathcal{B}}(\lambda)) \tag{178}$$

$$= (2\kappa + w_0 + w_{\mathcal{L}})\eta_{\mathcal{B}}(\lambda) + 2(\kappa + w_{\mathcal{L}})\eta_{\mathcal{L}}(\lambda). \tag{179}$$

Let

$$m_0 = 2(2\kappa + w_0 + w_{\mathcal{L}})(r + \kappa + w_{\mathcal{L}}) \tag{180}$$

$$m_1 = 2(\kappa + w_{\mathcal{L}})(r + \kappa + w_0)(r + \kappa + w_{\mathcal{L}}) \tag{181}$$

so that

$$(\eta_{\mathcal{L}}\widetilde{D}_{\mathcal{L}})'(\lambda) = m_0 c_{\mathcal{L}}(\lambda)c'_{\mathcal{L}}(\lambda) + m_1(c_{\mathcal{B}}(\lambda)c'_{\mathcal{L}}(\lambda) + c'_{\mathcal{B}}(\lambda)c_{\mathcal{L}}(\lambda)) \tag{182}$$

$$= (m_0 c_{\mathcal{L}}(\lambda) + m_1(c_{\mathcal{B}}(\lambda) - c_{\mathcal{L}}(\lambda)))c'_{\mathcal{L}}(\lambda) \tag{183}$$

$$= (m_1 c_{\mathcal{B}}(\lambda) + (m_0 - m_1)c_{\mathcal{L}}(\lambda))c'_{\mathcal{L}}(\lambda) \tag{184}$$

$$> 0, \tag{185}$$

which follows from the fact that $m_0 - m_1 = 2(r(\kappa + w_0) + (\kappa + w_{\mathcal{L}})^2)$. We conclude that $\partial_\lambda \sigma_{1,2}(\lambda; w_0, w_{\mathcal{L}}) > 0$. Finally, we consider

$$\begin{aligned}
\partial_\lambda \sigma_1^2(\lambda, \theta; w_0, w_{\mathcal{L}}) &= \psi(w_{\mathcal{L}})\beta_{\mathcal{L},\mathcal{L}}(\lambda)\mathcal{I}^{(3)}_{\mathcal{L},\mathcal{L}}(\theta) + \tau_V^{-1}(\beta_{\mathcal{B},\mathcal{B}}(\lambda) + 2\beta_{\mathcal{B},\mathcal{L}}(\lambda) + \beta_{\mathcal{L},\mathcal{L}}(\lambda))\mathcal{I}^{(1)}(\theta) \\
&\quad + 2\tau_V^{-1}(w_0 - w_{\mathcal{L}})\beta_{\mathcal{B},\mathcal{L}}(\lambda)\mathcal{I}^{(2)}_{\mathcal{L}}(\theta) && (186) \\
&= (\psi(w_{\mathcal{L}})\beta_{\mathcal{L},\mathcal{L}}(\lambda) + 2(\kappa + w_{\mathcal{L}})\tau_V^{-1}(w_0 - w_{\mathcal{L}})\beta_{\mathcal{B},\mathcal{L}}(\lambda))\mathcal{I}^{(3)}_{\mathcal{L},\mathcal{L}}(\theta) \\
&\quad + \tau_V^{-1}(\beta_{\mathcal{B},\mathcal{B}}(\lambda) + 2\beta_{\mathcal{B},\mathcal{L}}(\lambda) + \beta_{\mathcal{L},\mathcal{L}}(\lambda))\mathcal{I}^{(1)}(\theta). && (187)
\end{aligned}$$

Note that

$$\begin{aligned}
&\beta_{\mathcal{L},\mathcal{L}}(\lambda) + 2\beta_{\mathcal{B},\mathcal{L}}(\lambda) + \beta_{\mathcal{B},\mathcal{B}}(\lambda) \\
&\qquad = 2(\eta_{\mathcal{L}}(\lambda) + \eta_{\mathcal{B}}(\lambda))(\eta'_{\mathcal{L}}(\lambda) + \eta'_{\mathcal{B}}(\lambda)) && (188) \\
&\qquad = 2(\eta_{\mathcal{L}}(\lambda) + \eta_{\mathcal{B}}(\lambda))((r + \kappa + w_{\mathcal{L}})c'_{\mathcal{L}}(\lambda) + (r + \kappa + w_0)c'_{\mathcal{B}}(\lambda)) && (189) \\
&\qquad = 2(\eta_{\mathcal{L}}(\lambda) + \eta_{\mathcal{B}}(\lambda))((r + \kappa + w_{\mathcal{L}}) - (r + \kappa + w_0))c'_{\mathcal{L}}(\lambda) && (190) \\
&\qquad = 2(\eta_{\mathcal{L}}(\lambda) + \eta_{\mathcal{B}}(\lambda))(w_{\mathcal{L}} - w_0)c'_{\mathcal{L}}(\lambda) && (191) \\
&\qquad < 0. && (192)
\end{aligned}$$

Since $\mathcal{I}^{(1)}(0) = \mathcal{I}^{(3)}_{\mathcal{L},\mathcal{L}}(0) = \mathcal{I}^{(3)\prime}_{\mathcal{L},\mathcal{L}}(0) = 0$ and $\mathcal{I}^{(1)\prime}(0) = 1$, we have $\partial_\lambda \sigma_1^2(\lambda, 0; w_0, w_{\mathcal{L}}) = 0$ and

$$\partial_{\theta\lambda} \sigma_1^2(\lambda, 0; w_0, w_{\mathcal{L}}) = \beta_{\mathcal{L},\mathcal{L}}(\lambda) + 2\beta_{\mathcal{B},\mathcal{L}}(\lambda) + \beta_{\mathcal{B},\mathcal{B}}(\lambda) < 0. \qquad (193)$$

Therefore, $\partial_\lambda \sigma_1^2$ is strictly less than zero in a neighborhood of $\theta = 0$. ■

Internet Appendix

Table IA1: Appraisal, Price, and Default Risk: Repossession Costs

This table presents results from our tests of Corollary 2, which states that the difference between the lender's effective estimate of collateral value (i.e., estimate of collateral value less repossession costs) and equilibrium price is negatively associated with borrower riskiness. We assume repossession costs are 10% of the appraised value. In our ordinary least squares (OLS) regressions, the dependent variable is *log(0.90×A-P)* in Panel A and *log(P-0.90×A)* in Panel B. In our Poisson regressions, the dependent variable is *0.90×A-P* in Panel A and *P-0.90×A* in Panel B. *A* is *Appraised Value* (i.e., lender's value), and *P* is *Sale Price* (i.e., price). Our proxies for default risk are *Home Equity*, *LTV*, and *FICO*. We take the negative of *Home Equity* and *FICO* so that the sign of the predicted coefficient is the same across all variables. *PC1* is the first principal component of the three proxies. See Table 1 for variable definitions. All regression variables are winsorized at the 0.5% and 99.5% levels. *LTV*, *FICO*, and *PC1* are standardized to zero mean and unit standard deviation. Standard errors below coefficient estimates are adjusted for clustering at the zip code, year, and month levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

Panel A: Sale price at most as large as 90% of appraised value

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Home equity	-0.125***				-0.129***			
	(0.006)				(0.010)			
LTV		-0.143***				-0.135***		
		(0.007)				(0.006)		
FICO			-0.070***				-0.061***	
			(0.009)				(0.011)	
PC1				-0.147***				-0.147***
				(0.010)				(0.012)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	31.9%	31.7%	27.5%	28.0%				
Pseudo R^2					67.5%	67.4%	65.1%	65.6%
Observations	418,699	418,699	266,132	266,132	431,213	431,213	274,489	274,489

Panel B: Sale price at least as large as 90% of appraised value

	OLS				Poisson			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Home equity	-0.180***				-0.268***			
	(0.012)				(0.019)			
LTV		-0.128***				-0.122***		
		(0.003)				(0.005)		
FICO			-0.062***				-0.064***	
			(0.004)				(0.005)	
PC1				-0.201***				-0.227***
				(0.005)				(0.004)
Zip×Year×Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	63.3%	57.0%	55.5%	59.0%				
Pseudo R^2					77.5%	69.0%	70.2%	73.7%
Observations	20,702,445	20,702,445	12,477,035	12,477,035	20,726,204	20,726,204	12,493,944	12,493,944

Table IA2: Return Autocorrelation: Simple Averages

This table presents results from additional tests of Proposition 8, which we test by regressing *Log return* on the one-year lag of itself interacted with dummies for our different proxies for default risk. *Log return* is the annual change in log value of the non-seasonally adjusted FHFA HPI. Our proxies for default risk are *Home Equity*, *LTV*, and *Credit Score*. *Home Equity* is the simple average of quarterly home equity values (i.e., natural logarithm of home equity estimated from LTV and the loan amount at the time of loan origination) within a CBSA. *LTV* is the simple average of quarterly loan-to-value ratios within a CBSA. *Credit Score* is the simple average of quarterly credit scores within a CBSA. *PC1* is the first principal component of the three proxies. *Low*, *Mid*, and *High* are dummy variables indicating whether the proxy in that column is in the lowest, middle, or highest tercile within a given CBSA. Standard errors below coefficient estimates are adjusted for clustering at the CBSA, year, and quarter levels. Statistical significance at the 1%, 5%, and 10% levels is denoted by ***, **, and *.

	No Sort (1)	Home Equity (2)	LTV (3)	Credit Score (4)	PC1 (5)
ρ	0.68***				
	(0.08)				
$\rho \times$ Low		0.73***	0.54***	0.78***	0.91***
		(0.11)	(0.08)	(0.12)	(0.12)
$\rho \times$ Mid		0.73***	0.73***	0.65***	0.78***
		(0.09)	(0.09)	(0.07)	(0.10)
$\rho \times$ High		0.60**	0.98**	0.58**	0.51***
		(0.12)	(0.17)	(0.17)	(0.08)
Adjusted R^2	41.4%	41.8%	44.4%	42.1%	44.4%
Observations	8,772	8,772	8,772	8,772	8,772