FTG Working Paper Series

Smooth versus Harsh Regulatory Interventions and Policy Equivalence

by

Linda Schilling

Working Paper No. 00123-00

Finance Theory Group

www.financetheory.com

Smooth versus Harsh Regulatory Interventions and Policy Equivalence

Linda M. Schilling*
Olin Business School WUSTL, CEPR

May 28, 2023

Abstract

Policy makers have developed different forms of policy intervention for stopping, or preventing runs on financial firms. This paper provides a general framework to characterize the types of policy intervention that indeed lower the run-propensity of investors versus those that cause adverse investor behavior, which increases the run-propensity. I employ a general global game to analyze and compare a large set of regulatory policies. I show that common policies such as bailouts, Emergency Liquidity Assistance, and withdrawal fees either exhibit features that lower firm stability ex ante, or have offsetting features rendering the policy ineffective.

Key words: financial regulation, bank runs, global games, policy effectiveness, bank resolution, withdrawal fees, emergency liquidity assistance, lender of last resort policies, money market mutual fund gates, suspension of convertibility
JEL Classification: G28,G21,G33, G38, D82, D81, E61

---

*Olin School of Business at Washington University in St Louis, lindas@wustl.edu. 1 Snow Way Dr, St. Louis, MO 63130. I thank Nicolae Garleanu, Andreas Neuhierl, and seminar participants at Crei Barcelona, and Bocconi University for insightful comments.

1 Introduction

L'enfer est plein de bonnes volontés ou désirs
[*The road to hell is paved with good intentions*]
- Bernard of Clairvaux (1090 – 1153)

The prevention of runs on financial institutions such as banks, money market mutual funds, and, more recently, stablecoins and central bank digital currency (CBDC) concerns a vast academic literature[1] and policy institutions today (?). This paper contributes to a critical debate on financial regulation aimed at reducing a firm's run-propensity and its unintended consequences. The paper develops a flexible framework for analyzing the effectiveness of a large class of financial policy interventions at preventing runs on firms. Because the framework is general, I can identify features of regulation, and ultimately classify common policy regulation according to types that improve versus reduce firm stability.

The paper makes three contributions. The main contribution stems from characterizing policy interventions that improve versus deteriorate firm stability based on how the policy acts on the investors' withdrawal-contingent payoffs to roll over versus withdraw their funds. Policy-driven changes in the relative payoffs to roll-over versus withdrawal alter the investors' ex ante run-propensity, and thus the firm's proneness to runs. I determine two large classes of policy, "smooth" and "harsh." Both smooth and harsh policies can be of two types, "adverse" and "prudent." I show, the range of policy interventions that worsen stability ex ante is large and can be of two different types: "adverse smooth" and "adverse harsh", to be explained below. Among regulation that possibly worsens stability are bailouts and emergency liquidity assistance because both interventions have the potential to benefit the "wrong" investor group, that is, those that decide to withdraw.

To classify policy and determine how policy impacts firm stability, I consider the relative investor payoffs to roll over versus withdraw as a function of aggregate withdrawals, where high aggregate withdrawals implicate a run on the firm. Absent regulation and intervention (the status quo), the payoff difference (PI) to roll-over versus withdrawal is generally a continuous function of the aggregate withdrawals, see ?. I define "smooth policy intervention" as a regulation that acts on the PI by shifting relative incentives gradually while preserving the continuity of the PI in the withdrawals. In contrast, "harsh policy" *causes* discontinuities (jumps) in the PI at certain withdrawal thresholds, and possibly shifts these jump points as policy intensity picks up. For intuition on the difference between smooth and harsh intervention, any policy intervention needs to start and finish at some aggregate withdrawal threshold. These thresholds have an interpretation as withdrawal-contingent entry and exit points to intervention. A smooth policy may set or shift these entry and exit points to intervention but only in a way that preserves the

[1] See ??????????????????????

continuity of the PI in the withdrawals. That is, entry and exit to intervention shall not be too abrupt with regard to its impact on investor payoffs, otherwise a discontinuity in the PI arises, and policy intervention becomes harsh.

A smooth policy is not always beneficial to firm stability, and a harsh policy is not always detrimental. There are two types of policy intervention that monotonically improve firm stability ex ante, which I call "prudent (piecewise) smooth" and "prudent harsh," see Proposition 4.1 and Proposition 5.2. Prudent smooth policy strictly raises the PI to roll over versus withdraw over an interval of aggregate withdrawals (intervention interval) and nowhere lowers the PI. Prudent harsh policy is a policy that causes at least one upwards jump and no downwards jump of the PI, meaning that there exists a withdrawal threshold at which the policy intervention increases the favorability of roll-over versus withdraw in an ad-hoc way. A change in harsh policy can occur in two forms: Either in the form of a "piecewise smooth policy" that shifts payoffs gradually between jump points without causing additional jumps, and acts prudently if the PI is shifted upwards. Alternatively, a change in harsh policy can occur due to a shift in the jump-point. "Jump-shifts" act "prudently harsh", raising stability ex ante, only if the policy shifts a down-jump point of the PI up to a higher withdrawal level or an up-jump point down to a lower withdrawal level. An adverse policy that monotonically lowers firm stability ex ante can equally be of two types, either "adverse (piecewise) smooth," shifting the PI down, or "adverse harsh" by causing a down-jump in the PI, shifting a down-jump down to a lower or an up-jump upwards to a higher withdrawal level. These abstract concepts are brought to life in the application section 7 where I assess existing policy methods.

As the second contribution, the generality of the framework allows me to study the equivalence of policies, and provide conditions under which different policy types offset each other, see section 6. This equivalence analysis is important because many common policies belong to multiple classes, for instance, exhibiting adverse harsh and prudent piecewise smooth features such as Emergency Liquidity Assistance. Within the class of smooth policies the paper points out that bail-ins of investors that roll over act like bail-outs of withdrawing investors and vice versa, and both bail-ins and bail-outs can either increase or lower firm stability depending on the investor group they benefit. Therefore, generically a bailout does not improve firm stability, moreover, it lowers firm stability when paid to the withdrawing agent group. As an application of my equivalence result in subsection 7.1, I demonstrate that imposing and raising a fee on withdrawals is not an effective policy because it gives rise to adverse and prudent effects on firm stability that offset each other. There, I also show that lowering the entry threshold to the withdrawal fee is more effective, because it avoids these offsetting effects. Likewise, the provision of Emergency Liquidity Loans to a bank during a fire sale can in fact lower instead of raise bank stability ex ante because the loan constitutes a transfer from the roll-over to the withdrawing agent group, acting like a bail-in to roll-over agents and a bail-out of withdrawing agents, as discussed in section 7.2. As the second type of equivalence, I show that policy that acts harshly (causes or shifts jumps) can undo smooth policy and vice versa. As an application of this equivalence type, section 7.2 shows that providing and raising Emergency Liquidity Loans to a bank can lower instead of raise bank stability ex ante because the ELA provision causes a jump in the payoff difference function if the lender of last resort charges interest on the loan. Likewise, section 7.3 discusses that the suspension of convertibility of deposits can lower stability because it creates adverse jumps that offset the stabilizing effect of the intervention. The stability analysis of imposing and raising withdrawal fees or granting and raising an ELA loan at a varying entry threshold are contributions of their own in section 7.1 and 7.2.

Last, this paper makes a technical contribution by extending the jump-free **?** model to general payoff functions with finitely many jump points.

This framework is widely applicable. The firm I consider can be any institution that is exposed to its investors' decision whether to roll over funds or withdraw. Therefore, the firm can be a bank, a money market mutual fund (MMF), a central bank, a stable-coin, or a start-up that requires the roll-over of seed money. Funds can be short-term debt, long-term debt, commercial papers, seed money of start-ups, cryptocurrency and stablecoins, CBDC, or money market mututal fund shares. The framework solely requires that the payoffs to roll over versus withdraw be denominated in the real unit of account (consumption units). Therefore, payoffs need to be pinned down after an adjustment for inflation or an exchange rate.

The framework is general in that the types of regulation and contracts that are studied here solely need a description of the ex post payoffs to investors after the contract, asset returns, and regulation have been applied. More specifically, for the classification of regulation into classes that do improve versus those that lower firm stability, it is sufficient to observe how regulation acts on the investors' payoffs to roll over versus withraw funds, depending on the aggregate withdrawals of all firm investors. The types of regulation and policy interventions that are included in this framework are, though not limited to, bail-ins, bail-outs, emergency liquidity assistance (ELA) by a lender of last resort, suspension of convertibility of deposits or gates or withdrawal fees for money market mutual fund redemptions, and deposit insurance (guarantees), all possibly in a withdrawal contingent way.

The framework is specific in that it imposes sufficient structure on the payoffs to guarantee the selection of a unique equilibrium of the investor's coordination game in a global games framework (**??**), and thus a unique, model-implied ex ante run probability on the firm. For this purpose, I generalize the **?** framework to a setting that considers general withdrawal-contingent payoffs to investors, and allows for jumps in the payoff differences.

1.1 Literature

The paper contributes to three strands of literature, namely the literature on runs on financial firms, the literature on global games, and the literature on financial regulation to improve the resilience of the financial sector. The closest related papers are the bank run global game model in ?, the run model with a lender of last resort application in ?, the firm-regulator interaction with subsidies and runs in ?, and the book chapter on global games in ?.

This paper adds to the literature on bank and money market mutual fund runs and their prevention. In ?, a sufficiently conservative suspension policy deters runs completely. ? study the prevention of panic runs via suspension policies when depositors have asymmetric information. ? study bank runs with and without lender of last resort policies. ? consider ex-post optimal intervention delay when a run happens. ? study dynamic rumor-based bank runs with endogenous information acquisition. ? study the prevention of runs by allowing agents to report that a run is happening. ? studies mutual fund runs in a dynamic model but does not consider intervention or run prevention. ?? studies the impact of suspension of convertibility policies on bank stability. ? study the impact of bankruptcy code design on run incentives in a dynamic setting. Unlike all these papers, this paper analyzes a very general framework that allows for a wide range of policy interventions and contracts.

Unlike the majority of the mentioned papers, this paper employs a global games information environment (????????) for attaining a unique equilibrium which enables me to conduct unique comparative statics in the ex ante run likelihood under policy changes. In the context of runs on firms, global games have been employed as an equilibrium selection device by ???????????. This paper deviates from the existing global games literature by analyzing a general global games environment into which I build different types of regulation that impact firm stability. In doing so I build on the general structure in ? to generalize the payoff functions of the classic run model by ?. I then define types of policy by how they act on the payoff difference function under the constraint of maintaining the global games equilibrium selection. In doing so I explicitly allow for regulation that causes jumps in the payoff difference function while maintaining action single-crossing and (one-sided) strategic complementarity. In the global game by ?, a regulator can set transfers to investors to implement the efficient equilibrium whereas the firm can shirk the transfer by altering the contract with its investors. This paper differs from ? by focussing on different types of transfers to and across the coordinating investors. I show, depending on whether transfers are continuity-preserving or discontinuity-causing, positive or negative, they impact stability differently, either improving or deteriorating stability. I show that different types of transfers can, nevertheless, have equivalent effects on stability and I show that commonly applied intervention methods such as emergency loans, and the imposition of withdrawal fees are policies that exhibit mixed features, some improving and some lowering stability ex ante. **?** explicitly allows for moral hazard whereas I abstract from that. Similar to **?**, this paper studies how a firm's proneness to runs changes with policy. In **?**, however, the policy maker observes a payoff-relevant state realization which is not observed by the coordinating investors. Therefore, the policy conveys additional information which gives rise to equilibrium multiplicity. Here, in contrast, the policy does not serve as a signal, and a unique equilibrium attains. **?** and **?** consider the regulation of intermediary balance sheets to impact insolvency and illiquidity risk. I study regulation in a broader sense where I do not pin down balance sheets, contracts and regulation explicitly but rather consider very general payoffs to investors ex post of asset returns, contracts, seniority and regulation. This allows me to nest many common bank run models and regulation, and characterize stability improving regulation on a more abstract level without pinning down the regulation and contracts in detail.

With regard to the literature on unintended consequences of financial regulation, in a setting of self-fulfilling runs, **?** shows that if financial intermediaries expect bailouts in times of crises, the anticipation of bailouts causes intermediaries to choose illiquid and fragile asset positions. In the context of sovereign debt crises, **?** show that the prevention of sovereign default via bailouts in the short run may come at the cost of a higher default probability in the long run. **?** show that private leverage choices of banks become strategic complements if the policy response during crises is imperfectly targeted. This model features a simultaneous-move game, as in **?** and **?**. The withdrawal-contingent intervention policies considered here, however, resemble the literature on random and sequential withdrawals where each arriving depositor obtains a distinct allocation (**????**).

2 Model

I first introduce the model, and then discuss its assumptions in section 2.1.

There are three time periods, $t = 0, 1, 2$, and no explicit discounting. Implicitly, a discount factor can be accommodated via the payoffs to investors, as described below. There exists a firm, a regulator and a continuum of investors $i \in [0, 1]$. All of them are risk-neutral. The firm can be a bank, a money market mutual fund, a stablecoin, a central bank issuing CBDC, or a start-up that requires investors and the roll-over of funding seeds. Likewise one can think of the investors as depositors, investors in a money market mututal fund or general investors who at a future point in time need to decide whether to roll-over funds or withdraw. The regulator can represent the FDIC, the government or the lender of last resort (central bank). There is a single good in the economy that agents value for consumption. All payoffs are denominated in terms of that good.

At time zero, the investors are symmetric, and each is endowed with one unit to invest. All investors enjoy consumption at both $t = 1$ and $t = 2$. The firm requires funding for investment, and for that purpose collects endowments from the investors in $t = 0$. I assume that investing is individually rational to investors. Returns to scale are constant. The initial firm investment and thus funding via investors is normalized to one unit.

I do not model the firm and the regulator separately but rather think of them as one entity that jointly provides payoffs to investors. Therefore, in the benchmark model I do not model the firm's investment payoff structure, the contracts between investors and the firm, and the regulator's subsidy explicitly. Rather, I pin down investor payoffs conditional on the choice of action, *ex post* of firm revenue, contract payments and regulatory intervention. On an abstract level, I can collapse payoffs because, as I will outline below in the analysis, optimal investor behavior does not depend on the origin of payoffs rather than joint payoffs provided by the firm and the regulator conditional on an action. This stark abstraction has pros and cons. On the positive side, it allows me to analyze a very general policy framework that nests many common intervention methods, contracts, and asset payoff structures. But the collapse of firm-regulator payoffs requires me to abstract from moral hazard from the side of the firm towards its investors or between the firm and the regulator. The firm-regulator entity has aligned incentives to maximize firm stability, to be defined below. This set-up does nest a model where the firm and the regulator are modeled separately, as long as the firm faces no moral hazard problem towards the regulator or its investors, see the application section 7 for examples. For a nice example where the firm can shirk a regulatory intervention, see **?**.

I follow and outline the information structure in **?** but generalize firm (bank) and investor payoffs.

State Let $\theta \sim U[0, 1]$ denote the unobservable, random state of the economy. Generalizing **?**, as stated above, I do not impose a particular state-dependent firm asset payoff structure. Yet, I assume that the state realization is payoff relevant to investors. One may think of θ as parametrizing the payoff probability of a risky firm asset or a random asset return.

Contract and payoffs In $t = 0$, the firm offers a contract to the investors to raise funds for investment in the risky asset. All investors invest their endowment in the contract with the firm. At $t = 1$, an investor needs to decide on her *action*. She either "withdraws" her investment and thus opts for the short-term payoff $u_1(n, \theta)$ payable in $t = 1$, or she "rolls over" her investment until $t = 2$, opting for the payoff $u_2(n, \theta)$ payable in $t = 2$ where $n \in [0, 1]$ denotes the endogenous share of investors who withdraw in

$t = 1$ (aggregate withdrawals). One should think about the payoffs $u_1(n, \theta)$ and $u_2(n, \theta)$ not only as functions of firm asset payoffs, the contract and withdrawals but also ex post of firm profits and regulatory intervention, that is, the payment of bail-outs, bailins, suspension or withdrawal fees. The payoffs u_1 and u_2 are denominated in real terms. Therefore, if the firm is a stablecoin or a CBDC-issuing central bank, then u_1 and u_2 are ex post of a correction for the exchange rate and the price level (inflation). The payoff u_2 can be thought of incorporating a discount factor. The reason for why this generality is possible is because the investors' only care for final per period consumption and due to rational expectations. For roll-over incentives, only final real payoffs matter. The firm and the regulator jointly have deep pockets so that payoffs $u_1(n, \theta)$ and $u_2(n, \theta)$ at a given state θ and aggregate withdrawal level n are feasible, and this is common knowledge among all investors. Observe that the payoffs are not necessarily hard claims but can be state- and withdrawal-contingent. Therefore, the contract I am considering here is not necessarily a demand-deposit or debt contract. The payoffs satisfy monotonicity conditions in the state θ, and the aggregate withdrawals n, as summarized below in assumption 2.1. The functional forms of $u_1(n, \theta)$ and $u_2(n, \theta)$ are known to the depositors ex ante.

Signals Before the investors choose actions in $t = 1$, they observe noisy, private signals about the state θ,

$$\theta_i = \theta + \varepsilon_i. \tag{1}$$

The idiosyncratic noise term ε_i is independent of the state θ and is distributed iid according to the uniform distribution $\varepsilon_i \sim U[-\varepsilon, +\varepsilon]$.

Policy and Regulatory Intervention I assume that at $t = 0$, the regulator sets and commits to a policy parameter $p \in [0, \infty)$ where $p = 0$ corresponds to a committment to not interfere, or alternatively the absence of a regulatory institution. One can think of p as a policy intensity that is raised under policy intervention. The policy parameter is common knowledge among all investors. A change in p is supposed to act on the investors' payoffs u_1 and u_2 which is why, from now on, I subindex investor payoffs with p. For the first part of the paper, I study investor behavior for a general, given policy intensity $p \in [0, \infty)$, and then characterize different types of policy and policy changes by how they act on the investors' payoffs. Define the payoff difference of rolling over versus withdrawing as

$$v_p(n, \theta) = u_{2,p}(n, \theta) - u_{1,p}(n, \theta). \tag{2}$$

Note, that the aggregate withdrawals n and the state θ are random in $t = 0$. Following **?** section 2.2.2. and 2.2.3., I impose monotonicity conditions on the investor's relative payoffs that guarantee equilibrium existence and uniqueness. That is, this model tries to attain maximum generality with regard to the payoffs u_2 and u_1 and thus possible regulatory interventions but within the class of global games.

Assumption 2.1. *Fix policy intensity $p \in [0, \infty)$. It holds*
(1) (Strict state Monotonicity:) $\upsilon_p(n, \theta)$ is non-decreasing in θ, and strictly increasing in θ for all $\theta \in [\underline{\theta}_p, \overline{\theta}_p]$.
(2a) (Action single crossing:) For every state $\theta \in [\underline{\theta}_p, \overline{\theta}_p]$, there exists $n^*(p) \in (0, 1)$ such that $\upsilon_p(n, \theta) > 0$ for $n < n^*(p)$ and $\upsilon_p(n, \theta) < 0$ for $n > n^*(p)$.
(2b) (One-sided strategic complementarity:) For every state $\theta \in [\underline{\theta}_p, \overline{\theta}_p]$, whenever n is such that $\upsilon_p(n, \theta) > 0$, then $\upsilon_p(n, \theta)$ is strictly decreasing in n.
(3) (Uniform limit dominance:) There exist upper and lower regions of action dominance: There exist $\underline{\theta}_p, \overline{\theta}_p \in (0, 1)$ and $\epsilon > 0$ such that: if $\theta \in [0, \underline{\theta}_p]$, then withdraw is dominant, $\upsilon_p(n, \theta) < -\epsilon$, for all $n \in [0, 1]$ while for $\theta \in [\overline{\theta}_p, 1]$, roll-over is dominant $\upsilon_p(n, \theta) > \epsilon$, for all $n \in [0, 1]$.

Note that assumption 2.1 includes global strategic complementarity in actions. But the assumption imposes sufficiently strong additional structure to also guarantee equilibrium existence and uniqueness under one-sided strategic complementarity which is common in games of runs on financial institutions.

Timing

- In $t = 0$, the regulator sets and fully commits to her policy p without observing the state. The policy p is common knowledge among all agents, and the policy choice conveys no information. Then, the state θ realizes unobservably to all agents. All investors invest in the firm contract.

- In $t = 1$, all investors observe their private signal θ_i. Based on the signal and the policy, they decide whether to request withdrawal. The firm and the regulator jointly observe the aggregate withdrawal requests $n \in [0, 1]$, and depending on the policy p, allocate payoffs $u_1(n, \theta)$ to depositors who withdraw, where the state realization $\theta \in [0, 1]$ remains unobserved by all agents until $t = 2$.

- In $t = 2$, θ is revealed, and payoff $u_2(n, \theta)$ is paid to investors that chose roll-over.

The equilibrium concept is perfect Bayes Nash. Proofs that are not in the main text can be found in the appendix.

2.1 Discussion of model assumptions

Generically, I allow the payoff to withdrawal, $u_1(n, \theta)$, to depend on state θ since the payoff may be paid in $t = 2$ due to regulatory intervention even though the choice to withdraw was made in $t = 1$. One may consider here a mandatory deposit stay where a depositor chooses to withdraw but an intervention in $t = 1$ prevents her from doing so. If the payoff to withdraw is paid in $t = 1$, it cannot depend on θ since the state is revealed only later in $t = 2$. The payoff to roll over is paid in $t = 2$ and therefore can always depend on the state. I allow the payoffs to depend on aggregate withdrawals since in classic bank run models (**???**), regulatory intervention is triggered by high withdrawals, thus, altering the payoffs to all agents.

To gain intuition for Assumption 2.1, state monotonicity means that the action to "roll over" becomes relatively more favorable than withdraw for high state realizations.

One-sided strategic complementarity and single-crossing mean that, unless the state realizes in either of the dominance regions, for low withdrawals, roll over is optimal, but the optimality of roll over strictly declines in the withdrawals until a critical withdrawal level $n^*(p)$ is reached where the optimal response flips to "withdraw." For all higher withdrawals, withdraw is optimal, and the critical withdrawal level $n^*(p)$ is unique. To put these assumptions in context, in the bank run literature, at policy intensity p, the aggregate investor withdrawals determine whether a run occurs or not. The critical withdrawal level $n^*(p)$ is known as the critical withdrawal level at which the bank becomes illiquid, meaning for higher withdrawals $n \geq n^*(p)$ the bank is unable to fully serve depositors who roll over and withdrawal becomes optimal to depositors. To understand the single-crossing condition, note that policy impacts the relative favorability of roll-over versus withdrawal by altering the payoffs $u_{2,p}$ and $u_{1,p}$. Thus, changes in policy can or are supposed to cause changes in optimal behavior by investors. Alternatively, the threshold $n^*(p)$ can be understood as a regulatory intervention that occurs once withdrawals exceed $n^*(p)$, which may cause optimal investor behavior to switch at the intervention threshold, see section 5. In applications, the threshold $n^*(p)$ depends on the asset payoffs, budget constraints, the contracted investor's payoffs, the discount factor, and in case of nominal contracts, the price level or an exchange rate.[2]

The assumption on action single-crossing, introduces a coordination game among the investors.[3] The existence of dominance regions is important for the equilibrium selection

[2]In classic bank run appliations, for instance, a run occurs if aggregate cash withdrawals $nu_1(n,p)$ exceed a budget $B_1(p)$ available to early withdrawing agents. For attaining equilibrium uniqueness of the coordination game, a classic assumption yielding action single-crossing is that the product $nu_1(n,p)$ be strictly increasing in the aggregate withdrawals n. Therefore, at fixed policy p there exists a unique critical withdrawal level $\hat{n}(p)$ such that if and only if $n \geq \hat{n}(p)$ then $nu_1(n,p) \geq B_1(p)$. In that case, there exists a unique $n^*(p) \leq \hat{n}(p)$ for which $n^*(p)u_1(n^*(p),p) \leq B_1(p)$ and the payoff difference changes sign in $n^*(p)$.

[3]As one interpretation for $n^*(p)$ one can imagine depositors that finance a bank's investment in illiquid assets. The depositors have the possibility to withdraw from the bank at the interim stage if they believe that the asset quality θ will realize low. If the state θ realizes above the lower dominance region $\theta \in [\underline{\theta}, 1]$ and as long as the aggregate withdrawals are sufficiently low, $n < n^*$, the bank can finance all withdrawals by selling assets, and rolling over yields a higher payoff than withdraw. Therefore, $v_p(n,\theta) > 0$ and "roll over" is the best response to the aggregate action $n < n^*$. If however the withdrawals pick up, the bank needs to liquidate many illiquid assets, and the remaining investment is insufficient to pay a high payoff to depositors who roll over. That is, "withdraw" is the optimal response to high withdrawals $n > n^*(p)$, $v_p(n,\theta) < 0$.

argument. The subscript p clarifies that policy intensity impacts not only investor payoffs but can also determine the regions of states, $[0, \underline{\theta}_p]$ and $[\overline{\theta}_p, 1]$, for which investors have dominant actions.

Allover, assumption 2.1 is important to attain a unique coordination equilibrium and later, for maintaining equilibrium uniqueness under policy changes.

3 Status quo: Equilibrium Existence and Uniqueness

Any policy intervention is relative to a prevailing status quo. This benchmark status quo needs to be clearly defined so that I can compare equilibrium outcomes before and after a policy intervention or change in policy. A comparison of outcomes, in particular, requires that I start at a unique equilibrium and do not jump to multiple equilibria as policy changes.

Assumption 3.1. *Fix policy p. At p the payoff difference function $v_p(n, \theta)$ is continuous in $(n, \theta) \in [0, 1] \times [0, 1]$, and differentiable in $\theta \in (\underline{\theta}, \overline{\theta})$.*

Assumption 3.1 is important for equilibrium existence and uniqueness because it establishes continuity of the expected payoffs in the signal observed by investors. Because continuous functions on compact intervals are bounded, the assumption also implies that the payoff difference $v_p(n, \theta)$ is Lebesgue intergrable for all $(n, \theta) \in [0, 1] \times [0, 1]$. Depending on the policy type I introduce below, I may need to strengthen or complement assumption 3.1 for guaranteeing equilibrium existence and uniqueness.

One can imagine the status quo to be the case where a bank is on its own when faced with a run, that is, no regulator exists to intervene ($p = 0$). This status quo is for instance analyzed in **?**, and features a continuous payoff difference function, see also the applications section 7. Alternatively, an intervention mechanism may already be implemented, $p > 0$, in a way that the resulting payoff difference is continuous. I relax assumption 3.1 below in section 5 to allow for hash policy intervention that causes jumps, that is, discontinuous changes in the relative payoff to roll-over versus withdrawal. The following result is a version of **?** but for general payoffs.

Proposition 3.1 (Equilibrium Existence and Uniquness at status quo)
Fix policy intensity $p \in [0, \infty)$. Assume, the preferences of investors satisfy assumptions (2.1) and (3.1). As noise vanishes, $\varepsilon \to 0$, the investor's coordination game has a unique equilibrium, and the equilibrium is in trigger strategies. There exists a unique trigger signal $\theta^*(p)$ that makes an investor indifferent between rolling over the deposit or withdraw. For signals below the trigger $\theta_i < \theta^*(p)$ an investor optimally withdraws. For signals above the trigger $\theta_i > \theta^*(p)$, roll-over is optimal.

For tie-breaking reasons, I assume that an investor rolls over the investment whenever observing the equilibrium trigger, $\theta_i = \theta^*(p)$. Given an equilibrium trigger $\theta^*(p)$, the equilibrium withdrawals are described by a deterministic function of the state, $n(\theta, \theta^*)$, given in the appendix, equation (25). As is standard in global games theory, for a given policy parameter $p \in [0, \infty)$, the equilibrium trigger signal $\theta^*(p)$ is implicitly characterized as the zero to the expected payoff difference equation

$$H(p, \theta^*(p)) = \int_0^1 \upsilon_p(n, \theta(n, \theta^*(p)))\, dn \tag{3}$$

where $\theta(n, \theta^*(p))$ is the inverse of $n(\theta, \theta^*)$, that is, the state consistent with measure n withdrawals if all depositors play the equilibrium trigger strategy around θ^*,

$$\theta(n, \theta^*) = \theta^* + \varepsilon(1 - 2n), \theta^* \in [\underline{\theta} - \varepsilon,\ \overline{\theta} + \varepsilon] \tag{4}$$

By the single-crossing assumption, the optimality of roll-over versus withdraw switches if the aggregate withdrawals $n(\theta, \theta^*)$ exceed the critical withdrawal level $n^*(p)$. In the remaining part of the paper, I say that "a run on the firm occurs" if the withdrawals exceed the critical withdrawal level $n^*(p)$.[4] Given the trigger signal $\theta^*(p)$, a unique cut-off state $\theta_b(p) \in [\underline{\theta}, \overline{\theta}]$, the *critical state*, exists at which the aggregate withdrawals push the firm to the edge of a run:

$$n(\theta_b(p), \theta^*(p)) = n^*(p). \tag{5}$$

For a given trigger signal, state realizations below $\theta_b(p)$ imply lower signal realizations and thus higher aggregate withdrawals. If and only if $\theta < \theta_b(p)$, a run occurs because sufficiently many investors receive a signal below the trigger $\theta^*(p)$. Because the state is uniformly distributed, the ex-ante probability of a run equals θ_b. But as noise vanishes, $\varepsilon \to 0$, the equilibrium trigger $\theta^*(p)$ converges to the critical state $\theta_b(p)$. I therefore write:

Definition 3.1 (Bank stability)**.** *Bank stability increases in policy p if the ex-ante probability of a run $\theta_b(p)$ or equivalently*[5] *the equilibrium trigger $\theta^*(p)$ declines in p. In that case, policy acts "prudently."*

Generically, the regulator wants to design policy p in a way that reduces the ex ante run-likelihood. If an increase in policy intensity however increases the run likelihood, I call this phenomenon "preemptive investor behavior":

[4] In contrast, the bank run literature often defines a run as the incident where withdrawals reach the level at which the bank runs out of assets to liquidate, that is, as $u_2(n)$ hits zero. This however occurs at a withdrawal level $n > n^*$ where the optimal response has already switched to "withdraw".

[5] As noise vanishes, $\varepsilon \to 0$, the trigger and the critical state are undistinguishable and their derivatives coincide.

Definition 3.2 (Preemptive investor behavior)**.** *Investors preempt the regulator under policy p if an increase in policy intensity $p \geq 0$ impacts investor incentives adversely, lowering bank stability ex ante.*

The main focus of this paper is to determine what types of policy lower versus raise the probability of runs and thus firm stability. Note, generically, the objective to maximize stability is different from efficiency maximization.[6]

4 Smooth policy intervention

Having clarified that at a given status quo with policy intensity $p \in [0, \infty)$ a unique trigger equilibrium $\theta^*(p)$ exists, I now start the main analysis. Recall that at status quo p, the payoff difference $\upsilon_p(n, \theta)$ is continuous in the withdrawals under assumption 3.1. The following definition of policy is concerned with the maintenance of continuity as a regulatory intervention takes place (going from $p = 0$ to $p > 0$), or, as a policy intervention changes intensity (increasing p).

Definition 4.1 (Smooth policy intervention)**.** *Let $p \geq 0$. A regulator conducts "smooth policy intervention" via setting and increasing p if:*
(i) a marginal increase in policy p alters the payoffs ro roll-over or withdrawal in a way that preserves the continuity of the payoff difference function, $\upsilon_p(n, \theta)$, in the aggregate withdrawals $n \in [0, 1]$ for all $\theta \in [0, 1]$ and
(ii) the change in payoffs due to the marginal change in policy p preserves the properties of $\upsilon_p(n, \theta)$ stated in assumption 2.1 and 3.1.

The requirement that the policy intervention preserves the continuity of the payoff difference is crucial. It means the policy does not cause harsh changes in the investors' incentives at single withdrawal points.

The assumption that smooth policy intervention preserves the payoff properties stated in assumptions 2.1 and 3.1 is important for maintaining equilibrium uniqueness. It means, generically, a policy intervention must be carefully designed. Consider, for instance, increasing the relative favorability to "roll over" versus "withdraw" via the provision of a bailout. If roll over becomes as favorable as withdrawal for low but also for high aggregate withdrawals, equilibrium uniqueness is lost because the payoff difference function is no longer strictly decreasing in the withdrawals when positive, or lacks the single-crossing

[6] See for instance (**?**) where the provision of high deposit insurance can lead to inefficient losses to the deposit insurance fund because the depositors roll over their deposits for bad signals. For analyzing efficiency, one would need to explicitly model the asset's state-contingent payoffs and liquidation values which would impose additional structure on the investor payoffs and the economy. I prefer to keep the payoffs more general for now. An efficiency analysis can be reintroduced once the policy is explicit such as in the application section 7.

property. In that case, the global games equilibrium selection approach is no longer applicable, and the impact of policy on firm stability is undetermined.

Definition 4.2 (Prudent smooth policy intervention)**.** *A regulator conducts "prudent smooth policy intervention" via setting p if the policy intervention is smooth, and the policy intervention changes the relative payoffs to investors for aggregate withdrawals n in a non-empty, open interval $\mathcal{N}(p) \subseteq [0,1]$, called "intervention interval," such that:*
(i) $\frac{\partial}{\partial p}\upsilon_p(n,\theta) \geq 0$, for all withdrawals $n \in [0,1]$ and
(ii) $\frac{\partial}{\partial p}\upsilon_p(n,\theta) > 0$ for withdrawals $n \in \mathcal{N}(p)$.

I allow the intervention interval $\mathcal{N}(p)$ to depend on p, meaning that a change in policy p can widen the intervention interval, see for instance sections 7.1 and 7.2 where a change in policy lowers the entry threshold to imposing a withdrawal fee respectively an emergency liquidity loan. The intervention interval cannot depend on the state since otherwise the regulator's announcement of the interval in $t = 0$ would convey information on the state, which would give rise to equilibrium multiplicity, see (**?**). Technically, and included in the definition to prudent smooth intervention, a policy can act on two disjoint and disconnected intervals $\mathcal{N}_1(p)$ and $\mathcal{N}_2(p)$ simultaneously, meaning that the intervention interval becomes an intervention set $\mathcal{N}(p) = \mathcal{N}_1(p) \cup \mathcal{N}_2(p)$. Important for the definition of prudent smooth intervention is that there exists no subinterval of $[0,1]$, on which that same policy acts adverselely via $\frac{\partial}{\partial p}\upsilon_p(n) < 0$. Likewise, $\mathcal{N}(p)$ cannot be a point threshold since this creates discontinuity, see section 5 on harsh intervention.

A prudent smooth policy intervention marginally raises the relative favorability of "roll-over" versus "withdraw" by gradually shifting the according payoffs over the interval of withdrawals $\mathcal{N}(p)$ in a way that preserves the continuity of the payoff difference function $\upsilon_p(n)$. A smooth policy change can be attained by either increasing the payoffs to roll-over (bailout to investors who roll over), u_2, or equivalently by reducing the payoffs to withdraw (bail-in of investors that withdraw, suspension or withdrawal fee), u_1. This equivalence demonstrates the power of this approach using general payoffs: To the investors, only relative payoffs matter. Irrespective of whether policy marginally raises u_2 or lowers u_1, the investors react in the same way so that the impact on firm stability will be equivalent, see section 6 on policy equivalence.

Definition 4.3 (Adverse smooth policy intervention)**.** *A regulator conducts "adverse smooth policy intervention" via an increase in p if the policy intervention is smooth, and the open intervention interval $\mathcal{N}(p) \subset [0,1]$ satisfies:*
(i) $\frac{\partial}{\partial p}\upsilon_p(n,\theta) \leq 0$, for all withdrawals $n \in [0,1]$ and
(ii) $\frac{\partial}{\partial p}\upsilon_p(n,\theta) < 0$ for withdrawals $n \in \mathcal{N}(p)$.

While prudent smooth policy intervention gradually raises the favorability of roll-over versus withdrawal over the interval $\mathcal{N}(p)$, adverse smooth policy, perhaps by mistake, does the opposite. The adverse smooth policy increases u_1 via, for instance, a lender of last resort emergency liquidity provision, see section 7.2 or equivalently, by lowering u_2 via a bail-in of investors that roll over.

Obviously, there can exist policy interventions that are mixtures between prudent and adverse smooth policy in the sense that there exist intervals $\mathcal{N}_p(p)$ and $\mathcal{N}_a(p)$ such that the payoff difference of roll-over versus withdrawal, $v_p(n,\theta)$, strictly increases in p on $\mathcal{N}_p(p)$ but strictly declines on $\mathcal{N}_a(p)$. These cases are not clear-cut, and require a more thorough analysis, see for instance the application section 7.

The reader might wonder why adverse smooth policy is discussed here at all, given that is is simply the reversal of a prudent smooth policy. As it turns out, though, several applied policies have unintended consequences, as demonstrated in section 7, and it is important to have a label for the origin of these.

Both prudent and adverse smooth policies change relative payoffs conditional on the realization of withdrawals. Ex ante, because withdrawals are random, prudent and adverse smooth policy alter the expected value to “roll over” versus withdraw. As a consequence, the equilibrium trigger signal $\theta^*(p)$ needs to adjust. The change in the trigger, in return, alters ex ante firm stability.

As my first main result, the next result states how prudent and adverse smooth policy intervention impact the investor's incentive to run on the firm and thus firm stability.

Proposition 4.1 (Firm stability under Prudent and Adverse Smooth Policy)
Assume that in status quo p, assumptions 2.1 and 3.1 hold.
(i) Prudent smooth policy intervention according to Defintion 4.1 strictly improves ex ante firm stability: The trigger $\theta^*(p)$ and the critical state $\theta_b(p)$ strictly decline in p.
(ii) Adverse smooth policy intervention according to Definition 4.3 strictly lowers ex ante firm stability: the trigger $\theta^*(p)$ and the critical state $\theta_b(p)$ strictly increase in p.

The result says, if policy intervention raises the favorability of roll-over versus withdrawal without causing harsh changes in incentives (jumps), then policy improves bank stability monotonically, and preemptive behavior by investors does not arise. Adverse smooth policy intervention preserves continuity but raises the favorability of withdraw rather than roll-over. As a consequence, adverse smooth policy must lower stability as investors preempt the increase in policy intensity.

The preservation of continuity is crucial. Such smoothness of incentives requires in particular that entry and exit to an intervention do not occur too sudden at the boundaries to the intervention interval $\mathcal{N}(p)$. But there are intervention types where the immediacy of the intervention cannot preserve continuity. The case where intervention is harsh in a way that causes discontinuities in the payoff difference function is subject of the next section.

5 Harsh policy and preference-jumps

By Proposition 4.1, sufficiently smooth policy intervention that raises the favorability of roll-over versus withdrawal improves firm stability. Often, however, policy intervention is full-on once triggered. If such policy intervention causes jumps in the payoff difference function, or when a change in policy even shifts these jump points, the previous framework no longer applies, and cannot be used for measuring the effectiveness of policy. Therefore, I next introduce a general framework to study harsh policies that may cause stark alterations (jumps) in the investor's preferences at single withdrawal thresholds.

For that purpose, I generalize the status quo at a given policy $p \geq 0$ in a way that allows for jumps in the payoff difference function.

Assumption 5.1 (Discontinuous payoff difference)**.** *Fix policy $p \in [0, \infty)$.*
(i) The payoff difference function $\upsilon_p(n, \theta)$ is continuous in (n, θ) on $[0, 1] \setminus \{n_1, \ldots, n_k\} \times [0, 1]$, and differentiable in θ on $(\underline{\theta}_p, \overline{\theta}_p)$, where $n_1 < \cdots < n_k \in [0, 1]$, $k \in \mathbb{N}, k < \infty$ denote finitely many withdrawal thresholds at which the payoff difference function jumps.
(ii) For all jump-points $\{(n)_i\}_{i=1,\ldots,k}$ the left- and right-sided limits of the payoff difference function exist (are finite)

$$|\lim_{n \nearrow (n)_i} \upsilon_p(n, \theta(n, \theta_p^*))| =: c_{i,l} < \infty, \quad |\lim_{n \searrow (n)_i} \upsilon_p(n, \theta(n, \theta_p^*))| =: c_{i,r} < \infty \tag{6}$$

For intuition on the possible causes of these preference jumps, one can imagine a threshold intervention such as the suspension of convertibility, a sudden bail-in or a lender of last resort emergency liquidity provision that occurs when withdrawals realize above a particular withdrawal level.

Let m with $0 \leq m \leq k$ the number of policy-dependent withdrawal jump-thresholds, that shift with the policy. Let $(k - m)$ the number of policy-independent withdrawal jump-thresholds. For $m > 0$, without loss of generality, I reorder the policy-dependent jump points by $(n)_1 < \ldots, < (n)_m$. Set $(n)_0 = 0$ and $(n)_{k+1} = 1$. The renaming of jump points to $(n)_1, \ldots, (n)_m$ allows me to directly address all of the policy-dependent jump-points.

Before studying shifts in the jump points, I need to establish further conditions to attain equilibrium existence and uniqueness of the trigger equilibrium under jumps. I maintain assumption 2.1 but need to adopt the one-sided strategic complementarity assumption.

Assumption 5.2 (Single-crossing with jumps)**.** *The payoff difference function $\upsilon_p(n, \theta)$ is strictly decreasing in the aggregate withdrawals n whenever $\upsilon_p(n, \theta)$ is non-negative:*
(i) For withdrawals n between adjacent jump points (n_i, n_{i+1}), $\quad i = 0, \ldots, k$ it holds: whenever $\upsilon(n, \theta) \geq 0$ then $\upsilon(n, \theta) > \upsilon(n + h, \theta)$ for all $h > 0$ with $n + h < n_{i+1}$
(ii) If the left-sided limit of the payoff difference function in a jump point n_i, $\quad i = 1, \ldots, k$

is non-negative, $\lim_{n \nearrow n_i} \upsilon_p(n,\theta) \geq 0$*, then that jump point must be a downwards jump,* $c_{i,l} - c_{i,r} > 0$.

The assumptions (i) and (ii) of assumption 5.2 imply single-crossing of the payoff difference function while allowing for discontinuities. The requirement (ii) imposes that the payoff difference may jump upwards only across negative values of the payoff difference function. Jumps across positive values or from a positive to a negative value must be downwards jumps.

Assumptions 5.2 and 5.1 generalize the **?** environment further, beyond general payoffs, allowing for discontinuities of the payoff difference function. Both assumptions are necessary for preserving the equilibrium existence and uniqueness of a trigger equilibrium when allowing for the jumps.[7] Both the continuous environment studied in the previous section, and the **?** model are nested in the environment described in this section. Observe that assumption 3.1 is nested in assumption 5.1 when setting $k = 0$. Likewise, assumption 5.2 nests the standard one-sided strategic complementarity assumption in 5.2 for $k = 0$. Therefore, the status quo discussed in section 3 is contained in the generalized status quo environment of this section.

5.1 Harsh Policy Intervention

A smooth policy intervention shifts the payoff difference function gradually in the policy over a fixed interval of withdrawals and preserves contintuity in the withdrawals. In contrast, the following definition of a policy intervention captures the idea that policy may initiate or finish abruptly at some entry or exit threshold in a way that causes discontinuities of the payoff difference function.

Definition 5.1 (Harsh policy intervention)**.** *Fix the status quo policy* $p \geq 0$ *at which the payoff difference function* $\upsilon_p(n,\theta)$ *is continuous. A policy intervention is "harsh" if its implementation causes discontinuities (jumps) of the payoff difference function in the aggregate withdrawals* $n \in [0,1]$*, exhibiting at least one up- or downwards jump point* $n_i \in (0,1)$*,* $i = 1,\dots,k$*,* $k \geq 1$*. I call a policy intervention "adverse harsh" if it causes a downwards jump and "prudent harsh" if it causes an upwards jump of the payoff difference function in some withdrawal level.*

For an intuition of why I call downwards jumps adverseley and upwards jumps prudent, I refer the reader to Proposition 5.2 (i) and the discussion there below. One might be tempted to call this harsh intervention type "threshold intervention." Note, however

[7] By assumption 5.1, the payoff difference $\upsilon_p(n,\theta)$ is bounded in n over the interval $[0,1]$ because the jumps are finite and because the payoff difference is continuous over the intervals (n_i, n_{i+1}), $i = 0,\dots,k$. Therefore, and because the discontinuities have measure zero, the payoff difference function remains integrable over $[0,1]$.

that smooth policy intervention likewise starts at a threshold (left endpoint of $\mathcal{N}_p$) and ends at a threshold (right endpoint of $\mathcal{N}_p$).

In section 7, I show that the imposition of withdrawal fees, an ELA provision via the lender of last resort, or the suspension of convertibility of deposits are examples of harsh intervention.

The next result states that under the right monotonicity assumptions, the existence and uniqueness of equilibrium is preserved under harsh policy intervention.

Proposition 5.1 (Equilibrium existence and uniqueness under jumps)
Fix the status quo $p \geq 0$. Assume the payoff difference function $\upsilon_p(n, \theta)$ exhibits jump-points $\{(n)_i\}_{i=1,\ldots,k}$, and assumptions 2.1, 5.1 and 5.2 hold. As noise vanishes, $\varepsilon \to 0$, there exists a unique equilibrium and the equilibrium is in threshold strategies $\theta^*(p)$ where all investors withdraw if they observe a signal below the trigger and otherwise roll over.

The proof to Proposition 5.1 is a contribution to the global games literature beyond the general characterization in **?**. It generalizes the existence and uniqueness proof of the model in **?** to allow for finitely many jumps in the payoff difference function in addition to having general payoffs, subject to assumptions 5.1 and 5.2. Essentially the proof amounts to showing that expected relative payoffs conditional on a signal realization remain continuous, and strictly increasing in the signal when having finitely many jumps in withdrawal points of the payoff difference function.

Definition 5.2 (Change in harsh policy intensity)**.** *Fix policy $p \geq 0$, and let assumption 5.1 hold so that the payoff difference function $\upsilon_p(n, \theta)$ exhibits at least one jump point, $k > 0$. Denote by $\{n_1, \ldots, n_k\}$ the jump points of the payoff difference function. A change in harsh policy via p can occur in two kinds of ways:*

1. *Jump-shifts: The change in p alters at least one jump point, shifting it up or down, meaning the jump occurs at a higher or lower withdrawal level of the PI. That is, it holds $m > 0$ and $\frac{\partial}{\partial p}(n)_i \neq 0$ for $i \in \{1, \ldots, m\}$, $m \leq k$ (control of entry and exit points to harsh intervention)*

    (a) *Adverse jump-shift: A jump-shift is called "adverse" if jump i is a down-jump, $c_{i,l} - c_{i,r} > 0$ and the policy shifts the jump point downwards to a lower withdrawal level, $\frac{\partial}{\partial p}(n)_i < 0$, or jump i is an up-jump, $c_{i,l} - c_{i,r} < 0$, which the policy shifts upwards to a higher withdrawal level, $\frac{\partial}{\partial p}(n)_i > 0$.*

    (b) *Prudent jump-shift: A jump-shift is "prudent" if the policy shifts a down-jump to a higher or an up-jump to a lower withdrawal level.*

2. *Piecewise smooth policy: There exists an open interval of withdrawals $\mathcal{N}(p) \subset [0, 1]$ on which the payoff difference function is continuous in n, and differentiable in p such that: A change in p alters the payoffs to withdrawal or roll-over gradually along $n \in \mathcal{N}(p)$ in a way that preserves the continuity of the payoff difference on $\mathcal{N}(p)$.*

(a) *Prudent piecewise smooth policy: A piecewise smooth policy is called "prudent" if it pushes the PI upwards with*
(i) $\frac{\partial}{\partial p}\upsilon_p(n,\theta) \geq 0$*, for all withdrawals* $n \in [0,1] \setminus \{n_1, \ldots, n_k\}$ *and*
(ii) $\frac{\partial}{\partial p}\upsilon_p(n,\theta) > 0$ *for withdrawals* $n \in \mathcal{N}(p)$*.*

(b) *Adverse piecewise smooth policy: A piecewise smooth policy is called "adverse" if it pushes the PI downwards with*
(i) $\frac{\partial}{\partial p}\upsilon_p(n,\theta) \leq 0$*, for all withdrawals* $n \in [0,1] \setminus \{n_1, \ldots, n_k\}$ *and*
(ii) $\frac{\partial}{\partial p}\upsilon_p(n,\theta) < 0$ *for withdrawals* $n \in \mathcal{N}(p)$*.*

Again, the intervention set $\mathcal{N}(p)$ can be disconnected, that is, the union of several disjoint smaller open intervention intervals. The intervention set $\mathcal{N}(p)$ cannot contain any jump points.

Under a jump-shift, a change in (harsh) policy alters the payoff difference function only at discrete points, the jump points, while under a (piecewise) smooth policy intervention the payoff difference function is shifted over entire intervals of withdrawals, as in the case of smooth intervention policy. Every jump-shift is either prudent or adverse. In contrast, there exist mixtures of prudent and adverse piecewise smooth policies. While I do not formally define these, their analysis is included in Proposition 5.2 and its proof below.

Changes in harsh policy that constitute combinations of jump-shifts and piecewise smooth policy are common in applied settings, see section 7, which is why they deserve a definition of its own:

Definition 5.3 (Harsh combination policy change)**.** *A harsh combination policy change in* p *is a policy change that shifts a jump point and simultaneously shifts the payoff difference function gradually over some intervention set* $\mathcal{N}(p)$ *in a piecewise smooth way.*

A simple intuitive example is the case where the regulatory policy intervention starts harsh at a withdrawal threshold n_1, causing a jump, but simultaneously raises the payoff difference function on the interval $\mathcal{N}(p) = (n_1, 1]$, see subsection 7.3.

Section 7 demonstrates that ELA provision via a lender of last resort or the suspension of convertibility of deposits constitute harsh combination policy that causes and shifts a jump point while also conducting piecewise smooth policy on subintervals of withdrawals.

The next proposition is my second main result. It shows that changes in harsh policy via policy-driven jump-shifts can have adverse consequences for the investors' incentives to withdraw from the firm.

Proposition 5.2 (Stability under harsh policy change)
Let $\{(n_i)\}_{i=1}^m,\ m > 0$ *denote the policy-dependent jump points of the payoff difference function, of which each is either an up- or a down-jump point.*
(ia) If the policy change in p *causes an adverse jump-shift without acting piecewise smoothly, that is, weakly lowers all down-jump points, weakly raises all up-jump points,*

and either strictly lowers at least one down-jump point or strictly raises at least one up-jump point, or both, then the equilibrium trigger strictly increases, meaning that bank stability strictly declines.
(ib) If the policy change in p causes a prudent jump-shift without acting piecewise smoothly, then bank stability strictly increases.
(ii) If the policy change in p acts prudent piecewise smoothly and holds all jump points fixed, then bank stability strictly increases whereas if the change acts adverse piecewise smoothly bank stability declines.
(iii) Harsh combination policy: if a downward (upward) jump point declines (increases) fast in the policy and if the payoff difference makes large jumps in the policy-dependent thresholds $\{(n_i)\}_{i=1}^m$, *then the equilibrium trigger increases and stability drops in policy p even though the policy may simultaneously act in a prudent piecewise smooth way over some interval* $\mathcal{N}(p)$.

The consequences of such harsh policy "gone wrong" are severe for two reasons: First, harsh policy can cause deep payoff jumps, which in return have a large impact on the ex ante roll-over incentives of investors. The depth of a jump impacts stability to a comparable degree as the length of the intervention interval $\mathcal{N}(p)$ on which piecewise smooth policy is actively conducted, see section 6 on policy equivalence. The result in (ii) is essentially a restatement of Proposition 4.1 with the insight that the presence of jumps in the PI do not matter for whether a piecewise smooth policy acts prudent or adverse as long as the policy leaves the jump-points constant. The result in (iii) is part of a more general phenomenon which I discuss in more detail in section 6.2 which deals with policy equivalence and offsetting policies.

Proposition 5.2 not only concerns stability changes when altering existing harsh policy. It also covers stability changes when a harsh policy is imposed for the first time, transitioning from a continuous payoff difference (absent intervention, $p = 0$) to a discontinuous PI, under harsh policy. Generically, in bank run settings absent of regulatory policy ($p = 0$) the payoff difference function is continuous, that is, without jumps, see **?**. For analyzing the change in stability under the transition from no policy (continuity) to harsh policy (with jumps) one would study the framework above (with jumps), where the jump point is shifted from the boundary $n_i = 1$ (no policy intervention and no jumps) towards the interior, $n_i \in (0, 1)$ (harsh policy with jumps). We know that downwards shifts of downwards jumps constitute adverse harsh policy. Therefore, the imposition of harsh policy that causes a downwards jump in a previously continuous PI constitutes adverse harsh policy, and rationalizes the Definition 5.1. The other way around, the imposition of harsh policy that causes an upwards jump in a previously continuous PI constitutes prudent harsh policy.

The proof to Proposition 5.2 gives insight into why shifts in the jump points of the payoff difference function cause preemptive investor behavior. Therefore, I prove the proposition here in the text.

Proof. [Proposition 5.2] With jump points, I can rewrite the payoff difference equation as

$$H(p,\theta^*) = \int_0^{n_1} \upsilon_p(n,\theta(n,\theta^*))\,dn + \cdots + \int_{n_k}^{1} \upsilon_p(n,\theta(n,\theta^*))\,dn \tag{7}$$

To prove Proposition 5.2, recall that the equilibrium trigger θ^* is implicitly defined as the zero of the payoff indifference equation $H(\theta^*, p) = 0$, (7). From the proof to Proposition 4.1 we know $\frac{\partial H}{\partial \theta^*} > 0$. Using the implicit function theorem, the trigger declines in p if and only if the change in the payoff difference equation due to a change in p is positive, $\left\{\frac{\partial \theta^*}{\partial p} < 0\right\} \Leftrightarrow \{\frac{\partial H}{\partial p} > 0\}$. By the Leibniz rule for parameter integrals, the change in the payoff difference equation due to a change in p equals

$$\frac{\partial}{\partial p}H(p,\theta^*) = \int_{[0,n_1]\cap\mathcal{N}(p)} \frac{\partial}{\partial p}\upsilon_p(n,\theta(n,\theta^*))\,dn + \cdots + \int_{[n_k,1]\cap\mathcal{N}(p)} \frac{\partial}{\partial p}\upsilon_p(n,\theta(n,\theta^*))\,dn \tag{8}$$

$$+\sum_{i=1}^{m} \frac{\partial (n)_i}{\partial p}\left(\lim_{n\nearrow(n)_i} \upsilon_p(n,\theta(n,\theta^*)) - \lim_{n\searrow(n)_i} \upsilon_p(n,\theta(n,\theta^*))\right) \tag{9}$$

The integrals in (8) describe how a change in policy affects the payoff difference function over the intervention intervals $\mathcal{N}_i(p) = [n_{i-1}, n_i] \cap \mathcal{N}(p),\ i = 1, \ldots k+1, n_0 \equiv 0, n_{k+1} \equiv 1$ (adverse versus prudent piecewise smooth), while the summation term (9) describes how the jump points, e.g. entry and exit points, shift in the policy and whether jumps are up- or downwards jumps.

Concerning the proof of part (ii), under a pure piecewise smooth policy, a change in the policy either leaves all jump points constant or no jump points exist, so that the summation term (9) equals zero. The sign of the derivative $\frac{\partial}{\partial p}H(p,\theta^*)$ is, thus, solely determined by the sign of the terms in (8), and is positive only if the piecewise smooth policy is prudent. Further, if the piecewise smooth policy is prudent, then $\frac{\partial \theta^*}{\partial p} < 0$, and bank stability increases.

Concerning the proof of part (i), under a harsh policy change that is purely due to jump-shifts, there exists no interval of withdrawals $\mathcal{N}(p)$ over which the payoff difference changes gradually, and the integrals in (8) are all zero. Moreover, the payoff difference jumps in the withdrawal points $(n)_i$. Therefore, the left- and right-sided limits in each jump point are distinct, implying that the differences

$$\lim_{n\nearrow(n)_i} \upsilon_p(n,\theta(n,\theta^*)) - \lim_{n\searrow(n)_i} \upsilon_p(n,\theta(n,\theta^*)) \tag{10}$$

are non-zero. A difference is positive if the according jump point $(n)_i, i = 1, \ldots, k$, implies a down jump, whereas a difference is negative if the jump point implies an up-jump. The boundary derivatives in (9) are, thus, non-zero if at least one jump point is shifted by the policy.

If a difference (10) is positive (down jump), the boundary derivative in (9) is negative if the jump point strictly declines in the policy $\frac{\partial (n)_i}{\partial p} < 0$. If a difference (10) is negative (up jump), the boundary derivative in (9) is negative if the jump point strictly increases in the policy $\frac{\partial (n)_i}{\partial p} > 0$.

Therefore, a jump-shift strictly increases the trigger θ_p^* (lowers stability) if either all down-jump points (weakly) decline and or all up- jump points (weakly) increase in the policy parameter p, with at least one jump-shift being strict.

Concerning (iii), under a harsh combination policy, the intervention intervals $\mathcal{N}_i(p)$ are non-empty. Further, the gradual change in payoffs $\frac{\partial}{\partial p}\upsilon_p(n, \theta(n, \theta^*))$ over at least one of the intervention intervals is positive under a prudent piecewise smooth policy. Thus, at least one of the integrals in (8) is positive. Therefore, the trigger may decline (stability can improve) in the policy p, if the change in payoffs is stronger than the change in the jump point. If the jump points alter fast in the policy and if the intervention causes harsh changes in incentives (deep jumps) at the intervention points, stability can decline in the policy under an adverse jump-shift even though the relative incentives to roll over improve over the set of withdrawals $\mathcal{N}(p)$. $\square$

6 Policy Equivalence

The past sections introduced four different types of policy and their impact on stability. But often a policy acts smoothly and harshly at the same time, exhibiting prudent and adverse elements, see section 7 for examples. As a consequence, these different features of policy can (partially) offset each other, rendering the policy less effective with regard to its impact on stability than anticipated. Moreover, different policies can act alike.

6.1 Mixed smooth policies

A mixed smooth policy is a smooth policy that acts prudently over some intervention interval and adverseley over another. I will show next, that these mixed smooth policies are often ineffective, because the prudent and the adverse features partially offset each other. The dominating smooth policy is either the policy with the greater intervention interval or the more intense change in the PI.

Proposition 6.1 (Equivalence I: Mixed smooth policies across intervention intervals) *Consider a (piecewise) continuous payoff difference $\upsilon_p(n, \theta) = u_2(n, p) - u_1(n, p)$. Consider a policy that acts prudent respectively adverse piecewise smoothly on intervention intervals $\mathcal{N}_{p,i} \in [0, 1]$, $i = 1, 2, \ldots N$. Then the adverse effect can entirely offset the prudent effect on firm stability, depending on the length of the intervention interval and the* *average policy intensity over the intervention interval of either policy. Weak policy intensity over prolonged intervals can outweigh, offset or turn less effective intense intervention over short intervals and vice versa.*

Consider the case of piecewise smooth policy that acts prudently over an interval $\mathcal{N}_1 \subset [0,1]$ but adversely over an interval $\mathcal{N}_2 \subset [0,1]$, $\mathcal{N}_1 \cap \mathcal{N}_2 = \emptyset$. Then, the expected policy-driven change in the payoff difference equals $\frac{\partial}{\partial p}H(p,\theta^*) = \int_{\mathcal{N}_1(p)} \frac{\partial}{\partial p}\upsilon_p(n,\theta(n,\theta^*))\,dn + \int_{\mathcal{N}_2(p)} \frac{\partial}{\partial p}\upsilon_p(n,\theta(n,\theta^*))\,dn$. By assumption, both integrals have opposing signs, with the first integral being positive. By the mean value theorem for integrals, there exist points $\bar{n}_1 \in \mathcal{N}_1$ and $\bar{n}_2 \in \mathcal{N}_2$ at which the marginal payoff difference attains its average value over the corresponding intervention interval, so we can rewrite the expected change in the payoff difference as

$$\frac{\partial}{\partial p}H(p,\theta^*) = \sum_{i=1}^{2} |\mathcal{N}_i(p)| \, \frac{\partial}{\partial p}\upsilon_p(\bar{n}_i, \theta(\bar{n}_2,\theta^*)) \tag{11}$$

where $|\mathcal{N}_i(p)|$ is the length of the intervention interval i, and $\frac{\partial}{\partial p}\upsilon_p(\bar{n}_i,\theta(\bar{n}_2,\theta^*))$ is the average policy intensity over intervention interval $\mathcal{N}_i$. The sign of the expected change in the payoff difference, $\frac{\partial}{\partial p}H(p,\theta^*)$, and thus, whether bank stability increases or declines equals the sign of the policy that has the greates intervention interval or the greatest policy intensity over that interval, or both. More importantly, because the signs of the terms are opposite, the prudent policy does not act as strongly as if the adverse effect had been absent. In fact, the adverse effect can cancel the prudent effect entirely. The argument easily generalizes to more than two intervention intervals. One example of such a mixed smooth policy is given in section 7.2 which shows that raising the ELA loan acts prudently on one withdrawal range but adversely on another. If the ELA entry threshold is not chosen wisely, raising the ELA loan backfires, and the adverse smooth effect dominates the prudent one, see Corollary 7.3

6.1.1 Equivalence of smooth policies over same intervention interval

A special case of mixed smooth policy equivalence occurs if policy acts ambiguously over the *same* intervention interval. Consider a fixed intervention interval $\mathcal{N}_p$. We know by Proposition 4.1 that a prudent smooth policy gradually shifts the payoff difference to roll-over versus withdrawal upwards along $\mathcal{N}_p$. But because the payoff difference $\upsilon_p(n,\theta) = u_2(n,p) - u_1(n,p)$ is a difference, an upwards shift of the difference can occur in two kinds of ways, either by augmenting the first term u_2, that is paying a bailout to investors that roll over, or by reducing the second term u_1, meaning to bail-in depositors that withdraw:

Proposition 6.2 (Equivalence IIa: Cost-reducing smooth policy within $\mathcal{N}_p$)
Fix an intervention interval $\mathcal{N}_p$ and consider the (piecewise) continuous payoff difference

$\upsilon_p(n,\theta) = u_2(n,p) - u_1(n,p)$. *Consider a bailout provision $\varepsilon(n) \geq 0$ which is paid to investors that roll over along the intervention interval $\mathcal{N}_p \subset [0,1]$ in a piecewise smooth way. Then the ex ante stability improvement attained via this bailout is equivalent to a stability improvement that would have been attained if instead of the bailout the regulator had bailed in the withdrawing investors by the same amount $\varepsilon(n)$.*

The proof is easy to see: It holds

$$\begin{aligned} \upsilon_p(n,\theta) + \varepsilon(n) &= (u_2(n,p) + \varepsilon(n)) - u_1(n,p) && \text{bailout of roll-overs in } t = 2 \\ &= u_2(n,p) - (u_1(n,p) - \varepsilon(n)) && \text{bail-in of withdrawals in } t = 1 \end{aligned}$$

Therefore the impact on the equilibrium trigger θ_p^* and thus ex ante stability $\theta_b(p)$ is the same. From a policy designer's perspective, the result is important to reduce policy costs while keeping effectiveness with regard to stability constant. A bail-in can be accomplished with a zero government budget, whereas bailouts require a budget and thus taxpayer money. The argument also works the other way around. It holds $\upsilon_p(n,\theta) - \varepsilon(n) = u_2(n,p) - (u_1(n,p) + \varepsilon(n)) = (u_2(n,p) - \varepsilon(n)) - u_1(n,p)$. Thus,

Proposition 6.3 (Equivalence IIb: Unintended smooth policy within $\mathcal{N}_p$)
Fix the intervention interval $\mathcal{N}_p$ and consider the (piecewise) continuous payoff difference $\upsilon_p(n,\theta)$. A bailout provision $\varepsilon(n) \geq 0$ paid to withdrawing investors over $\mathcal{N}_p$ has the same detrimental effect on firm stability as a bailin by $-\varepsilon(n) \leq 0$ of investors that roll over.

Such a bailout to withdrawing investors is for instance paid implicitly and explicitly when a lender of last resort (LOLR) provides an emergency liquidity loan, see the analysis in section 7.2. As the LOLR raises the liquidity loan it is implicity bailing in investors that roll-over whom need to repay the loan. Beyond that one might think that a larger emergency loan allows the firm to survive larger runs which should stabilize the firm. But also here, the opposite is true. A larger loan increases the payoff to withdraw because the chance of being served the face value increases. Therefore, the ELA provision constitutes an explicit bailout of withdrawing investors which, by the equivalence, acts like a bailin of investors that roll over, both harming stability.

The effect is likewise active when considering a raise of a withdrawal fee, see Corollary 7.1 of section 7.1.1. Raising the fee allows the bank to slow down asset liquidations which increases the payoff to roll-over. On the other hand, raising the fee allows the bank to survive greater runs which increases the payoff to withdrawal because the likelihood of being served the face value goes up. Both effects exactly offset each other so that raising the fee has zero impact on stability in a particular withdrawal range even though the fee is imposed.

Note, the result in Proposition 6.3 is somewhat at odds with the result in **?** where a social planner decides to liquidate additional assets in $t = 1$ when learning that a run is happening. In the context of this model, the additional asset liquidation increases payoffs to early withdrawing agents, which raises their ex ante incentive to withdraw in the first place. The ex ante run likelihood goes up. In **?** such additional liquidation is socially optimal since there exist impatient agent types that might not have withdrawn yet and who would receive zero utility when not being allowed to withdraw.

6.2 Equivalence of smooth and harsh policy

Consider the case where a policy intervention shifts a jump point $n_k(p)$ of the payoff difference $\upsilon_p(n,\theta)$ and at the same time acts piecewise smoothly on an intervention interval $\mathcal{N}_p \subset [0,1]$. The policy-driven change in the expected payoff difference equals $\frac{\partial}{\partial p}H(p,\theta^*) = \int_{\mathcal{N}(p)} \frac{\partial}{\partial p}\upsilon_p(n,\theta(n,\theta^*))\,dn + \frac{\partial n_k}{\partial p}(\lim_{n\nearrow n_k}\upsilon_p(n,\theta(n,\theta^*)) - \lim_{n\searrow n_k}\upsilon_p(n,\theta(n,\theta^*)))$. Bank stability increases, that is, $\theta^*(p)$ declines, in policy change p if $\frac{\partial}{\partial p}H(p,\theta^*)$ is positive. Using the intermediate value theorem for integrals, there exists a withdrawal threshold $\bar{n} \in \mathcal{N}(p)$ in the intervention interval at which the marginal payoff difference, $\frac{\partial}{\partial p}\upsilon_p(n,\theta(n,\theta^*))$, attains its average speed. As a consequence, I can rewrite the overall policy-driven change in the expected payoff difference as

$$\frac{\partial}{\partial p}H(p,\theta^*) = |\mathcal{N}(p)| \times \frac{\partial}{\partial p}\upsilon_p(\bar{n},\theta(\bar{n},\theta^*)) \tag{12}$$

$$+ \frac{\partial n_k}{\partial p}\left(\lim_{n\nearrow n_k}\upsilon_p(n,\theta(n,\theta^*)) - \lim_{n\searrow n_k}\upsilon_p(n,\theta(n,\theta^*))\right) \tag{13}$$

where $|\mathcal{N}(p)|$ is the length of the intervention interval, $\frac{\partial}{\partial p}\upsilon_p(\bar{n},\theta(\bar{n},\theta^*))$ is the average change of the PI over intervention interval $\mathcal{N}(p,\theta)$, $\frac{\partial n_k}{\partial p}$ is the speed of the policy-driven shift in jump point n_k, and $(\lim_{n\nearrow n_k}\upsilon_p(n,\theta(n,\theta^*)) - \lim_{n\searrow n_k}\upsilon_p(n,\theta(n,\theta^*)))$ is the depth of the jump in n_k. When evaluating the impact of the (piecewise) smooth versus the jump-shifting component of policy on $\frac{\partial}{\partial p}H(p,\theta^*)$ and thus firm stability, the length of the intervention interval has a comparable impact as the depth of the jump. Likewise, the average policy-driven change of the PI over the intervention interval has a comparable effect on stability as the speed at which the jump-point shifts with policy. The length of the intervention interval is always positive whereas the depth of the jump is positive for down-jumps and negative for up-jumps. Also, the shift in the jump-point and the shift in the PI can both go in either direction. As a corollary of Proposition 5.2 (iii),

Corollary 6.1 (Equivalence III: Ineffective adverse harsh policy)
Consider a (piecewise) continuous payoff difference $\upsilon_p(n,\theta)$ with a jump point $n_k(p)$. Consider a harsh combination policy that acts piecewise smoothly on an intervention interval $\mathcal{N}_p \in [0,1]$ and simultaneously shifts the jump point $n_k(p)$. Assume the policy shifts the jump-point and the payoff difference on $\mathcal{N}_p$ in opposite ways such that exactly one shift acts prudently. If the intervention interval is short relative to the jump depth,

and if the jump-point shifts fast relative to the change in the PI, the jump-shift can offset or outweigh the piecewise smooth effect of policy, and in either case, make it less effective than had the jump been absent.

Section 7.2 provides an example, because an ELA provision causes a jump in the payoff difference function at the ELA entry point if ELA is costly. Lowering the ELA entry point adversely shifts the jump-point but simultaneously acts prudently piecewise smooth because asset liquidation can be paused sooner. The jump-shift makes lowering the ELA entry point a less effective policy because it partially offsets the prudent effect.

7 Applications

This section discusses several common policy interventions to provide examples of smooth, and harsh policies as well as policy equivalence. To construct the examples, I need to define a status quo where policy is absent. For that purpose, I next describe a risk-neutral version of the banking model in **?** (GP) which serves as the benchmark model before policy intervention is introduced.

Benchmark before policy intervention (Goldstein-Pauzner)

There exists a continuum of depositors $[0, 1]$. Unlike in GP, all depositors are risk-neutral and can consume in $t = 1$ and $t = 2$ (are "patient"). Let $\theta \sim U[0, 1]$ parametrize the random, unobservable state of the economy, and let $\overline{\theta} \in (0, 1)$ an upper threshold state close to 1. Besides storage, there exists a risky asset in the economy to shift consumption across time. For every unit investment, if the state realizes in $\theta \in [0, \overline{\theta})$ the asset pays $R > 2$ in $t = 2$ with probability $p(\theta)$ and otherwise zero, and in case of liquidation in $t = 1$ pays 1 like storage. If the state realizes high in $\theta \in [\overline{\theta}, 1]$, the asset pays R already in $t = 1$ and with probability $p(\theta) = 1$. The function $p(\theta)$ is positive, strictly increasing, and differentiable in θ for $\theta \in [0, \overline{\theta})$ and is constant at 1 for $\theta \in [\overline{\theta}, 1]$. The bank offers a demand-deposit contract to depositors to raise funds for investment in the risky asset. Following GP, assume the contract offers a short-term coupon $r_1 > 1$ in the case a depositor withdraws the deposit[8] in $t = 1$, and offers a long-term coupon $\frac{R(1-nr_1)}{1-n}$ in the case a depositor rolls over the deposit to $t = 2$, where $n \in [0, 1]$ is the endogenous measure of depositors who withdraw in $t = 1$. Risk-sharing imposes a payoff externality: As long as withdrawals are low, $n < 1/r_1$, the bank can service all withdrawal requests by liquidating assets. But if the withdrawals reach the threshold $n_{Ill} := 1/r_1$, the bank can no longer finance all withdrawals by liquidation, and becomes illiquid (bank run).

[8] GP show that risk-sharing, that is, setting $r_1 > 1$ is socially optimal with risk-averse, and some impatient depositors even though it gives rise to runs. I impose risk-sharing even though agents are risk-neutral and patient here to keep the possibility of runs alive.

In that case, the depositors who roll over receive zero. The depositors who withdraw queue in front of the bank. With probability $\frac{1}{nr_1}$, a withdrawing depositor is early in the queue and receives the face value of the deposit r_1, whereas with probability $1-\frac{1}{nr_1}$ she is late in the queue and receives zero. The payoff difference function in the liquid case $n \in [0, n_{Ill})$ equals $\upsilon_L(n) = p(\theta)\ \frac{R(1-nr_1)}{1-n} - r_1$ whereas in the illiquid case $n \in [n_{Ill}, 1]$, $\upsilon_{Ill}(n) = 0 - \left(\frac{n_{Ill}}{n} \times r_1 + (1 - \frac{n_{Ill}}{n}) \times 0\right)$.

7.1 Prudent smooth policy intervention via withdrawal fees

The following example is to the best of my knowledge new to the literature[9], and analyzes the marginal change of firm stability when the regulator imposes a withdrawal fee $c \in (0, r_1)$ as soon as the aggregate withdrawals $n \in [0,1]$ exceed a cutoff $n_c \geq 0$. The firm can be a bank, a money market mutual fund (MMF) or a stablecoin. Henceforth, I call the firm a bank.

Assume the imposition of the withdrawal fee attains before the bank becomes illiquid, $n_c < 1/r_1$. The imposition of a withdrawal fee constitutes a 2-dimensional policy tool (n_c, c) because the intervention threshold and the fee can be move independently of one another. I discuss changes in either policy variable. As long as the endogenous aggregate withdrawals realize below the intervention threshold n_c, no fee is imposed and the payoff difference function equals

$$\upsilon_L(n) = \underbrace{p(\theta)\ \frac{R(1-nr_1)}{1-n}}_{u_2(n)} - \underbrace{r_1}_{u_1},\ n \in [0, n_c). \tag{14}$$

As soon as the withdrawals are high enough to trigger the fee, $n \geq n_c$, the claim of a withdrawing investor is reduced by the amount of the fee. Importantly, in my example, the reduced claim allows the bank to reduce the speed of its asset liquidation for servicing withdrawals.[10] The reduced speed of asset liquidations pushes the illiquidity threshold of the bank up from $n = 1/r_1$ (when never imposing a fee) to

$$n_{Ill}(c) \equiv n_c + \frac{(1-r_1 n_c)}{(r_1 - c)}, \tag{15}$$

meaning the bank can now survive larger runs, that is, stays liquid for a greater range of withdrawals. If the withdrawals are high enough to trigger the fee but low enough so that

[9]The imposition of fees to prevent MMF runs has previously been studied in **?** and **?** in a Diamond-Dybvig (1983) style model. **?** studies first best implementation via gates and fees when investors can incur liquidity shocks. There, the probability of a run is, however, not uniquely determined so that a marginal change in bank stability due to a marginal change in the fee or the threshold cannot be analyzed.

[10]One could alternatively design payoffs to instead redistribute the fee from the withdrawing depositors to depositors who roll over but the original idea of withdrawal fees is to reduce asset liquidations.

the bank remains liquid, $n \in [n_c, n_{Ill})$, a withdrawing investor receives the face value r_1 if she is sufficiently early in the queue so that she is served before the fee is imposed. The probability of that event is n_c/n. If she is late in the queue, with probability $1 - n_c/n$, she is served after the fee is imposed, and receives the face value reduced by the fee. The payoff difference for $n \in [n_c, n_{Ill})$, thus, becomes

$$\upsilon_{L,c}(n) = \underbrace{p(\theta)\ \frac{R(1 - n_c r_1 - (n - n_c)(r_1 - c))}{1 - n}}_{u_2(n,\theta)} - \underbrace{\left(\frac{n_c}{n} r_1 + \frac{n - n_c}{n}(r_1 - c)\right)}_{u_1(n)}. \tag{16}$$

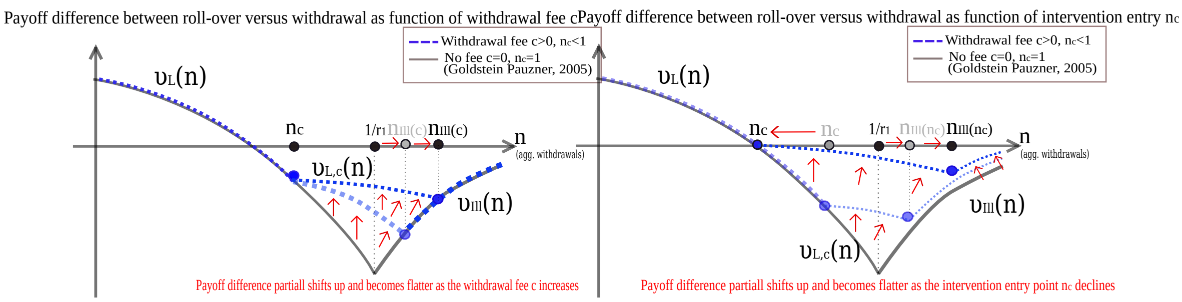
As soon as the bank becomes illiquid, $n \in [n_{Ill}, 1]$, investors who roll-over receive zero. Investors who withdraw receive the face value r_1 if they are early in the queue before the withdrawal fee is triggered, they receive the reduced face value $r_1 - c$ if they withdraw after the fee is imposed but before the bank becomes illiquid, and otherwise receive zero. The payoff difference becomes

$$\upsilon_{Ill}(n) = \underbrace{0}_{u_2} - \underbrace{\left(\frac{n_c}{n} r_1 + \frac{n_{Ill}(c) - n_c}{n}(r_1 - c) + \frac{n - n_{Ill}(c)}{n} \times 0\right)}_{u_1(n)}. \tag{17}$$

7.1.1 Analysis: Raising the withdrawal fee

I first consider a change in the withdrawal fee, holding the intervention threshold constant, and consider a change in the intervention threshold in the next subsection. The imposition of the constant withdrawal fee constitutes smooth intervention: the payoff difference function jumps neither at the intervention threshold $n = n_c$, where the imposition of the fee is triggered, nor at the illiquidity threshold $n = n_{Ill}$. To determine whether this smooth intervention acts prudent or adverse, consider the withdrawal range over which the fee is imposed but the bank is not yet illiquid, $n \in [n_c, n_{Ill})$. An increase in the withdrawal fee raises the payoff difference to roll over versus withdraw, $\frac{\partial}{\partial c}\upsilon_{L,c}(n) > 0$, for two reasons. First, the fee reduces the payoff to withdraw directly and, second, it slows down the required asset liquidation for servicing further withdrawals which increases the roll-over payoff at the margin. The fee, thus, simultaneously acts like a bail-in of investors that withdraw and a bail-out to investors that roll-over, in comparison to the benchmark where no intervention exists. Next consider the withdrawal range where the bank is illiquid, $n \in [n_{Ill}, 1]$. The allover impact on payoffs is zero in this withdrawal range, $\frac{\partial}{\partial c}\ \upsilon_{Ill}(n) = 0$, but, there are two effects at play here that cancel each other out: First, as in the case of the range $[n_c, n_{Ill})$, increasing the fee reduces the payoff to withdraw directly. On the other hand, the fee pushes the illiquidity threshold $n_{Ill}(c)$ up because the additional slow down of asset liquidations allows the bank to survive larger runs. Perhaps surprisingly, the latter effect acts *against* bank stability because it increases the expected payoff to withdraw[11] because the positive payoff upon withdrawing $r_1 - c$ is attained with a greater probability. In fact, this latter effect exactly undoes the stability improving first effect, both effects offset each other such that the payoff difference stays exactly constant, see the section on policy equivalence 6.1.1.

Consequentially, the interval on which intervention is effective is not $[n_c, 1]$ but the smaller interval $\mathcal{N}_c = [n_c, n_{Ill}]$, meaning the imposition of the withdrawal fee is not effective for preventing runs on $n \in [n_{Ill}, 1]$ even though the fee is imposed in this range, see Figure 1a versus 1b. As a Corollary of Proposition 4.1(i), I obtain:



(a) The payoff difference function $v(n)$ shifts up in the range $[n_c, n_{Il}]$ the larger the withdrawal fee c.

(b) The payoff difference function $v(n)$ shifts up in the range $[n_c, 1]$ as the intervention entry point n_c is lowered.

Figure 1

Corollary 7.1 (Raising the withdrawal fee)
Assume the regulator imposes a fee on withdrawals, $c \in [0, r_1)$, if the aggregate withdrawals exceed threshold $n_c \in (0, 1/r_1)$. A policy change that raises the withdrawal fee c holding n_c constant constitutes prudent smooth intervention on $\mathcal{N}_c = [n_c, n_{Il}]$, and, thus, increases bank stability ex ante monotonically. The larger fee allows the bank to survive greater runs which is, however, a feature that acts against bank stability by increasing the expected payoff to withdraw, which reduces the effectiveness of the intervention.

The feature that an increased survival range acts against bank stability is not unique to withdrawal fees, see section 7.2 on raising the Emergency Liquidity Assistance by a lender of last resort. To study the stability change when transitioning from not imposing to imposing a withdrawal fee at n_c, one can study the case $c \to 0$, because the PI under fees converges to the PI of the Goldstein-Pauzner setting when not imposing a fee, where convergence is in $\mathcal{L}^1$.

7.1.2 Altering the entry point to policy intervention

Next, I consider a change in policy by lowering the intervention point n_c while leaving the fee constant. Lowering the intervention point allows the bank to reduce asset liquidations

[11]Here, expectation is taken over the range of possible withdrawals $n \in [0, 1]$.

sooner, and consequentially, the bank can survive larger runs. Therefore, the illiquidity threshold $n_{Ill}(n_c)$ rises as the intervention point n_c declines. Because the payoff difference is continuous in n_c and $n_{Ill}(n_c)$, a change in n_c will not create or shift any jumps so that a decline in n_c constitutes smooth policy intervention, if at all.

To evaluate how a change in the intervention threshold n_c effects bank stability, consider the change in the payoff differences due to an increase in n_c. It holds $\frac{\partial}{\partial n_c}\upsilon_{L,c}(n) < 0$ because as the fee is imposed later overall more asset liquidation is required which lowers the roll-over payoff. Further, $\frac{\partial}{\partial n_c}\upsilon_{Ill}(n) < 0$ because as the intervention is delayed, a withdrawing depositor is served with a higher probability which increases the expected payoff to withdraw. The intervention interval when altering the intervention entry threshold equals $\mathcal{N}_p = [n_c, 1]$ and is thus larger than the intervention interval when raising the fee. As a consequence, altering the intervention entry point is potentially the more effective prudent smooth policy in comparison to raising the withdrawal fee. Thus, as a Corollary to Proposition 4.1,

Corollary 7.2 (Lowering the entry point to impose the withdrawal fee)
Assume the regulator imposes a fee on withdrawals, $c \in [0, r_1)$, if the aggregate withdrawals exceed threshold $n_c \in (0, 1/r_1)$. A policy that lowers the intervention entry threshold n_c constitutes prudent smooth policy, and, thus, raises bank stability ex ante monotonically. The intervention interval equals $\mathcal{N}_p = [n_c, 1]$, and is larger than the intervention interval of a policy that raises the withdrawal fee.

7.2 Harsh combination policy: Emergency Liquidity Assistance

The following example is, to the best of my knowledge, also new to the literature, and complements the analysis of lender of last resort policies given in **?**, section 6. I discuss the connection below.

Assume that instead of imposing a withdrawal fee, there exists a lender of last resort (LOLR) that is willing to lend a bounded amount of emergency liquidity assistance (ELA) $B > 0$ at gross rate $r > 1$ once the bank is perceived as facing a run, and before the bank becomes illiquid. Assume the bank is perceived as facing a run if the withdrawals exceed a threshold $n_B \in (0, 1/r_1)$. Akin to the imposition of a withdrawal fee, ELA provision is a 2-dimensional policy tool (n_B, B). Until ELA is triggered, the bank services withdrawals by liquidating assets. Once ELA is active, the bank no longer needs to liquidate assets, but can draw on the liquid resources B to repay the face value r_1 to withdrawing depositors. The borrowed amount B needs to be repaid with interest in $t = 2$ by the depositors who roll over. Assume that the asset's return is high enough to repay ELA as long as withdrawals are sufficiently low, $R > B(r-1)$, see the discussion on insolvency below. The borrowed funds allow the bank to fully repay withdrawing depositors for a larger range of withdrawals, meaning the ELA provision defers the illiquidity of the bank, pushing the illiquidity threshold up from threshold $n = 1/r_1$ to $n_{Ill}(B) = \frac{(1+B)}{r_1}$. If the ELA provision is sufficiently large, with $B \geq r_1 - 1$, then illiquidity of the bank is ruled out, $\frac{1+B}{r_1} \geq 1$. I henceforth assume that the ELA provision is partial, $B < r_1 - 1$, because I want to understand how the withdrawal incentives of depositors change as the ELA provision increases from zero onwards. Because ELA is partial, the bank is forced to resume the liquidation of assets once the resources B are used up, that is, for $nr_1 > n_B r_1 + B$. I call the withdrawal threshold at which all funds B are used up and liquidation resumes $n_{res}(B) \equiv n_B + \frac{B}{r_1}$. The payoff difference before ELA is triggered equals

$$v_L(n) = \underbrace{p(\theta)\ \frac{R(1-nr_1)}{1-n}}_{u_2(n)} - \underbrace{r_1}_{u_1},\ n \in [0, n_B). \tag{18}$$

Once the ELA intervention starts, asset liquidation is halted as long as ELA is sufficient to serve withdrawals. The payoff difference[12] on $[n_B, n_{res})$ becomes

$$v_{L,B}(n) = \underbrace{p(\theta)\ \max\left(\frac{R(1-n_B r_1) + B - (n-n_B)r_1 - rB}{1-n}, 0\right)}_{u_2(n)} - \underbrace{r_1}_{u_1}. \tag{19}$$

I assume that the bank borrows the entire funds B, and cannot borrow a withdrawal-contingent amount. Borrowed funds that are not utilized to repay withdrawing agents in $t = 1$ are invested in storage, and jointly with the returns on the asset are used to repay the loan to the LOLR in $t = 2$. I apply the max operator in (19) and following because the bank has limited commitment, and because the bank becomes insolvent before it becomes illiquid in $t = 1$. This is an observation that the literature has made before, see also **?**: The ELA loan allows more withdrawals at the expense of agents who roll-over, meaning the loan is a transfer from the roll-over depositors whom need to repay the loan to withdrawing depositors. Ultimately, this is the reason why the ELA provision is a double-edged sword, lowering illiquidity risk in the short-run at the expense of raising credit risk in the long run. A policy that imposes withdrawal fees, in contrast, constitutes a transfer from the withdrawing to the roll-over agent group. Thus, its impact on stability will turn out to be very different from ELA's impact.

I henceforth assume $n_B < \frac{R-B(r-1)}{Rr_1-B(r-1)}$ so that the acceptance of ELA, $n \geq n_B$, does not cause the bank's insolvency right away. This assumption can be rationalized by demanding that ELA is provided only to illiquid but solvent banks, as in **?**. If the withdrawals are so high that the ELA funds B are insufficient to cover all withdrawals, $n \geq n_{res}$, the bank is forced to resume the liquidation of assets and the payoff difference becomes

$$v_{L,B+}(n) = \underbrace{p(\theta)\ \max\left(\frac{R(1-nr_1+B) - rB}{1-n}, 0\right)}_{u_2(n)} - \underbrace{r_1}_{u_1},\ n \in [n_{res}, n_{Ill}). \tag{20}$$

until the bank becomes illiquid for $nr_1 \geq 1 + B$. As soon as the bank becomes illiquid, the

[12] Observe, if the ELA intervention threshold n_B is chosen too high, then the bank is insolvent before all funds are utilized.

payoff difference becomes

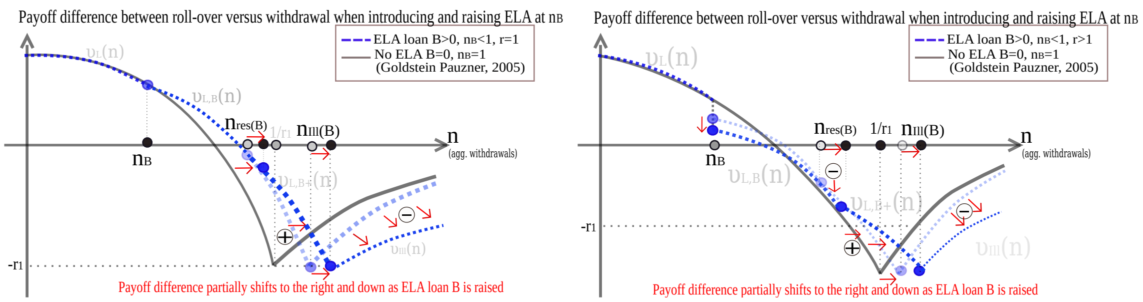
$$v_{Ill}(n) = \underbrace{0}_{u_2} - \underbrace{\left(\frac{n_{Ill}(B)}{n} \times r_1 + \left(1 - \frac{n_{Ill}(B)}{n}\right) \times 0\right)}_{u_1(n)},\ n \in [n_{Ill}(B), 1] \tag{21}$$

because a withdrawing depositor is served the face value only if she is early in the queue.

7.2.1 Analysis: Raising the ELA provision

First, observe that the ELA provision $B > 0$ causes a downward jump of the payoff difference function as the withdrawals hit the ELA entry point n_B if the LOLR charges interest on the loan, $r > 1$, see Figure 2b: $\lim_{n \nearrow n_B} v_L(n) - \lim_{n \searrow n_B} v_{L,B}(n, \gamma) = p(\theta)\frac{(r-1)B}{1-n_B} > 0$. Thus, ELA constitutes harsh policy intervention if the LOLR charges positive net interest $r > 1$. The depth of the jump increases with the ELA loan B because more interest becomes due in $t = 2$. If the LOLR charges no interest, $r = 1$, then no jump occurs in the entry threshold n_B, see Figure 2a. There is no jump at the ELA exit point n_{res} where the funds are used up. Therefore, raising the ELA provision constitutes piecewise smooth policy when holding the jump point n_B constant.

I next analyse how an increase in B affects the payoff difference function, holding the ELA entry point n_B fixed. I discuss shifting the ELA entry point in the next subsection. Once ELA is triggered, $n \in [n_B, n_{res})$, the payoff to roll-over declines in the ELA provision because depositors who roll-over need to repay more funds with interest to the LOLR given survival of the bank, $\frac{\partial}{\partial B} v_{L,B}(n, \gamma) < 0$, see Figure 2b. This effect negatively affects bank stability, it, however, becomes void if the LOLR charges zero interest, $r = 1$. Thus, a rise in the ELA provision impacts the roll-over incentives adversely piecewise smooth in the range $n \in [n_B, n_{res})$ if interest is charged, $r > 1$, and otherwise has no impact.



(a) Zero interest Ela loan $r = 1$ (b) Interest on ELA loan, $r > 1$

Figure 2: When an ELA loan B is provided at threshold n_B, the payoff difference function $v(n)$ shifts to the right, allowing the bank to survive larger runs as B increases (the illiquidity threshold n_{Ill} rises). The payoff difference function $v(n)$ shifts up for all $n \in [n_{res}, n_{Ill}]$, but the PI shifts down over the range $n \in [n_B, n_{res}]$ and $[n_{Ill}, 1]$ because ELA is expensive and because given bank illiquidity, the payoff to withdraw increases with the ELA provision because the likelihood of getting served in the queue goes up. If the LOLR charges interest on the ELA loan, $r > 1$, a jump in the PI occurs at n_B. The depth of the down-jump increases with B.

In the withdrawal range for which the ELA provision is used up but the bank is not illiquid yet, $n \in [n_{res}, n_{Ill})$ the change in relative payoffs due to an increase in ELA $\frac{\partial}{\partial B} \upsilon_{L,B+}(n,\gamma)$ can go in either direction: On the one hand, as the lender of last resort (LOLR) raises the ELA provision, the depositors who roll-over need to repay more funds and interest to the LOLR given survival. This negative effect does not vanish if the LOLR charges zero interest. On the other hand, as more ELA is provided, the liquidation of assets can be deferred for longer. Overall, whether the payoff to roll over increases or declines with the ELA provision in this withdrawal range depends on whether the return on the asset R exceeds the cost of the ELA loan r. A sufficient and reasonable condition for the latter, $\frac{\partial}{\partial B} \upsilon_{L,B+}(n,\gamma) > 0$, is that the LOLR charges lower interest on the ELA loan than the return on the asset, $r \leq R$, for instance, $r = 1$ (zero net interest). The withdrawal threshold at which the bank needs to resume the asset liquidations, n_{res}, shifts upwards as more ELA is provided, see Figures 2b and 2a. Likewise, the bank's illiquidity is deferred: the threshold n_{Ill} increases, as the LOLR provides more ELA. That is, the withdrawal interval for which the ELA funds are used up but the bank is not illiquid yet, $n \in [n_{res}(B), n_{Ill}(B)) = \mathcal{N}_B$, shifts upwards with the ELA funds B but maintains its length constant. Even though the payoff difference is continuous at n_{Ill}, the rise in the illiquidity threshold n_{Ill} matters directly for incentives because it increases the probability that a depositor is served the face value when withdrawing, once the bank is illiquid: Because the ELA provision pushes the illiquidity point $n_{Ill}(B)$ upwards, it holds $\frac{\partial}{\partial B} \upsilon_{Ill}(n) < 0$. That is, the increase in the ELA provision constitutes adverse piecewise smooth policy and has a negative effect on the roll-over incentives, acting like a bail-in of depositors that roll over in this withdrawal range, see on the equivalence of policy in section 6. The intervention interval equals $\mathcal{N}_B = [n_B, 1]$ for positive net interest $r > 1$, and equals $\mathcal{N}_B = [n_{res}, 1]$ for zero net interest, $r = 1$. Allover by Proposition 5.2 (ii),

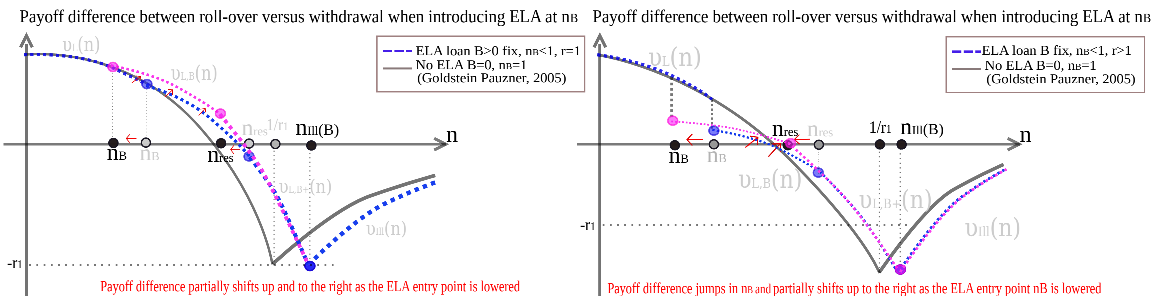
Corollary 7.3 (Increasing the ELA funds)
Assume the LOLR provides an ELA loan $B > 0$ at interest $r \in [1, R)$ if the withdrawals realize above a threshold $n_B \in (0, 1/r_1)$. A policy that raises the ELA provision B, holding the entry threshold n_B constant, constitutes a mixed smooth policy, acting prudently on $[n_{res}, n_{Ill}]$, adversely on $[n_{Ill}, 1]]$, and for $r > 1$, adverseley on $[n_B, n_{res})$. The policy of providing and raising an ELA loan B lowers ex ante bank stability if the entry threshold n_B is set too close to the illiquidity point $1/r_1$, even if the ELA loan is granted at zero net interest $r = 1$.

The Corollary implies that an ELA provision is an unfortunate policy because it implies the possibility of making things worse ex ante.

Proof. [Corollary 7.3] Because providing and raising the ELA loan is a mixed policy, I need to consider the expected change in the payoff difference for determining the overall impact of policy on stability. Via equation (8), and with a policy variable $p = B$ it holds $\frac{\partial}{\partial B} H(B, \theta^*) = -\int_{n_B}^{n_{res}} p(\theta(n,\theta^*)) \, \frac{(r-1)}{1-n} \, dn + \int_{n_{res}}^{n_{Ill}} p(\theta(n,\theta^*)) \left(\frac{R-r}{1-n}\right) dn - \int_{n_{Ill}}^{1} \frac{1}{n} \, dn$. Observe that at the margin there is no jump or jump-shift in $\frac{\partial}{\partial B} H(B, \theta^*)$, because for $B \to 0$ there is no jump in the payoff difference whereas for $B > 0$, the raise in B does not shift the jump point. This feature changes when the policy variable switches to the intervention entry point n_B, see the next subsection. If the intervention threshold n_B is chosen below but close to the original illiquidity threshold (absent intervention) $1/r_1$, then by Lebesgue's dominated convergence theorem, the only prudent effect on stability via the ELA provision vanishes, $\lim_{n_B \to 1/r_1} \int_{n_B+B/r_1}^{(1+B)/r_1} p(\theta(n,\theta^*)) \left(\frac{R-r}{1-n}\right) dn \to 0$, whereas all the adverse effects on stability remain. This holds even for net interest zero $r = 1$. Thus, bank stability strictly declines as the LOLR increases the ELA provision B. $\square$

7.2.2 Lowering the ELA entry point

I next discuss how a policy intervention that lowers the entry point to ELA, n_B, affects bank stability, holding the liquidity provision B fixed. To determine the overall change in incentives, I need to consider the shift in the jump point as well as changes in the payoff difference function $v_B(n)$ due to changes in n_B. First, we know that the payoff difference function jumps down in the ELA entry point n_B if the LOLR charges interest $r > 1$ because as ELA is granted, the depositors that roll over additionally owe the interest on the ELA loan. Lowering the entry point, thus, affects the roll-over incentives adversely by Proposition 5.2(i). The ELA exit point n_{res} depends on the ELA entry point but the payoff difference function is continuous in n_{res}, so its boundary derivative vanishes. Concerning changes in the payoff difference function $v_B(n)$, when ELA is active and asset liquidation has not resumed yet, $[n_B, n_{res})$, lowering the ELA entry point raises the PI because the bank can stop costly asset liquidations sooner, it holds $\frac{\partial}{\partial n_B} v_{L,B}(n,\gamma) < 0$, independently of interest r. Thus, *lowering* the entry point acts prudent piecewise smooth on the intervention interval $[n_B, n_{res})$. For $r > 1$, the downwards shift of the down-jump point n_B and the prudent piecewise smooth effect on $[n_B, n_{res})$ act against one another.



(a) Zero interest Ela loan $r = 1$: no jump at threshold n_B

(b) Interest on ELA loan, $r > 1$, causes a jump at ELA entry point n_B

Figure 3: When lowering the ELA entry point n_B holding the loan amount B constant, the interval $[n_B, n_{res}]$ over which ELA is active shifts down but maintains its length. The PI over $[n_B, n_{res}]$ declines slower and thus shifts up as n_B shifts down. The illiquidity threshold is unchanged. If $r > 1$, the depth of the down-jump increases with n_B and lowering n_B causes an adverse jump-shift which acts against bank stability, lowering the effectiveness of ELA.

Corollary 7.4 (Lowering the ELA entry point)
Consider the provision of an ELA loan B at entry point n_B at interest rate $r \geq 1$. Independently of whether interest is charged on the loan or not, lowering the ELA entry threshold n_B raises bank stability ex ante by acting prudent (piecewise) smoothly on the intervention interval $\mathcal{N}(n_B) =$

$[n_B, n_{res}) = [n_B, n_B + B/r_1)$. *If the LOLR charges interest on the loan, $r > 1$, the ELA provision causes a jump in the PI at the ELA entry point n_B. In that case, lowering the ELA entry jump point n_B constitutes harsh combination policy, giving rise to an additional adverse jump-shift. The adverse jump-shift is always weaker than the main, prudent piecewise smooth effect but makes the policy less effective to improve bank stability ex ante.*

Proof. [Corollary 7.4] To determine the overall effect of this harsh combination policy on stability, I need to consider the expected change in the PI when raising n_B. It turns out that the adverse shift in the jump point is always weaker than the prudent smooth effect: Because $R > r$, it holds $\int_{n_B}^{n_B+B/r_1} p(\theta(n, \theta^*)) \frac{r_1(R-1)}{1-n} dn > p(\theta(n_B + B/r_1, \theta^*)) \frac{B(r-1)}{1-n_B} \to p(\theta(n_B, \theta^*)) \frac{B(r-1)}{1-n_B}$ as $\varepsilon \to 0$. Therefore, the overall change in the expected PI when raising n_B is negative for any ELA interest rate $r \geq 1$: $\frac{\partial}{\partial n_B} H(n_B, \theta^*) \leq 0$. By Proposition 5.2(ii), thus, bank stability strictly increases as the entry threshold to ELA, n_B is *lowered*. □

We can compare the policy that raises the ELA loan B to a policy that lowers the entry threshold n_B.[13] Both policies have their issues. Lowering the ELA entry threshold always improves on stability relative to a higher threshold while the policy of granting and increasing ELA, by setting $B > 0$, can backfire. This makes it seem as if lowering the entry threshold is the better policy. But intuitively, without ELA provision, lowering the entry threshold becomes meaningless:

Proposition 7.1
If the ELA provision is small, lowering the entry threshold has no impact on stability.

As a consequence of the results in this section, the provision of ELA should be critically reassessed. An intervention via imposing a withdrawal fee at a low threshold, for instance, has an unambiguous positive effect on stability ex ante and no downsides.

Proof. [Proposition 7.1] This is straight forward to see: the lowering of the entry threshold is not a strong policy since it only acts on the interval $[n_B, n_{res}]$, and the length of this interval $|n_{res} - n_B| = B/r_1$ hinges on the size of the ELA loan. Even for $r = 1$, $0 \geq \lim_{B \to 0} \frac{\partial}{\partial n_B} H(n_B, \theta^*) = 0$. □

I next compare the model and analysis here to the LOLR model in **?**, section 6. As the first difference to my example, in **?** the ELA provision is full: The amount is always large enough to fend off the run in $t = 1$ but ELA is only provided to solvent banks. In their model, the LOLR observes the asset return realization, and provides sufficient funds only if the bank is capable of repaying in $t = 2$. Thus, a run under ELA never occurs if the bank is solvent. In my example above, the ELA provision is generically not sufficient to entirely fend off the run, because I am interested in how a gradual increase of ELA funds impacts the run incentives of all agents, taking into account that the ELA provision requires a repayment of the loan by depositors that

[13] We can do that because the policy-driven change in ex ante bank stability θ^* due to a change in policy $p = B$ versus $p = n_B$ only differs in the numerator $\frac{\partial}{\partial B} H(B, \theta^*)$ versus $\frac{\partial}{\partial n_B} H(n_B, \theta^*)$ and not in the denominator $-\frac{\partial}{\partial \theta^*} H(B, \theta^*) = -\frac{\partial}{\partial \theta^*} H(n_B, \theta^*)$, recall the proof to Proposition 5.2.

roll-over. Moreover, in my model, ELA is triggered only once withdrawals are high enough so that the bank always needs to liquidate some assets. The ELA entry threshold acts as a second policy variable. In **?**, the ELA provision is instantaneous, and entirely prevents asset liquidation if the bank is solvent.

As the second difference, in my ELA model, the roll-over decisions are made by the depositors and are not delegated to fund managers. Rather, in line with **?** and **?**, a depositor who rolls over has a claim on the bank's returns. In the context of my ELA example, this modeling choice implies that ELA constitutes a transfer from depositors who roll over to depositors that withdraw because the ELA loan needs to be repaid before returns are paid to roll-over depositors.

7.3 Harsh policy via Suspension and Budget Interdependence

The imposition of withdrawal fees or an ELA provision, discussed above, provide examples where the intervention threshold and the policy-implied budget transfer across agent groups can be set independently of one another. I next present an intervention type, the suspension of convertibility followed by the bank's resolution under receivership (in short "receivership resolution"), where the policy jointly pins down the intervention threshold and the transfer. As a consequence, receivership intervention is particularly tricky to handle when it comes to designing stability-maximizing policy. The following example is based on the analysis of suspension interventions followed by resolution under receivership in **??** for the special case of zero deposit insurance.

As previously, the depositors can withdraw the face value of their deposit r_1 at the interim period, and the bank finances withdrawals by liquidating assets. As the standard bank run externality, high withdrawals reduce the remaining bank investment and thus the payoffs to depositors that roll over. This payoff externality via the withdrawals creates interdependence of budgets available to the withdrawing and the not withdrawing agent group which in return leads to a reduction of the policy variables: A regulator observes withdrawals at the bank level, and has the authority to stop runs by suspending the convertibility of deposits before the bank becomes illiquid. The regulator sets the intervention delay $p \in [0, 1]$ as the policy variable, where $1 - p$ denotes the measure of cash withdrawals the regulator tolerates until intervention. The regulator intervenes to stop the run once the cash withdrawals reach $1 - p \in [0, 1]$, and thus imposes the $t = 1$ budget constraint $nr_1 \leq 1 - p$. Policy p pins down the critical suspension entry threshold

$$n_c(p) := \frac{1-p}{r_1} \in (0, 1/r_1) \tag{22}$$

at which the regulator intervenes. Absent regulatory intervention, $p = 0$, the bank is illiquid if the cash withdrawals reach the liquidation value of the asset, $nu_1 \geq 1$. Therefore, 1 is the maximum budget to early withdrawing investors. The policy contingent budget available to early withdrawing investors is given as $G_1(p) = \max(1 - p, 0)$. The remaining investment in the asset accrues interest until $t = 2$. The budget to late withdrawing agents is given as $G_2(p) = H\,(1 - \min(nr_1, 1 - p))$. The budgets to early and late withdrawing agents are interdependent: As policy intensity p increases, the regulator tolerates fewer withdrawals until intervention, thus, the budget to early withdrawing agents $G_1(p)$ declines whereas the budget to agents that roll over, $G_2(p)$, increases. As I will explain next, this budget interdependence makes the suspension policy a harsh combination policy. To determine the payoff difference function: If the aggregate cash withdrawals remain below the policy dependent budget, $nr_1 \leq G_1(p)$, then no policy intervention occurs. In that case, investors who withdraw receive r_1, and the investors who roll over receive an equal share of the budget in $t = 2$, u_2 satisfies $(1-n)u_2(n,\theta) = G_2(p)$. Thus whenever $n \leq n_c(p)$ (no policy intervention "np"), the payoff difference equals

$$\upsilon_{np}(n,\theta) = \underbrace{p(\theta)\frac{H(1-nu_1)}{1-n}}_{u_2(n)} - \underbrace{r_1}_{u_1}. \tag{23}$$

If the cash withdrawals however reach or exceed the budget G_1, the regulator intervenes, stops the run, takes over control of the remaining assets, and continues the investment of the remaining asset share p at a reduced return $r \in (0, H)$ that is likewise subject to aggregate risk, $p(\theta)$. The regulator's reduced effectiveness in managing assets implies a costliness of intervention, which in return creates a jump of the payoff difference function in the intervention threshold n_c, see below. Withdrawals that would exceed budget G_1 are no longer served. Instead, these agents enter a regulatory procedure, a "mandatory deposit stay," jointly with the agents that rolled over. Under a mandatory deposit stay, all these investors share the proceeds of remaining investment. The proportion p of the asset that was protected by intervention matures, and yields a policy-dependent, risky pro rata share to agents under the mandatory stay $u_p = p(\theta)\,\frac{rp}{1-G_1(p)/r_1}$. where $G_1(p)/r_1 = 1 - n_c(p)$ is the share of depositors that may withdraw before policy intervention occurs. Conditional on policy intervention, $n > n_c(p)$, the payoff difference equals

$$\upsilon_p(n,\theta) = \underbrace{p(\theta)\,\frac{pr}{1-G_1(p)/r_1}}_{u_2} - \underbrace{\left(\frac{G_1(p)/r_1}{n}\, r_1 + \left(1 - \frac{G_1(p)/r_1}{n}\right)\, p(\theta)\,\frac{pr}{1-G_1(p)/r_1}\right)}_{u_1(n)} \tag{24}$$

where $\frac{G_1(p)/r_1}{n}$ is the probability that an investor who requests withdrawal is served the face value r_1 and thus does not enter the mandatory stay. The payoff difference conditional on intervention is always negative because for states for which withdrawal is not dominant, $\theta \in (\underline{\theta}, 1]$, it must hold $r_1 - \frac{p(\theta)\,pr}{1-G_1(p)/r_1} > 0$.[14]

7.3.1 Analysis

The budget interdependence creates a harsh policy combination: The intervention jump threshold $n_c(p)$ depends on and shifts in policy intensity p, and generically constitutes a discontinuity. By $r < H$, the payoff difference jumps down in $n = n_c$: $\lim_{n\nearrow n_c(p)} \upsilon_{np}(n,\theta) - \lim_{n\searrow n_c(p)} \upsilon_p(n,\theta) = p(\theta)\frac{(H-r)p}{1-\frac{1-p}{r_1}} > 0$. As the regulator tolerates fewer withdrawals until in-

[14]For all states in the lower dominance region $\theta \in [0, \underline{\theta})$ withdrawal (by definition) is dominant, meaning the payoff difference is negative for all realizations of n. For all states between the upper and lower dominance region $\theta \in [\underline{\theta}, \overline{\theta}]$ the sign of the payoff difference function depends on the realization of the aggregate withdrawals n. If for some $\theta \in (\underline{\theta}, 1]$ it held $r_1 - \frac{p(\theta)\,pr}{1-G_1(p)/r_1} < 0$, then also $r_1 - \frac{p(\theta)\,H(1-nr_1)}{1-n} < 0$ for all $n < n_c(p)$, contradicting that withdrawal is not dominant, see (??) for the construction of the lower dominance region for this example.

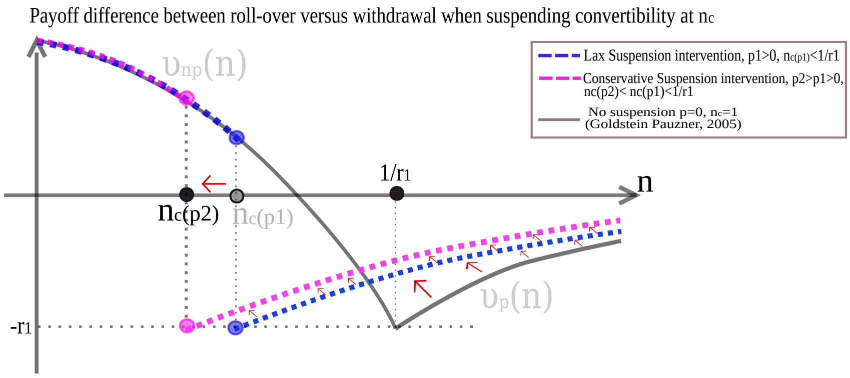


Figure 4: Assume the regulator suspends the convertibility of deposits as withdrawals exceed $n_c(p)$. The intervention causes a down-jump in $n_c(p)$ simultaneously to an upwards shift of the PI in the range $\mathcal{N}_p = [n_c(p), 1]$. As fewer withdrawals are tolerated, p increases from p_1 to p_2, corresponding to a lower intervention entry threshold $n_c(p_2) < n_c(p_1)$, and thus a lower jump point (adverse harsh), as well as an additional upwards shift of the PI on $\mathcal{N}_p = [n_c(p), 1]$ (prudent piecewise smooth).

tervention, p increases, and the down-jump point $n_c(p)$ declines (comes forward), implying an adverse harsh effect on bank stability via Proposition 5.2(ia). Simultaneously, a policy that tolerates fewer withdrawals acts prudent piecewise smoothly on the intervention interval $\mathcal{N}(p) = (n_c, 1]$ because it increases the budget $G_2(p)$ to investors that roll-over by lowering the budget available to investor that withdraw. Consequently, the payoff difference function $\upsilon_p(n, \theta)$ shifts upwards in p conditional on intervention.

Corollary 7.5 (Schilling (2019): Suspension of convertibility and receivership)
Assume a regulator sets a policy $p \in (0, 1)$ whereby it stops runs by suspending the convertibility of deposits if the cash withdrawals at the bank exceed the level $1 - p$, that is, for withdrawals above a threshold $n_c(p) \in (0, 1/r_1)$. Lowering the suspension entry threshold n_c constitutes harsh combination policy. If r is large and close to H, lowering the entry threshold improves stability ex ante. But if r is low, lowering the entry threshold can deteriorate stability ex ante.

Similar to an ELA provision, the suspension of convertibility is a policy that can backfire, and a policy that imposes withdrawal fees is the the safer policy with regard to assuring a positive impact on bank stability. The online appendix gives an additional prudent smooth policy example, namely partial deposit insurance where the intervention interval is the full range $\mathcal{N}(\gamma) = [0, 1]$.

8 Conclusion

This paper provides a general framework to analyze the effectiveness of policy interventions with regard to their capacity to prevent or ease runs on firms such as banks, money market mutual funds, or stablecoins. The paper establishes two different classes of policy based on how the policy acts on the investor's payoffs, "smooth" or "harsh", where both smooth and harsh policy can be of the type "prudent" or "adverse." Every real-world policy exhibits at least one of the four features and the according effect on payoffs. For each class and type I determine how it impacts the investors' ex ante run propensity and, thus, firm stability. The range of policies that lower bank stability ex ante is large and are either adverse smooth or adverse harsh, meaning either they lower the favorability of roll-over versus withdrawal gradually or in a way that gives rise to discontinuities in the withdrawal contingent relative payoff difference.

I then show that common policies such as emergency liquidity provision (ELA) by a lender of last resort, the imposition of withdrawal fees or the suspension of convertibility belong to multiple classes, and thus have mixed effects on stability. I show that if a policy belongs to multiple classes, and exhibits the according features, it can become ineffective with regard to improving stability since different features are equivalent to one another or can offset each other. Bailins can act like bailouts and can both improve or deteriorate stability. An ELA provision can lower stability, and the imposition of withdrawal fees is partially ineffective in lowering firm stability *because* it allows firms to survive greater runs, thus, acting like a bailout to withdrawing investors which is equivalent to a bail-in of investors that roll-over.

9 Appendix

9.1 Existence and Uniqueness of Equilibria (no jumps)

Proof. [Proposition 3.1] The proof largely follows **?**. I first show existence and uniqueness of a trigger equilibrium: Fix policy $p \geq 0$. Assume all investors follow the same strategy that maps signals to actions. Moreover, assume the investors follow a threshold strategy around θ^* (for sake of brevity, in this proof I suppress the dependence of θ^* on the policy p). Then the measure of agents that run at each state θ and threshold θ^* is deterministic and continuous in either argument,

$$n(\theta, \theta^*(p)) = \mathbb{P}(\theta_i < \theta^* | \theta) = \begin{cases} \frac{1}{2} + \frac{\theta^*(p) - \theta}{2\varepsilon}, & \theta \in [\theta^*(p) - \varepsilon, \theta^*(p) + \varepsilon] \\ 1, & \theta < \theta^*(p) - \varepsilon \\ 0, & \theta > \theta^*(p) + \varepsilon \end{cases} \tag{25}$$

Given a signal θ_i and threshold signal θ^*, an agent holds the following expectation over the payoff difference

$$H(\theta_i, n(\cdot)) = \frac{1}{2\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} (u_{2,p}(n(\theta, \theta^*), \theta) - u_{1,p}(n(\theta, \theta^*), \theta)) \, d\theta \tag{26}$$

The function $H(\theta_i, n(\cdot))$ is continuous in signal θ_i because by assumption 3.1, the payoff difference is Lebesgue integrable, because the functions $g_1(\theta_i) = \theta_i + \varepsilon$ and $g_2(\theta_i) = \theta_i - \varepsilon$ are continuous, because compositions of continuous functions are continuous, and because continuous functions on bounded intervals are bounded. By the same argument, an agent's expected payoff difference when observing the trigger signal θ^*,

$$H(\theta^*, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon} \int_{\theta^* - \varepsilon}^{\theta^* + \varepsilon} (u_{2,p}(n(\theta, \theta^*), \theta) - u_{1,p}(n(\theta, \theta^*), \theta)) \, d\theta, \tag{27}$$

is continuous in θ^*. Also, $H(\theta^*, n(\cdot, \theta^*))$ is strictly increasing in signal θ^*, as long as $\theta^* < \overline{\theta}_p + \varepsilon$, because for larger signals θ^* the expectation is taken over a higher range of fundamentals $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$. By assumption 2.1, the payoff difference is strictly increasing in θ for all $\theta \in [\underline{\theta}_p, \overline{\theta}_p]$, whereas the function $n(\theta, \theta^*)$ is evaluated at the same values due to a shift in the argument θ^* and the range of fundamentals. By the existence of an upper and lower dominance region, assumption 2.1, we know that $\upsilon_p(n(\theta, \theta^*), \theta^*) < 0$ for all $\theta \in [0, \underline{\theta}_p]$, whereas $\upsilon_p(n(\theta, \theta^*), \theta^*) > 0$ for all $\theta \in [\overline{\theta}_p, 1]$. Thus, $H(\theta^*, n(\cdot, \theta^*)) < 0$ for all $\theta^* \in [0, \underline{\theta}_p - \varepsilon]$ and $H(\theta^*, n(\cdot, \theta^*)) > 0$ for all $\theta^* \in [\overline{\theta}_p + \varepsilon, 1]$. Allover, because $H(\theta^*, n(\cdot, \theta^*))$ is continuous and strictly increasing in θ^*, is positive for high and negative for low values of θ^*, there must exist a unique threshold signal θ^* that satisfies

$$H(\theta^*, n(\cdot, \theta^*)) = 0 \tag{28}$$

To show that θ^* is an equilibrium, that is, $H(\theta_i, n(\cdot, \theta^*)) < 0$ for $\theta_i < \theta^*$ and $H(\theta_i, n(\cdot, \theta^*)) > 0$ for $\theta_i > \theta^*$, the proof in **?** applies. Using an interval decomposition, they show that for $\theta_i < \theta^*$, it must follow $H(\theta_i, n(\cdot, \theta^*)) < 0$ because this expected value is taken over a lower range of fundamentals than the expected value $H(\theta^*, n(\cdot, \theta^*))$, and because the payoff difference function satisfies single-crossing in n by assumption 2.1. Last, it remains to show that there exist no non-threshold equilibria. By assumption 2.1 and 3.1, $\upsilon_p(n, \theta)$ is strictly increasing in $\theta \in [\underline{\theta}_p, \overline{\theta}_p]$, is strictly decreasing in n whenever positive, and satisfies single-crossing. Therefore, the proof in (**?**) applies. □

9.2 Comparative statics under smooth intervention

Proof. [Proposition 4.1] By Proposition 3.1, for given $p > 0$ there exists a unique equilibrium trigger θ^* which is implicitly defined as the zero to

$$H(p, \theta^*) \equiv \int_0^1 \upsilon_p(n, \theta(n, \theta^*))\, dn = 0 \tag{29}$$

For sake of brevity, I suppress the dependence of θ^* on the policy p. The implicit function theorem delivers how θ^* changes as a function of p. By assumption, $\upsilon_p(n, \theta)$ is increasing in the state θ while $\theta(n, \theta^*)$ is strictly increasing in θ^*. Thus, $\frac{\partial H}{\partial \theta^*} > 0$. Next, since $\upsilon_p(n, \theta)$ is continuous in n, the boundary derivatives are zero, and we have $\frac{\partial H}{\partial p} = \int_0^1 \frac{\partial}{\partial p} \upsilon_p(n, \theta(n, \theta^*))\, dn = \int_{n \in \mathcal{N}(p)} \frac{\partial}{\partial p} \upsilon_p(n, \theta(n, \theta^*))\, dn$ which is positive under a prudent and negative under an adverse smooth policy. Altogether, $\frac{\partial \theta^*}{\partial p} = -(\frac{\partial H}{\partial p})/(\frac{\partial H}{\partial \theta^*}) < 0$ if and only if the policy is prudent. □

9.3 Equilibrium Existence and uniqueness with jumps

Proof. [Proposition 5.1] To show existence and uniquenss of a trigger equilibrium, assume again that all investors follow the same strategy that maps signals θ_i to actions. Assume that investors follow a threshold strategy around θ^*. Then the measure of agents that run at each state, $n(\theta, \theta^*)$ is deterministic. Observe that $n(\theta, \theta^*)$ is at one for $\theta < \theta^* - \varepsilon$, because all agents oberve signals below the trigger signal and withdraw. Further, $n(\theta, \theta^*)$ is strictly decreasing in state θ for

$\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$, and attains zero for $\theta > \theta^* + \varepsilon$. Therefore, as θ increases in $[\theta^* - \varepsilon, \theta^* + \varepsilon]$, n transitions through all jump points $n_1, \dots, n_k$ of the payoff difference function.

Consider the inverse of $n(\theta, \theta^*)$, $\theta(n, \theta^*)$, as given in (4). Let $\theta_1, \dots, \theta_k$ the states for which $n(\theta, \theta^*)$ attains the jump points, that is, $\theta_1 = \theta(n_1, \theta^*), \dots, \theta_k = \theta(n_k, \theta^*)$. In this proof, I call these states the "jump-states", and address them using the subscript θ_j, not to be confused with signal θ_i. Note, due to $n_1 < \dots < n_{k-1} < n_k$, I have $\theta_k < \theta_{k-1} < \dots < \theta_1$. Set $\theta_0 = 1$ and $\theta_{k+1} = 0$. Note that in a trigger equilibrium around θ^*, it holds that $\theta_1, \dots, \theta_k \in (\theta^* - \varepsilon, \theta^* + \varepsilon)$ because $n(\theta, \theta^*)$ is continuous and because $n(\theta^* - \varepsilon, \theta^*) = 1$, $n(\theta^* + \varepsilon, \theta^*) = 0$. Then $[0, 1] = \cup_{j=0}^{k} [\theta_{j+1}, \theta_j]$, and for every signal θ_i and $\varepsilon > 0$, it holds $[\theta_i - \varepsilon, \theta_i + \varepsilon] \subset \cup_{j=0}^{k} [\theta_{j+1}, \theta_j]$. I want to partition the interval $[\theta_i - \varepsilon, \theta_i + \varepsilon]$ by the jump states it contains, by considering $[\theta_i - \varepsilon, \theta_i + \varepsilon] \cap \left(\cup_{j=0}^{k} [\theta_{j+1}, \theta_j]\right)$. Let $n \in \{0, 1, \dots, k\}$ the number of jump states contained in the interval $[\theta_i - \varepsilon, \theta_i + \varepsilon]$. If $n = 0$, then there exists no partition by jump points and I write $[\theta_i - \varepsilon, \theta_i + \varepsilon] \cap \left(\cup_{j=0}^{k} [\theta_{j+1}, \theta_j]\right) = [\theta_i - \varepsilon, \theta_i + \varepsilon]$.

If $n \geq 1$, I address the jump states in this interval directly by calling them $\theta_{j_1}, \dots \theta_{j_n}$, where θ_{j_1} is the smallest one among them, and thus, θ_{j_n} the largest, and where because of the reverse numbering of the jump states, it holds $j_1 \leq k$ and $j_n \geq 1$. This yields a partition of $[\theta_i - \varepsilon, \theta_i + \varepsilon]$ according to $[\theta_i - \varepsilon, \theta_i + \varepsilon] \cap \left(\cup_{j=0}^{k} [\theta_{j+1}, \theta_j]\right) = [\theta_i - \varepsilon, \theta_{j_1}] \cup [\theta_{j_1}, \theta_{j_2}] \cup \dots \cup [\theta_{j_n}, \theta_i + \varepsilon]$.

By assumption 5.1, the payoff difference function is continuous on all open intervals $[\theta_i - \varepsilon, \theta_{j_1})$, $(\theta_{j_1}, \theta_{j_2})$, $\dots (\theta_{j_n}, \theta_i + \varepsilon]$. Further by assumption 5.1, the right and left sided limits of the payoff difference function exist at each jump state θ_{j_i}, $i = 1, \dots m$,

$$|\lim_{\theta \nearrow \theta_{j_i}} \upsilon_p(n(\theta, \theta^*), \theta)| = |\lim_{n \searrow n_{j_i} \equiv n(\theta_{j_i}, \theta^*)} \upsilon_p(n, \theta(n, \theta^*))| =: c_{i,r} < \infty \tag{30}$$

$$|\lim_{\theta \searrow \theta_{j_i}} \upsilon_p(n(\theta, \theta^*), \theta)| = |\lim_{n \nearrow n_{j_i} \equiv n(\theta_{j_i}, \theta^*)} \upsilon_p(n, \theta(n, \theta^*))| =: c_{i,l} < \infty \tag{31}$$

Given a signal θ_i, the true state must be located in $[\theta_i - \varepsilon, \theta_i + \varepsilon]$. If this interval contains jump states, $n \geq 1$, an agent's expected payoff difference to roll over versus withdraw when observing signal θ_i can therefore be rewritten as

$$H(\theta_i, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon} \left(\int_{\theta_i - \varepsilon}^{\theta_{j_1}} \upsilon_p(n(\theta, \theta^*), \theta)\, d\theta + \int_{\theta_{j_1}}^{\theta_{j_2}} \upsilon_p(n(\theta, \theta^*), \theta)\, d\theta + \dots + \int_{\theta_{j_n}}^{\theta_i + \varepsilon} \upsilon_p(n(\theta, \theta^*), \theta)\, d\theta \right) \tag{32}$$

If an investor observes the trigger signal $\theta_i = \theta^*$, the interval of possible states $[\theta^* - \varepsilon, \theta^* + \varepsilon]$ contains all jump states, $n = k$, and her expected payoff difference equals

$$H(\theta^*, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon} \left(\int_{\theta^* - \varepsilon}^{\theta_{j_1}} \upsilon_p(n(\theta, \theta^*), \theta)\, d\theta + \int_{\theta_{j_1}}^{\theta_{j_2}} \upsilon_p(n(\theta, \theta^*), \theta)\, d\theta + \dots + \int_{\theta_{j_k}}^{\theta^* + \varepsilon} \upsilon_p(n(\theta, \theta^*), \theta)\, d\theta \right) \tag{33}$$

I first argue, there exists a unique θ^*, that satisfies $H(\theta^*, n(\cdot, \theta^*)) = 0$. To see that, note that $H(\theta^*, n(\cdot, \theta^*))$ is strictly increasing in θ^* for $\theta^* < \overline{\theta} + \varepsilon$, because by assumption 2.1 $\upsilon_p(n(\theta, \theta^*), \theta)$ is non-decreasing and is strictly increasing in θ for $\theta \in [\underline{\theta}_p, \overline{\theta}_p]$. Further, $H(\theta^*, n(\cdot, \theta^*)) > 0$ for $\theta^* \in [\overline{\theta}_p + \varepsilon, 1]$, and $H(\theta^*, n(\cdot, \theta^*)) < 0$ for $\theta^* \in [0, \underline{\theta}_p - \varepsilon]$. Last,

Lemma 9.1. *$H(\theta^*, n(\cdot,\theta^*))$ is continuous in θ^**

Because $H(\theta^*, n(\cdot,\theta^*))$ is strictly increasing and continuous in θ^*, exceeding 0 for high values of θ^* and undercutting 0 for low values of θ^*, there exists a unique θ^* with $H(\theta^*, n(\cdot,\theta^*)) = 0$, the candidate for a trigger equilibrium.

It remains to show that θ^* is an equilibrium. That is, one needs to show that for all signals $\theta_i < \theta^*$ it follows $H(\theta_i, n(\cdot,\theta^*)) < 0$ whereas for all $\theta_i > \theta^*$ it follows $H(\theta_i, n(\cdot,\theta^*)) > 0$. By assumption 2.1, $\upsilon_p(n,\theta)$ is positive for high values of θ, negative for low values of θ, and satisfies single-crossing. Therefore, for this part, the existence proof on page 1313 in **?** also applies here. They show, if $\theta_i < \theta^*$, then $H(\theta_i, n(\cdot,\theta^*)) < 0 = H(\theta^*, n(\cdot,\theta^*))$. This holds because $\upsilon_p(n,\theta)$ is positive for high values of θ, negative for low values of θ, crosses zero only once, and because agent i forms expectations about the payoff difference over a lower range of fundamentals than for $\theta_i = \theta^*$. Likewise for $\theta_i > \theta^*$. Allover, there exists a unique threshold equilibrium around trigger θ^*.

No non-threshold equilibria

It remains to show that there are no non-threshold equilibria. I follow the notation in **?**: A mixed strategy for investor i is a measurable function $s_i : [-\varepsilon, 1+\varepsilon] \to [0,1]$ that maps the investor's private signal into a probability to withdraw. A strategy profile is then denoted by $\{s_i\}_{i\in[0,1]}$. A state realization θ generates random signals $\theta_i = \theta + \varepsilon_i$ in the range $[\theta-\varepsilon, \theta+\varepsilon]$. The signals jointly with the strategy profile $\{s_i\}_{i\in[0,1]}$ generate the aggregate withdrawals $\tilde{n}(\theta)$ at state θ which is a random variable. For a given state θ, define the cumulative distribution function of $\tilde{n}(\theta)$ as

$$F_\theta(n) = \mathbb{P}(\tilde{n}(\theta) \le n|\theta) = \mathbb{P}\left(\int_{i\in[0,1]} s_i(\theta+\varepsilon_i)di \le n|\theta\right) \tag{34}$$

where the probability is measured with respect to the signal noise distribution $\varepsilon_i \sim U[-\varepsilon, \varepsilon]$. An investor's expected payoff difference when observing signal θ_i and given a strategy profile $\{s_i\}_{i\in[0,1]}$ can, via the law of iterated expectation, be written as

$$H(\theta_i, \tilde{n}(\cdot)) = \frac{1}{2\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon}\left(\int_0^{n_1} \upsilon(\theta,n)\,dF_\theta(n) + \cdots + \int_{n_k}^{1} \upsilon(\theta,n)\,dF_\theta(n)\right)d\theta \tag{35}$$

where $n_1, \ldots n_k$ are the jump points of $\upsilon(\theta,n)$ in the aggregate withdrawals n, and where the inner integrals of (35) are well-defined Lebesgue-Stieltjes integrals by assumption 5.1. The non-existence proof in **?** fully applies, because

Lemma 9.2. *$H(\theta_i, \tilde{n}(\cdot))$ is continuous in signal θ_i*

and because by the assumptions 2.1, and 5.2, the payoff difference function $\upsilon(n,\theta)$ satisfies single-crossing in n. Moreover, $\upsilon(n,\theta)$ is strictly decreasing in n whenever positive in the sense of assumption 5.2 and because $\upsilon(n,\theta)$ strictly increases in the state for state realizations in $[\underline{\theta}, \overline{\theta}]$. The proofs to Lemmata 9.1 and 9.2 can be found in the online appendix. □

10 Supplementary Appendix

10.1 Additional Applications

10.1.1 Prudent smooth policy intervention via providing and raising partial Deposit Insurance (Guarantee)

The next regulatory policy I discuss is the provision of an increasing share of deposit insurance. I show, raising the partial deposit insurance provision constitutes prudent smooth intervention, thus raising bank stability ex ante by Proposition 4.1. I consider partial insurance because if insurance is full there is no policy parameter to alter.[15] The following example revisits **?** for the special case where there is no suspension of convertibility $a = 1$ (laissez-faire) but where the regulator provides partial deposit insurance, described by the share $\gamma \in (0, 1)$. The resulting model is essentially the just-described **?** model, enriched by a partial deposit guarantee. The example nests the risk-neutral version of the **?** model when setting $\gamma = 0$.

Assume, deposits are insured up to the amount $\gamma \in [0, 1)$, $\gamma \leq r_1$. Insurance alters the depositors' payoffs in the following way in comparison to the benchmark: In the case of a bank run $n \geq 1/r_1$, the depositors who roll over receive a positive payoff $\gamma \geq 0$, and the depositors who withdraw receive the face value r_1 with probability $\frac{1}{nr_1}$ (early in the queue) and receive the insured fraction γ with probability $1 - \frac{1}{nr_1}$ (late in the queue). Absent a run, if the asset does not pay off then the deposit insurance repays the depositors the insured share of their deposit.

To pin down payoffs, for a given state realization $\theta \in [0, \overline{\theta})$[16], and in the case where the bank remains liquid (L) in $t = 1$, $n < 1/r_1$, the payoff difference between roll-over and withdrawal equals

$$v_L(n, \gamma) = \underbrace{\left(p(\theta) \max\left(\frac{R(1 - nr_1)}{1 - n}, \gamma\right) + (1 - p(\theta)) \times \gamma\right)}_{u_2(n,\theta)} - \underbrace{r_1}_{u_1} \tag{36}$$

In the case where the bank becomes illiquid (Ill), $n \geq 1/r_1$, the payoff difference becomes

$$v_{Ill}(n, \gamma) = \underbrace{\gamma}_{u_2} - \underbrace{\left(\frac{1}{nr_1} \times r_1 + \left(1 - \frac{1}{nr_1}\right) \times \gamma\right) = \frac{1}{nr_1}(\gamma - r_1)}_{u_1(n)} \tag{37}$$

The payoff difference function is continuous in n for every insurance choice $\gamma \in [0, 1)$. Thus, the provision of partial deposit insurance constitutes smooth policy intervention. Further, increasing the share of deposit insurance provision γ constitutes prudent smooth intervention:

In the liquid case, $n < 1/r_1$, it holds $\frac{\partial}{\partial \gamma} v_L(n, \gamma) > 0$. Similarly, in the illiquid case, $n \geq 1/r_1$,

[15] Considering partial insurance is reasonable, because from different models we know that full insurance does not lead to efficient allocations due to moral hazard because depositors stop monitoring the bank (**?**) or because of inefficient continuation of investment because depositors liquidate the bank too seldom (**??**). The literature that analyzes the economics of deposit insurance is large, and the example here serves to provide one example where deposit insurance acts smoothly on payoffs. For a different analysis of partial insurance, see **?** who analyze optimal insurance provision in the case of asymmetric deposits and lump-sum deposit insurance in a **?** model.

[16] For states in $[\overline{\theta}, 1]$ all depositors roll over because this is the dominant action, see (**?**). We therefore exclude these states from the analysis here.

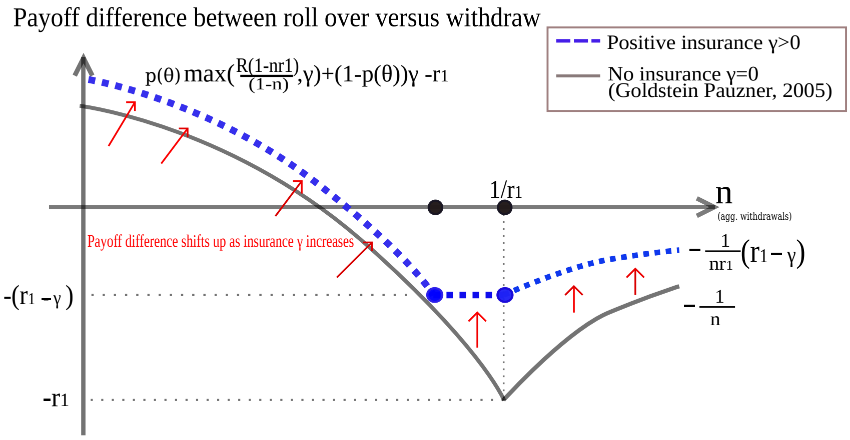


Figure 5: The payoff difference function $\upsilon(n)$ shifts up the more insurance coverage γ is provided.

$\frac{\partial}{\partial\gamma}\upsilon_{Ill}(n,\gamma) > 0$. Allover, $\frac{\partial}{\partial\gamma}\upsilon(n,\gamma) > 0$ for all $n \in [0,1]$, and the intervention interval is given as $\mathcal{N}_\gamma = [0,1]$. As a Corollary of Proposition 4.1(i), I obtain:

Corollary 10.1 (Raising partial deposit insurance is prudent smooth policy)
An increase of partial deposit insurance $\gamma \in [0,1)$ constitutes prudent smooth policy intervention. In the unique equilibrium, ex ante bank stability improves in the guaranteed share $\gamma \in (0,1)$.

10.2 Proofs of Lemmata

Proof. [Proof Lemma 9.1] Consider two triggers θ_x^* and θ_y^*. Without loss of generality, $\theta_x^* < \theta_y^*$, and I can write $\theta_y^* = \theta_x^* + d$, $d > 0$. I want to show: $\lim_{d \to 0} H(\theta_y^*, n(\cdot, \theta_y^*)) = H(\theta_x^*, n(\cdot, \theta_x^*))$. As the state θ increases in $[\theta^* - \varepsilon, \theta^* + \varepsilon]$, the function $n(\theta, \theta^*)$ crosses all jump points $n_1, \ldots n_k$. The according jump states, however, depend on the trigger θ^*: By $\theta_x^* < \theta_y^*$, we have $n(\theta, \theta_x^*) \leq n(\theta, \theta_y^*)$. Because we require for all jump points $j = 1, \ldots k$

$$n(\theta_j^x, \theta_x^*) = n_j = n(\theta_j^y, \theta_x^*), \tag{38}$$

and because $n(\theta, \theta_x^*)$ is increasing in the trigger but decreasing in the state it follows $\theta_j^x < \theta_j^y$ for all $j = 1, \ldots k$. Further, note that $n(\theta_j^x, \theta_x^*) = n(\theta_j^y, \theta_x^*)$ implies that $\theta_x^* - \theta_y^* = \theta_j^x - \theta_j^y$ for all j. That is, $\theta_y^* = \theta_x^* + d$ implies $\theta_j^y = \theta_j^x + d$. Therefore,

$$2\varepsilon\, H(\theta_y^*, n(\cdot, \theta_y^*)) = \int_{\theta_y^* - \varepsilon}^{\theta_{j_1}^y} \upsilon(\theta, n(\theta, \theta_y^*)) d\theta + \cdots + \int_{\theta_{j_k}^y}^{\theta_y^* + \varepsilon} \upsilon(\theta, n(\theta, \theta_y^*)) d\theta \tag{39}$$

$$= \int_{\theta_x^* + d - \varepsilon}^{\theta_{j_1}^x + d} \upsilon(\theta, n(\theta, \theta_y^*)) d\theta + \cdots + \int_{\theta_{j_k}^x + d}^{\theta_x^* + d + \varepsilon} \upsilon(\theta, n(\theta, \theta_y^*)) d\theta \tag{40}$$

$$= \int_{\theta_x^* - \varepsilon}^{\theta_{j_1}^x} \upsilon(\theta + d, n(\theta + d, \theta_y^*)) d\theta + \cdots + \int_{\theta_{j_k}^x}^{\theta_x^* + \varepsilon} \upsilon(\theta + d, n(\theta + d, \theta_y^*)) d\theta \tag{41}$$

$$= \int_{\theta_x^* - \varepsilon}^{\theta_{j_1}^x} \upsilon(\theta + d, n(\theta, \theta_x^*)) d\theta + \cdots + \int_{\theta_{j_k}^x}^{\theta_x^* + \varepsilon} \upsilon(\theta + d, n(\theta, \theta_x^*)) d\theta \tag{42}$$

where the last step follows from $n(\theta + d, \theta_y^*) = n(\theta + d, \theta_x^* + d) = n(\theta, \theta_x^*)$. Therefore,

$$|H(\theta_x^*, n(\cdot, \theta_x^*)) - H(\theta_y^*, n(\cdot, \theta_y^*))| \tag{43}$$

$$= \frac{1}{2\varepsilon} \Big| \int_{\theta_x^* - \varepsilon}^{\theta_{j_1}^x} \left(\upsilon(\theta, n(\theta, \theta_x^*)) - \upsilon(\theta + d, n(\theta, \theta_x^*))\right) d\theta \tag{44}$$

$$+ \cdots + \int_{\theta_{j_k}^x}^{\theta_x^* + \varepsilon} \left(\upsilon(\theta, n(\theta, \theta_x^*)) - \upsilon(\theta + d, n(\theta, \theta_x^*))\right) d\theta \Big| \tag{45}$$

$$\leq \frac{1}{2\varepsilon} \Big(\int_{\theta_x^* - \varepsilon}^{\theta_{j_1}^x} |\upsilon(\theta, n(\theta, \theta_x^*)) - \upsilon(\theta + d, n(\theta, \theta_x^*))|\, d\theta \tag{46}$$

$$+ \cdots + \int_{\theta_{j_k}^x}^{\theta_x^* + \varepsilon} |\upsilon(\theta, n(\theta, \theta_x^*)) - \upsilon(\theta + d, n(\theta, \theta_x^*))|\, d\theta \Big) \tag{47}$$

The payoff difference function $\upsilon(\theta, n(\theta, \theta_x^*))$ is continuous between the jump points, implying $\lim_{d \to 0} |\upsilon(\theta, n(\theta, \theta_x^*)) - \upsilon(\theta + d, n(\theta, \theta_x^*))| = 0$. Moreover, the payoff difference function is bounded by assumption 5.1. Thus, $|H(\theta_x^*, n(\cdot, \theta_x^*)) - H(\theta_y^*, n(\cdot, \theta_y^*))| \to 0$ as $d \to 0$ by Lebesgue's dominated convergence theorem. □

Proof. [Proof Lemma 9.2] To show continuity of $H(\theta_i, \tilde{n}(\cdot))$ in signal θ_i, I show that for $h > 0$, $\lim_{h \to 0} |H(\theta_i + h, \tilde{n}(\cdot)) - H(\theta_i, \tilde{n}(\cdot))| = 0$. Observe that for a small $h > 0$, the intervals

$[\theta_i - \varepsilon, \theta_i + \varepsilon]$ and $[\theta_i + h - \varepsilon, \theta_i + h + \varepsilon]$ overlap. Therefore,

$$
\begin{aligned}
&H(\theta_i + h, \tilde{n}(\cdot)) - H(\theta_i, \tilde{n}(\cdot)) \qquad (48)\\
&= \frac{1}{2\varepsilon} \int_0^1 (\mathbf{1}_{\{\theta \in [\theta_i + h - \varepsilon, \theta_i + h + \varepsilon]\}} - \mathbf{1}_{\{\theta \in [\theta_i - \varepsilon, \theta_i + \varepsilon]\}}) \left(\int_0^{n_1} \upsilon(\theta, n)\, dF_\theta(n) + \cdots + \int_{n_k}^1 \upsilon(\theta, n)\, dF_\theta(n) \right) d\theta \\
&= \frac{1}{2\varepsilon} \int_0^1 (\mathbf{1}_{\{\theta \in [\theta_i + \varepsilon, \theta_i + h + \varepsilon]\}} - \mathbf{1}_{\{\theta \in [\theta_i - \varepsilon, \theta_i + h - \varepsilon]\}}) \left(\int_0^{n_1} \upsilon(\theta, n)\, dF_\theta(n) + \cdots + \int_{n_k}^1 \upsilon(\theta, n)\, dF_\theta(n) \right) d\theta
\end{aligned}
$$

where I have used that on $[\theta_i - \varepsilon, \theta_i + \varepsilon] \cap [\theta_i + h - \varepsilon, \theta_i + h + \varepsilon]$ the indicator functions cancel out to zero. For every state θ, the Lebesgue-Stieltjes integrals $\left(\int_0^{n_1} \upsilon(\theta, n)\, dF_\theta(n) + \cdots + \int_{n_k}^1 \upsilon(\theta, n)\, dF_\theta(n) \right)$ exist, that is, are bounded by assumption 5.2. Further, as $h \to 0$, it holds $\mathbf{1}_{\{\theta \in [\theta_i + \varepsilon, \theta_i + h + \varepsilon]\}} \to \mathbf{1}_{\{\theta \in [\theta_i + \varepsilon, \theta_i + \varepsilon]\}} = 0$ almost everywhere. Likewise, for $h \to 0$, $\mathbf{1}_{\{\theta \in [\theta_i - \varepsilon, \theta_i + h - \varepsilon]\}} \to \mathbf{1}_{\{\theta \in [\theta_i - \varepsilon, \theta_i - \varepsilon]\}} = 0$ almost everywhere. Thus, $|\mathbf{1}_{\{\theta \in [\theta_i + \varepsilon, \theta_i + h + \varepsilon]\}} - \mathbf{1}_{\{\theta \in [\theta_i - \varepsilon, \theta_i + h - \varepsilon]\}}| \to 0$, and $|\mathbf{1}_{\{\theta \in [\theta_i + \varepsilon, \theta_i + h + \varepsilon]\}} - \mathbf{1}_{\{\theta \in [\theta_i - \varepsilon, \theta_i + h - \varepsilon]\}}| \leq 1$. Thus, by Lebesgue's dominated convergence theorem, $\lim_{h \to 0} |H(\theta_i + h, \tilde{n}(\cdot)) - H(\theta_i, \tilde{n}(\cdot))| = 0$. $\square$