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Providing Incentives with Private Contracts

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Abstract

Agents working together to produce a joint output care about each other's incentives. Because real world contracts are typically private information, observed only by their direct signatories, agents are vulnerable to the principal opportunistically reducing the power of other agents' incentives. When agents are sufficiently skilled, the principal can mitigate this commitment problem by making the most skilled one "team-leader," with authority to write other agents' contracts. This endogenous hierarchy, never optimal with public contracts, raises effort, output, and compensation, but distorts effort allocation due to rent extraction. Our model applies to bank syndicates, venture capital, organizational design, and outsourcing.

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1 Introduction

Teamwork is pervasive in modern economies. When multiple agents work together on a project, they care about how much effort each of them will put in, and hence about the incentives that each of them received from the principal. Yet directly observing the incentives of other agents working on the same project is often not possible, because in many settings, contracts are bilateral and seen only by the parties signing them.[1] The privacy of contracts makes teamwork more difficult to incentivize. In this paper, we show that contractual privacy has important consequences for the way in which team projects are organized. In particular, settings with private contracting are likely to be more hierachical than they would be were contracts public.[2] Our results can be used to study, including to pay transparency and hierarchy within an organization; the delegation of control over incentive structures in market-based settings such as investment banking syndicates, and venture capital partnerships; and to the outsourcing or subcontracting of production decisions.

We consider a model in which efforts are complementary in production, so each agent needs to be sure that the other team members have strong incentives in order to find it worthwhile to put in high effort himself.[3] The principal retains the profit from the project that is not used to pay agents, and this, coupled with the privacy of contract offers, creates a commitment problem. The principal would like to promise each agent that she will provide high bonuses to the other agents because, due to the complementarity of effort, each agent will work harder, knowing that the others are also working hard. But the principal can always privately renege on such cheap talk when compensation contracts are private and not contingent on the incentives provided to other agents on the team. Each agent rationally expects that the principal will behave opportunistically, thus making it more expensive for the principal to incentivize agents in teams than if bonus structures were transparent.

We can make a preliminary observation, therefore, that private contracting will lead to suboptimal pay and effort provision by agents as compared to the second-best when contracts are public. We then

[1]While compensation contracts can in principle be verified in a court of law, such verification is costly, particularly for third parties. Even though it might be possible to make contracts public on some occasion, contracting parties could always renegotiate privately afterwards (Aghion, Dewatripont, and Rey (1994)). Recent evidence suggests that the majority of employees do not know how much money their peers make, nor do they know the compensation budget offered to their boss (IWPR (2017); Cullen and Perez-Truglia (2020)).

[2]In this paper, we take the privacy of contracts as given and focus on organizational design. For recent contributions rationalizing the privacy of contracts within a given organizational structure, see Cullen and Pakzad-Hurson (2019) and Halac, Lipnowski, and Rappoport (2021).

[3]The privacy of contracts will also matter when agents' efforts are substitutes. We choose to work with complementarities, both for tractability, and because we think that positive externalities are a realistic feature of many interesting team-based settings. Gryglewicz and Mayer (2022) study a dynamic model where a financial intermediary and a firm make substitute efforts. DeMarzo and Kaniel (2021) study a model with individual outputs where agents care about each others' compensation relative to their own, and their effort exerts negative externalities on other agents.

turn our attention to features of the real world environment that mitigate this problem. In particular, the contracting parties observe the bonus contract—so, if the principal were to delegate contracting to one of the agents, then that agent-delegate would observe the bonus provided to the other agent (the "sub-agent"). While delegation of contracting does not in itself affect how many contracts are observed within an organization (since we assume that all contracting remains bilateral), it does affect who observes those contracts. The impact of delegation on the distribution of information about compensation can be beneficial to the principal: it improves the transparency of incentives to the agents making effort choices.

Our first main result is that delegating contracting to one of the agents, rather than keeping it centralized in the principal's hands, always raises total compensation and thus helps to restore incentives. The reason for the improvement in incentives is precisely that it allows transparency of contracting in the places where it is most important: between agents working together on a joint project. It allows one of the agents to observe the other's (steep) incentives, with positive feedback effects on effort. However, the problem with delegated contracting is that when the principal relinquishes control of incentive provision, providing only a "budget" for total compensation without stipulating its distribution, this leads to skewed incentives. Those who now have responsibility for distributing the compensation budget extract excessive rents, resulting in the power of incentives being more unequal than would be optimal in a second-best world. This means that the ability to use delegation to commit to stronger incentives is beneficial for the principal only when agents are skilled enough, and similar enough. In this case, on the one hand, effort complementarities are most important and fear of expropriation is most damaging, and on the other, rent extraction is most limited. A subsidiary result here is that if agents are heterogeneous, the principal should put the more skilled agent in charge of determining the allocation of the compensation budget, making him the "team leader", so that the excessive incentives are paid out to the agent whose effort responds more strongly to them.

Our second, and perhaps most surprising, main result, is that increasing transparency is not the only factor driving the principal's decision to delegate contracting. We show that delegation of contracting can still be an optimal choice for the principal *even when the observability of contracts is deliberately held constant between delegated and centralized structures*. To this end, we let the contract that is observable under delegation (i.e., the contract between the team leader and the team member) be also observable under centralization of contracting. The reason why delegation can still be optimal in this case is subtle. Although the two agents now observe the same contracts, transparency remains incomplete, so the principal still suffers from a commitment problem related to the contract of the agent which remains private. The commitment problem arising from this incomplete transparency results in skewed pay in centralized as well as delegated settings, but the skew is different, and sometimes better for the principal, with delegation. Thus, delegation is beneficial not only because it improves transparency (our first result) but also because it sometimes mitigates the deleterious impact of remaining opacity (our second result).

The intuition for our second result is as follows. When there are complementarities between agents, incentive contracts serve a dual purpose—they not only directly motivate the recipient, but they also indirectly motivate the teammates of the recipient, who work harder because their effort is complementary to the effort of the agent receiving the public bonus. When contracting is centralized, other things being equal, the principal sets higher pay for agents whose bonuses are publicly observed, because these bonus serve both purposes. By contrast, when compensation choices are delegated, the agent making those choices will, because of his inevitable self-seeking behavior, receive higher pay. This skew in incentives is better for the principal when the principal is able to put the more skilled agent in charge of contracting, as long as that agent's subordinate is sufficiently skilled and sufficiently similar to the agent-delegate himself. Rent-seeking by the latter will be then be mitigated, and moreover, the pay skew is in the principal's preferred direction (towards the more skilled agent). So, in a world where contracts are not fully transparent, delegation of contracting can be an optimal response even when it does not affect which or how many contracts are publicly observed by agents.

Our theory applies to any setting in which different agents must work together on a joint output and contracts are imperfectly observed. This includes many prominent economic and financial applications: teamwork within organizations; the outsourcing of projects by corporations; venture capital; investment banking syndicates for IPOs and bond underwriting; and loan syndications. Interestingly, many of these applications feature hierarchies, which are at least partly the choice of those involved. Corporations can choose whether to employ a team in house to undertake a project (which would correspond to centralized contracting, where all agents are directly compensated by the principal) or to outsource a project (corresponding to delegation, where the principal deals with only one agent, who contracts with all the other agents). Firms running a "bake off" for a desirable IPO or loan syndication can choose whether to have a single bookrunner or lead bank, which will then be allowed to select and compensate the other participating banks out of the agreed spread, or to insist on multiple lead managers. While there are surely many reasons other than the privacy of contracts bearing on why these various settings feature hierarchies,[4] our theory helps us to understand in each case the implications of the hierarchy for the incentives of teams which are organized in this way.

[4]There is a large theoretical literature exploring the causes and consequences of delegation within organizations, which we survey in section 8 below. However, this literature has not, to our knowledge, touched on the question of why or under what circumstances incentive or compensation decisions should be delegated. The outsourcing literature, discussed in section 7, has also left aside the specific issue of contract pay-delegation because it mostly assumes a two-level production process where compensation is determined by ex post Nash bargaining. The finance literature, discussed in the same section, has almost universally taken the hierarchical structure as given.

In particular, if the principal at the top (investor, issuing firm, ...) delegates the power to make contracting decisions down the hierarchy to a lead bank or general partner, there will be both a cost and a benefit. The benefit is that incentives for the agent to whom the contracting has been delegated will certainly increase, leading to an increase in that agent's effort beyond what could be attained without delegation. The cost is that incentives for agents further down the hierarchy will be lower than the principal would desire — because of rent extraction by the agent who is "middleman" — and may or may not be higher than they would be if the principal were able to contract directly with these sub-agents. The benefits of such delegation or subcontracting are likely to offset the costs as long as the skills of the agents in the hierarchy are high, similar to one another, and their efforts are complementary, which seems to be true of many of the settings described above.

The remainder of the paper is organized as follows. Section 2 presents the economic setup. Section 3 solves for the optimal contract under centralized contracting, while Section 4 does the same under delegated contracting. In Section 5 we compare the two contracting schemes and find conditions under which it is optimal to delegate to one agent the authority to compensate the other agent. Section 6 studies optimal delegation when the observability of contracts is kept constant across contracting schemes (i.e., only one compensation contract is publicly observable with centralized or delegated contracting). In Section 7 we discuss financial and other applications of our theory. We defer our discussion of related literature to Section 8, before concluding in Section 9. Appendix A contains the proofs, while the Online Appendices B and C present the derivation of the optimal contracts in a centralized contracting scheme with two or one observable contracts, respectively.

2 Economic Setting

We consider an economy with two dates. At date 0, a principal makes an investment in a risky project and needs to hire two agents to implement it. The two agents can individually exert effort to increase the project's expected output, which is realized at date 1. The effort that each agent $i = 1, 2$ exerts at date 0, denoted by e_i for $i = 1, 2$, is *unobservable* to the other agent and to the principal. All three players in this economy are risk-neutral and have limited liability.

Technology. We denote the output of the risky project by X and we assume that it follows a Bernoulli distribution. The project either succeeds or fails. The probability of success of the project, denoted by π, is affected by the effort choices of the two agents:

$$X(e_1, e_2) = \begin{cases} 1 & \text{with prob.} \quad \pi(e_1, e_2) \\ 0 & \text{with prob.} \quad 1 - \pi(e_1, e_2). \end{cases} \tag{1}$$

Given the realized output in the two states, $\pi(e_1, e_2)$ coincides with the expected output of the project, $\mathbb{E}[X(e_1, e_2)]$.

Effort. For tractability, we model the probability of success π as a Cobb-Douglas function of the two agents' effort choices:

$$\pi(e_1, e_2) = e_1^{\alpha_1} e_2^{\alpha_2}, \tag{2}$$

where $\alpha_i \in (0, 1)$ represents the elasticity of π with respect to agent i's effort e_i. This effort elasticity measures the agent's ability to "transform" effort into output and so can be interpreted as the agent's *skill* level. The probability function π is strictly increasing and concave in the effort level of each agent, $\partial\pi/\partial e_i > 0$ and $\partial^2\pi/\partial e_i^2 < 0$. This implies that additional effort increases the expected output with diminishing returns. Notably, our specification exhibits complementarity between the agents' effort levels, $\partial^2\pi/\partial e_1 \partial e_2 > 0$, so that one agent's effort is more productive the higher is the effort exerted by the other. Effort from both agents is needed for the project to succeed since $\pi(0, e_2) = \pi(e_1, 0) = 0$. Moreover, to guarantee that π is a well-defined probability function, we restrict the effort choice e_i to be continuous in $[0, 1]$.[5]

Exerting effort is costly for the agents, and we assume that they have a quadratic cost function,

$$c_i = \frac{e_i^2}{2}, \tag{3}$$

which is strictly increasing and convex in the effort choice e_i.[6] Finally, we normalize each agent's reservation utility (i.e., their outside options) to zero.[7]

[5] Cobb-Douglas functions are commonly used to represent the effect of the levels of inputs to production on total output. In the context of agency theory, Bhattacharyya and Lafontaine (1995), for instance, consider a downstream Cobb-Douglas production function for a franchising business, where the inputs of production are the effort levels of the franchisee and the franchisor. In the context of venture capital, Repullo and Suarez (2004) adopt a specification similar to ours in to capture the effort complementarities between an entrepreneur and a venture capitalist.

[6] Our model can easily accommodate a more general heterogeneous cost function such as $c_i = {e_i}^{\kappa_i}/\kappa_i$, where κ_i captures the elasticity of the effort cost with respect to the effort level of agent i. Heterogeneous costs of effort ($\kappa_1 \neq \kappa_2$) do not add additional insights to the problem since, as we discuss in the next sections, the optimal contracts only depend on the ratios α_i/κ_i. We therefore normalize $\kappa_1 = \kappa_2 = 2$ and we analyze the implications of heterogeneous agents by use of different effort elasticities ($\alpha_1 \neq \alpha_2$).

[7] The setting can be extended to allow for positive reservation utilities for each agent. When agents have positive reservation utilities, the results become stronger in the sense that the principal's temptation to cut promised bonuses in

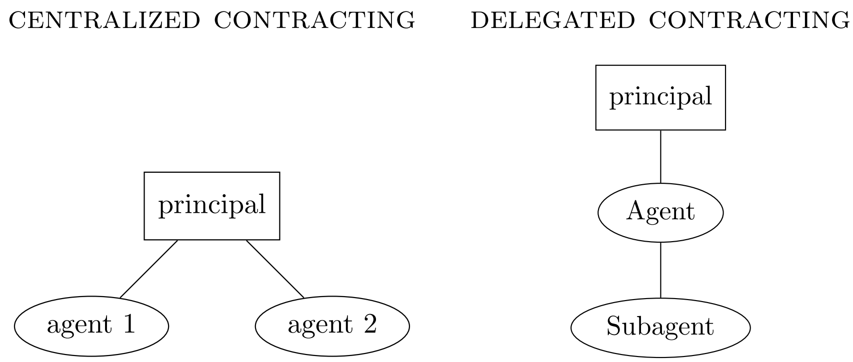

Figure 1: Contracting Schemes

Contracts. The principal needs both agents to implement the project, but she can choose whether to contract directly with both agents, or whether instead to hire only one agent and let this agent hire (and hence write a contract with) the other agent. We refer to the two contracting schemes as *centralized contracting* and *delegated contracting*, respectively. In the latter scheme, for ease of exposition, we will refer who does the hiring as the *Agent* and to the agent who is hired by the other agent as the *Subagent*. Depending on the setting, one can think of the Agent as the manager, or the general contractor, and the Subagent as the worker, or subcontractor. Importantly, when the agents are heterogeneous ($\alpha_1 \neq \alpha_2$), the principal also chooses with which agent she will contract directly if she decides to adopt a delegated contracting scheme. Figure 1 illustrates the structure of the two contracting schemes. The centralized contracting scheme corresponds to a flat hierarchy whereas the delegated contracting scheme is a steeper hierarchy.

In both contracting schemes, only two contracts are written and we are interested in exploring the case in which these bilateral contracts are private information to the parties signing them. This means that each contract is observable only by the two parties who sign it. Therefore, while in the case of centralized contracting it is the principal who observes two contracts, in the case of delegated contracting it is the Agent. Our model, therefore, features two types of hidden actions: unobservable effort and unobservable contracts. Any enforceable contract is based on what is observable by all three players, which is a realization x of the project's output $X(e_1, e_2)$.

For a given realization $x \in \{0, 1\}$ of the project's output, we denote by $b(x)$ the principal's *total compensation budget* for the agents, and by $\phi_i(x)$ the fraction of the budget that is allocated to agent

an ex post opportunistic way becomes worse, and so the need to delegate contracting to improve observability becomes stronger. It can be shown that with strictly positive reservation utilities, delegated contracting will sometimes be optimal even when agents' efforts are substitutes rather than complements (details available from the authors on request). For simplicity, we will work with a model with zero reservation wages.

i, where $\sum_{i=1}^{2} \phi_i(x) = 1$. It follows that agent i's contingent compensation is equal to $\phi_i(x)b(x)$. Since in the low state of the world the project fails and does not deliver any output, and the principal and agents all have limited liability, the compensation budget and hence the payments to the agents are all equal to zero in that state. Therefore, in what follows we drop the dependence of the contracts on the cashflow x and we simply refer to $b(1)$ and $\phi_i(1)$ as b and ϕ_i.

In the centralized contracting scheme, the principal chooses both the size of the compensation budget b and its allocation between the two agents ϕ_i. The word "centralized" indicates that both these decisions are retained by the principal. By contrast, in the delegated contracting scheme, the principal only sets b, the fraction of output that will be used for compensation, whereas the decision regarding the division of the compensation budget ϕ_i is delegated to the Agent.

Note that our separation of promised payments to agents into budget b and share ϕ_i is only for expositional convenience.[8] The offers that agents actually receive are in dollar terms, so that under centralized contracting, an agent is promised a certain dollar amount (equal to $\phi_i b$) when the project succeeds, but can infer neither ϕ_i nor b from this offer. Similarly, under delegated contracting, the Subagent receives a dollar offer and from this can infer nothing about the total compensation budget nor the pay of the Agent. The Agent, on the other hand, observes the dollar amount that the principal will pay him if the project succeeds and the dollar amount he promises the subagent in that case, and hence both his own and the Subagent's incentive pay.

Payoffs. The expected payoff of the principal, denoted by v, is given by the project's payoff if it succeeds minus the compensation budget, times the probability of success,

$$v = (1 - b)e_i^{\alpha_i} e_j^{\alpha_j}. \tag{4}$$

The expected payoff of agent i, denoted by u_i, is given by his expected compensation minus his cost of effort,

$$u_i = (\phi_i b)e_i^{\alpha_i} e_j^{\alpha_j} - \frac{e_i^2}{2}. \tag{5}$$

Before studying the optimal choice that the principal makes at date 0 between the two contracting schemes, we first characterize the optimal contracts in two schemes separately, and discuss the fundamental role played by the observability of these contracts.

[8] In particular, this represention will allow us to nicely distinguish the two effects of delegation: the gain from observability, which comes in the form of a larger budget, from the cost from rent extraction, which comes in the form of a distortion in the shares of the budget allocated to each agent.

3 Centralized Contracting

In this section we consider the centralized contracting scheme in which the principal contracts directly with both agents. We model the interaction between the principal and the two agents as a noncooperative game. The sequence of events is as follows:

(i) The principal makes two simultaneous take-it-or-leave-it offers to the two agents. Each offer includes a compensation level, contingent on the success of the project.

(ii) Each agent observes only his own offer and decides whether to accept the offer and, in that case, how much effort to exert.

(iii) If the project succeeds, the principal uses the project's output to pay the agents the compensation specified in the accepted contracts and collects the residual. If the project fails, no player receives anything.

We solve for the optimal contracts by working backwards. First, we take as given the principal's choice of compensation budget b and allocation ϕ_i, and derive each agent i's optimal effort choice e_i which maximizes his expected payoff u_i given his belief about the other agent's effort. Second, given the two agents' optimal effort choices (e_1, e_2), we derive the optimal choices of b and ϕ which maximize the principal's expected payoff v. Notice that since contracts are not publicly observable, an agent's choice of effort cannot be contingent on the effort level exerted by the other agent, nor on the contract privately signed by the other agent. This creates a role for beliefs about effort levels and contracts that has been missing from the literature so far but which is central to our analysis.

Before proceeding to the formal analysis, let us highlight the importance of being able to observe the contracts of other complementary agents contributing to the same project. Suppose the principal were able to make and commit to publicly observable contract offers, and consider the pair of optimal *public contracts* that the principal would choose in this case. (These are second best contracts in that the agents can perfectly observe each others' contracts but still effort itself is not contractible; they are derived in the Online Appendix B). Now suppose that when contracts are private, the principal tries to offer agent i his optimal public contract, and promises agent i that she will also offer agent j the latter's optimal public contract. Would the agents in this setting continue to exert the same effort they would if contracts were indeed public? The answer is no.

To see why, suppose that one of the agents were to exert the level of effort associated with public contracts; then, it would be optimal for the principal to deviate with the other agent and write him a better contract which economizes on incentive payments. The principal can be made better off and the other agent no worse off by this deviation, because the second agent's compensation contract is not a best response to the first agent's. Or, to put it another way, when the second agent puts in more effort, he exerts a positive externality on both the principal and the first agent (who are both more likely to receive a positive payoff at date 1), and the principal's optimal bilateral contract with the second agent internalizes the exernality on the principal and the second agent but not on the first agent. Anticipating the "opportunistic" behavior of the other pair, each agent demands more compensation to exert a given effort than they would if contracts were public. If contracts were public, each agent could confidently expect higher effort from the other agent (given the observed contract), raising the productivity of their own effort, and so making higher effort more worthwhile for a given level of compensation. With private contracts, therefore, incentive provision in direct contracting schemes is more expensive than with public contracts.

In solving the centralized contracting game, we will be looking for a Perfect Bayesian equilibrium (PBE) with strictly positive effort.[9] Formally, after receiving the compensation offer $\phi_i b$ from the principal, agent i solves for his optimal level of effort e_i, taking as given agent j's conjectured effort level $\hat{e}_j$. In equilibrium, each agent's conjecture about the effort level exerted by the other agent must be correct and correspond to the equilibrium effort level, denoted by e_i^C (C for centralized contracting): $\hat{e}_i = e_i^C$, for $i = 1, 2$.[10]

The optimal level of effort exerted by agent i, therefore, is a function of his compensation and the conjectured effort of agent j:

$$e_i(\phi_i b, \hat{e}_j) = \arg\max_{e_i} \; (\phi_i b)\pi(e_i, \hat{e}_j) - \frac{e_i^2}{2}, \tag{6}$$

$$= \left(\alpha_i \phi_i b \hat{e}_j^{\alpha_j}\right)^{\frac{1}{2-\alpha_i}}. \tag{7}$$

As in standard principal-agent problems with moral hazard, agent i's optimal effort increases with the compensation $\phi_i b$ he receives when output is high. Moreover, agent i exerts more effort the higher the effort he believes agent j's will exert, reflecting the complementarities between their effort choices.

[9]An equilibrium with zero effort from both agents also exists. However, it is Pareto dominated by the equilibrium with strictly positive effort levels analyzed in this paper. The recent literature on unique-implementation (e.g., Segal (2003), Winter (2004), Halac, Lipnowski, and Rappoport (2021), Halac, Kremer, and Winter (2022)) focuses on mechanisms that rule out Pareto dominated equilibria.

[10]This solution approach has been widely used in the industrial organization literature (e.g., Crémer and Riordan (1987), Horn and Wolinsky (1988), Hart and Tirole (1990), O'Brien and Shaffer (1992), Laffont and Martimort (2000), Rey and Tirole (2007)), in finance (e.g., DeMarzo and Kaniel (2021)), and applied to a wide variety of other settings (e.g., Segal (1999)).

The principal's problem is to choose the compensation budget b and its allocation between the two agents $(\phi_i, 1-\phi_i)$ so as to maximize the expected residual output from the project v, subject to the two agents' incentive compatibility (IC) and individual rationality (IR) constraints,

$$\big(b(\hat{e}_i, \hat{e}_j), \phi_i(\hat{e}_i, \hat{e}_j)\big) = \underset{b,\, \phi_i}{\arg\max} \;\; (1-b)\; \pi\big(e_i(\phi_i b, \hat{e}_j), e_j((1-\phi_i)b, \hat{e}_i)\big). \tag{8}$$

The IC constraint of agent i is given by the optimal effort choice in (7). Agents' IR constraints are always satisfied given their outside options of 0.[11] Since the principal rationally takes into account (through the IC constraints) each agent's conjecture of the other agent's effort level, the optimal budget and allocation are a function of the agents' beliefs. Imposing this equilibrium condition after solving the optimization problem in (8), we obtain the optimal contracts in the centralized contracting scheme, which the following proposition characterizes.

Proposition 1. *With centralized contracting, the optimal compensation budget and allocation are*

$$b^C = \frac{2(\alpha_i + \alpha_j - \alpha_i\alpha_j)}{4 - \alpha_i\alpha_j}, \tag{9}$$

$$\phi_i^C = \frac{1}{2} + \frac{1}{2}\left(\frac{\alpha_i - \alpha_j}{\alpha_i + \alpha_j - \alpha_i\alpha_j}\right), \tag{10}$$

respectively. It follows that:

(i) the compensation budget b^C increases with both effort elasticities, α_i and α_j;

(ii) the allocation ϕ_i^C increases with α_i, decreases with α_j and is larger than 1/2 iff $\alpha_i > \alpha_j$;

(iii) agent i's compensation $\phi_i^C b^C$ increases with α_i and decreases with α_j.

In our setting, the effort elasticities of the two agents (α_1, α_2) are the only drivers of the optimal contracts. Proposition 1 shows that the higher are the agents' skill levels, the larger the total compensation budget. When the agents are more productive, the principal finds it more worthwhile to increase the probability of success of the project (through stronger incentives) at the cost of a lower residual output $(1-b)$. Moreover, it is optimal for her to pay the agent with the higher skill more. The higher an agent's skill, the larger the fraction of the compensation budget allocated to him. This implies that an increase in the skill of one agent induces two competing effects on the dollar compensation of the other agent: a positive effect through an increase in the total budget, and

[11] We assume that the principal can always choose not to implement the project, so we are only interested in contracts which can generate a non-negative profit for the principal ($b < 1$).

a negative effect through a decrease in the fraction of the budget he receives. Proposition 1 reveals that the latter effect always dominates.

The optimal budget and allocation obtained in Proposition 1 are based on the maintained assumption that contracts are not publicly observable. In the Online Appendix B, we derive the optimal public contracts in a centralized contracting scheme. These are second-best contracts and are denoted by (b^*, ϕ^*). The next corollary provides the comparison.

Corollary 1. *Under centralized contracting, the optimal compensation budget is lower with private contracts than with public contracts, $b^C < b^*$, while the fraction of the budget allocated to the most skilled agent is higher, $\phi_i^C > \phi_i^*$ if $\alpha_i > \alpha_j$. Overall, both agents receive lower compensation and exert lower effort when contracts are private, $\phi_i^C b^C < \phi_i^* b^*$, for any i.*

Part of the intuition for the lower compensation and effort has already been explained above—it is not credible for the principal to propose the second-best public information contracts when contracts are private as the agents are aware that these second-best contracts are not best responses to one another; the principal will deviate and offer contracts with lower compensation. To see the same result in a different way, note that with public contracts, there are two reasons why the principal sets a relatively high compensation for (say) agent 1. First, increasing agent 1's bonus has a direct effect on agent 1's effort. But second, increasing agent 1's bonus, by making agent 1 work harder in equilibrium, also increases agent 2's effort. This gives the principal an extra reason to increase agent 1's compensation when contracts are public which is absent when contracts are private as agent 2 does not then observe the increase in agent 1's compensation. The same argument applies to any potential increase in agent 2's compensation. So compensation and effort are lower when contracts are private.

When contracts are private, the principal also skews compensation towards the more skilled agent relative to what she would when contracts are public. This is because the direct effect of increasing compensation on effort is now the driving force behind the principal's choice (the indirect effect on the other agent's effort, mentioned above, is now absent). The more skilled agent's effort is more responsive to increases in compensation than the unskilled agent's (for whom the cost of effort is higher) and so it makes sense to concentrate more of the budget on the more skilled agent.

4 Delegated Contracting

In this section we analyze a delegated contracting scheme in which the principal sets only the total compensation budget. She contracts with only one Agent, promising to pay to him the total budget if the project succeeds. That Agent then contracts with a Subagent, and agrees to pay the Subagent a dollar amount if the project succeeds. The payment to the Subagent will be drawn from the total compensation budget paid by the principal to the Agent and so must be less than this total compensation budget by limited liability of the Agent. As with centralized contracting, we model the interactions between the principal and the Agent, and between the Agent and the Subagent as a noncooperative game. The sequence of events is as follows:

(i) The principal makes a take-it-or-leave-it offer to the Agent. The offer includes a compensation budget, contingent on the success of the project. The Agent decides whether to accept the offer or not. Contracting between the principal and the Agent is not observed by the Subagent.

(ii) After signing a contract in stage one, the Agent makes a take-it-or-leave-it contract offer to the Subagent. The offer includes a compensation level, contingent on the success of the project. The Subagent decides whether to accept the offer or not. Contracting between the Agent and the Subagent is not observed by the principal.

(iii) If the Agent and the Subagent have accepted contracts in stage one and stage two, respectively, they decide how much effort to exert.

(iv) If the project succeeds, the principal uses the project's output to pay the Agent according to the contract signed in stage one and collects the residual. The Agent uses what he receives from the principal to pay the Subagent according to the contract signed in stage two and keeps the residual part of the budget. If the project fails, no player receives anything.

Similarly to the centralized contracting case, we look for an equilibrium with strictly positive effort choices, and solve the principal's contracting problem by working backwards. However, since it is now the Agent who decides how to allocate the budget, there are two differences between the principal's problem in the delegated contracting scheme and in the centralized contracting scheme. First, the principal no longer has direct control over the allocation of the budget; and second, the Agent now observes the contract that has been signed by the Subagent when he chooses his own effort. The Subagent, however, still does not observe the contract between the principal and the Agent, so he is in essentially the same situation as that in the centralized contracting scheme.

The optimal level of effort exerted by the Subagent, therefore, is a function of his compensation and the conjectured effort of the Agent:[12]

$$e_S(\phi_S b, \hat{e}_A) = \arg\max_{e_S} \ (\phi_S b)\pi(\hat{e}_A, e_S) - \frac{e_S^2}{2}, \tag{11}$$

$$= (\alpha_S \phi_S b \hat{e}_A^{\alpha_A})^{\frac{1}{2-\alpha_S}}. \tag{12}$$

Properties of the Subagent's optimal effort e_S parallel those characterizing the optimal effort of an agent in the centralized contracting scheme (i.e., e_i in (7)).

What is different from the centralized contracting scheme is that the Agent observes *both* contracts in the delegated contracting scheme. As a result, the Agent plays a best response to the Subagent conjecture of the his effort by exerting an optimal effort level which is a function of the budget he received from the principal, the allocation he has offered to the Subagent, and the Subagent's beliefs:

$$e_A(b, \phi_A, \hat{e}_A) = \arg\max_{e_A} \ (\phi_A b)\pi\big(e_A, e_S((1-\phi_A)b, \hat{e}_A)\big) - \frac{e_A^2}{2}, \tag{13}$$

$$= \Big(\alpha_A \phi_A \left(\alpha_S (1-\phi_A)\hat{e}_A^{\alpha_A}\right)^{\frac{\alpha_S}{2-\alpha_S}} b^{\frac{2}{2-\alpha_S}}\Big)^{\frac{1}{2-\alpha_A}}. \tag{14}$$

For a given allocation of the budget, the Agent's optimal effort level increases with the size of the budget b. However, the budget allocation ϕ_A has a nonlinear effect on the Agent's effort choice. This is due to the complementarity in effort provision. On one hand, a large ϕ_A means the Agent keeps more budget for himself which provides greater incentives for the Agent to exert more effort. On the other hand, if he keeps too much, that will leave too little for the Subagent. As a result, the Subagent will exert low effort, which, given complementarity, discourages the Agent from making much effort himself.

We continue solving the problem by backward induction. For a given budget b, specified in the contract between the principal and the Agent, we solve for the optimal budget allocation ϕ_A that maximizes *the Agent's* expected payoff, conditioning on the best responses $e_A(b, \phi_A, \hat{e}_A)$ in (14) and $e_S(\phi_S b, \hat{e}_A)$ in (12):

$$\phi_A(b, \hat{e}_A) = \arg\max_{\phi_A} \ (\phi_A b)\pi\big(e_A(b, \phi_A, \hat{e}_A), e_S((1-\phi_A)b, \hat{e}_A)\big) - \frac{e_A(b, \phi_A, \hat{e}_A)^2}{2}. \tag{15}$$

[12] In the delegated contracting scheme, we use subscript A and S when referring to the Agent and the Subagent, respectively.

We then solve for the optimal compensation budget b that maximizes the principal's expected residual output, conditioning on the chain of best responses $\phi_A(b, \hat{e}_A)$ in (15), $e_A(b, \phi_A, \hat{e}_A)$ in (14) and $e_S(\phi_S b, \hat{e}_A)$ in (12):

$$b(\hat{e}_A) = \arg\max_b \ (1-b)\pi\big(e_A(b, \phi_A(b, \hat{e}_A), \hat{e}_A), e_S((1-\phi_A(b, \hat{e}_A))b, \hat{e}_A)\big). \tag{16}$$

As in the centralized contracting scheme, both the Agent's and the Subagent's IR constraints are always satisfied given their outside options of 0. In equilibrium, the Subagent's conjecture about the effort level exerted by the Agent must be correct and correspond to his equilibrium effort level, denoted by e_A^D (D for delegated contracting): $\hat{e}_A = e_A^D$. Imposing this equilibrium condition after solving the optimization problem in (16), we obtain the optimal contracts in the delegated contracting scheme, which are presented in the following proposition.

Proposition 2. *With delegated contracting, the optimal compensation budget and allocation are*

$$b^D = \frac{2(\alpha_A + \alpha_S) - \alpha_A\alpha_S}{4}, \tag{17}$$

$$\phi_A^D = \frac{1}{2} + \frac{1-\alpha_S}{2}, \tag{18}$$

respectively. It follows that:

(i) the compensation budget b^D increases with both effort elasticities, α_A and α_S;

(ii) the allocation ϕ_A^D is independent of α_A, decreases with α_S and is always larger than 1/2;

(iii) the Agent's compensation $\phi_A^D b^D$ increases with α_A and decreases with α_S iff $\alpha_A > \frac{2-2\alpha_S}{2-\alpha_S}$;

(iv) the Subagent's compensation $(1-\phi_A^D)b^D$ increases with both α_A and α_S.

Proposition 2 reveals that, as in the centralized contracting scheme, the higher the skill level of the agents (i.e., Agent and Subagent in this case), the larger the compensation budget. The intuition remains the same: the principal gives stronger incentives (by increasing the budget) when the agents are more productive. Interestingly, though, with our Cobb-Douglas specification for the probability of success, the allocation of the budget, chosen optimally by the Agent, is independent of the Agent's skill. Instead, the Agent always keeps at least half of the budget for himself, and then gives the Subagent a fraction of the second half of the budget, depending on the subagent's skill. The Subagent's share of the whole budget equals skill level, α_S. The more skilled the Subagent, the lower the rents that the Agent extracts from the compensation budget.

An increase in the Agent's skill induces the principal to increase the compensation budget, but does not affect the allocation. Therefore, contingent on the success of the project, the dollar compensation of both the Agent and the Subagent are increasing in α_A. An increase in the Subagent's skill, instead, generates not only a higher budget, but also a more balanced allocation, since ϕ_A^D approaches 1/2 when α_S approaches 1. While this always increases the dollar compensation of the Subagent in the high state of the world, it can decrease that of the Agent. This is because the decrease in rent extraction may dominate the increase in the budget.

We define the rent extraction of the Agent as the additional fraction of the compensation budget that the Agent keeps for himself, compared to the second-best allocation $\phi_A^* = \alpha_A/(\alpha_A + \alpha_S)$. Denoting the rent extraction by Δ_A, it follows that

$$\Delta_A \equiv \phi_A^D - \phi_A^* = \alpha_S \left(\frac{1}{\alpha_A + \alpha_S} - \frac{1}{2} \right) \tag{19}$$

is always strictly positive since $\alpha_i \in (0, 1)$ for any i. Moreover, the rent extraction Δ_A always decreases with the skill of the Agent, α_A, whereas it decreases with the skill of the Subagent, α_S, only when the Subagent is sufficiently skilled. The trade-off between the rent extraction distortion, which results in an inefficient allocation of compensation between the two agents, and the observability gain, which increases incentives, is the key driver of the optimal choice between centralized and delegated contracting, and is the focus of the next section. For now, we summarize our empirical predictions regarding compensation, effort and outputs for delegated versus centralized hierarchies in the following corollary.

Corollary 2. *Compared to the centralized contracting scheme, in the delegated contracting scheme:*

(i) the Agent always receives higher compensation upon success and exerts higher effort;

(ii) the Subagent receives higher compensation upon success iff the Agent's skill level is high enough, $\alpha_A > \bar{\alpha}_A^c(\alpha_S)$, and exerts higher effort iff $\alpha_A > \bar{\alpha}_A^e(\alpha_S)$, where the thresholds $\bar{\alpha}_A^c(\alpha_S)$ and $\bar{\alpha}_A^e(\alpha_S)$ decrease in α_S, and are such that $\bar{\alpha}_A^e(\alpha_S) < \bar{\alpha}_A^c(\alpha_S)$;

(iii) the probability of success, and hence the expected output, is higher iff the Agent's skill level is high enough, $\alpha_A > \bar{\alpha}_A^\pi(\alpha_S)$, where the threshold $\bar{\alpha}_A^\pi(\alpha_S)$ decreases in α_S, and is such that $1/2 < \bar{\alpha}_A^\pi(\alpha_S) < \bar{\alpha}_A^e(\alpha_S)$.

Compared to centralized contracting, both the compensation budget and its allocation to the Agent are larger in the delegated contracting scheme. Therefore, in this scheme, the Agent receives a higher compensation if the project succeeds, $\phi_A^D b^D > \phi_A^C b^C$, which in turn makes him exert higher effort in equilibrium. Regarding the effort level and compensation of the Subagent, as well as the probability of success of the risky project, we have two competing effects. The larger budget under delegated contracting tends to increase them, but the lower budget allocation (due to the Agent's rent extraction) tends to decrease them. When the Agent is skilled enough ($\alpha_A > \bar{\alpha}_A^{\pi}(\alpha_S)$), the increase in his effort under delegated contracting increases the probability of success of the risky project, even when the Subagent's effort decreases ($\bar{\alpha}_A^{\pi}(\alpha_S) < \alpha_A < \bar{\alpha}_A^{e}(\alpha_S)$). When the Agent is more skilled ($\alpha_A > \bar{\alpha}_A^{e}(\alpha_S)$), the impact of the Agent's higher effort increases the Subagent's effort, even when the Subagent's compensation is lower under delegated contracting ($\bar{\alpha}_A^{e}(\alpha_S) < \alpha_A < \bar{\alpha}_A^{c}(\alpha_S)$). This happens because the complementarity of effort provision. As the Agent's skill further increases ($\alpha_A > \bar{\alpha}_A^{c}(\alpha_S)$), the positive effect through the higher the compensation budget dominates, increasing the Subagent's compensation as well. The three thresholds identified in Corollary 2 are decreasing in the Subagent's skill because when α_S increases, the compensation budget and the Subagent's allocation increase more when the principal chooses to delegate.

5 Optimal Contracting Scheme

Having analyzed the optimal (private) contracts in the centralized and delegated contracting schemes, in this section we discuss the optimal choice that the principal makes between the two hierarchical structures. In particular, we show that when the agents' skills are sufficiently high, the principal prefers to contract with only one agent, delegating to that agent the power to compensate the other agent. Specifically, we provide formal conditions under which delegated contracting is optimal.

5.1 Optimal Hierarchy with Delegation

Before comparing the principal's expected payoff under the two contracting schemes, we first establish with whom, among the two agents, the principal prefers to contract directly, when relying on delegated contracting. This pins down which agent plays the role of Agent, and which Subagent, in this contracting scheme. Naturally, when the agents are homogeneous ($\alpha_1 = \alpha_2$), the principal is indifferent as to whether she contracts directly with agent 1 or agent 2. However, when the agents are heterogeneous ($\alpha_1 \neq \alpha_2$), the principal is not indifferent but strictly prefers one of them as the Agent with whom to contract directly. Lemma 1 below summarizes the principal's optimal choice.

Lemma 1. *With delegated contracting, the principal always delegates to the more skilled agent:* $\alpha_A = \max\{\alpha_1, \alpha_2\}$ *and* $\alpha_S = \min\{\alpha_1, \alpha_2\}$.

Without loss of generality, in what follows we consider the case $\alpha_1 > \alpha_2$, i.e., agent 1 is more skilled than agent 2. Lemma 1 reveals that, in the delegated contracting scheme, the principal finds it optimal to contract directly with agent 1 and to delegate to him the power to contract with agent 2. In other words, it is optimal for the principal to have the more skilled agent be the Agent, and consequently the less skilled one be the Subagent. There are two opposing effects behind this result. For a given rent extraction Δ, the principal would prefer to contract with the less skilled agent, i.e., agent 2. This is because agent 2 is relatively more exposed to opportunism if he does not observe the contract signed by the other agent, as agent 1's effort is more important overall. Effort complementarity is stronger for the less skilled agent because the impact of the effort of the more skilled agent on the less skilled one is larger than vice versa.[13]

The second, countervailing, effect is induced by the different rents that the two agents would choose to extract if given the role of the Agent. In particular, given the optimal contract in Proposition 2, it follows that agent 2, the less skilled agent, would extract more rents than agent 1: $\Delta_2 = (1 - \alpha_1/2) - \phi_2^* > \Delta_1 = (1 - \alpha_2/2) - \phi_1^*$. So, the allocation distortion induced by the rent exaction of agent 2 is larger, and makes the principal inclined to contract directly with agent 1. Lemma 1 shows that overall the second effect dominates: the higher rent extraction distortion from delegating to agent 2 is more detrimental to the principal than the lower observability gain through effort complementarity from delegating to agent 1. This makes it optimal for the principal to delegate to the most skilled agent.

We illustrate these effects in Figure 2, where we plot the expected payoff of the principal in the delegated contracting scheme, as a function of a generic allocation of the budget to agent 1,

$$v^D = K(b^D)\,[\phi_1^{\alpha_1}(1-\phi_1)^{\alpha_2}]^{\frac{1}{2-\alpha_1-\alpha_2}} \,. \tag{20}$$

The function $K(\cdot)$ does not depend on the allocation of the budget, and is given explicitly in (A.18). Importantly, since the optimal compensation budget b^D in (17) is independent of the principal's choice of Agent, $K(b^D)$ is constant across the two possible cases of delegation: (i) the principal delegates to agent 1 ($A = 1$), (ii) the principal delegates to agent 2 ($A = 2$). For each of these cases, we mark with a solid dot the expected payoff of the principal corresponding to the optimal (delegated) contracts:

$$v^D_{A=1} = K(b^D)\Bigg[\underbrace{(\phi_1^* + \Delta_1)}_{\phi^D_{A=1}}{}^{\alpha_1}(1-\phi_1^*-\Delta_1)^{\alpha_2}\Bigg]^{\frac{1}{2-\alpha_1-\alpha_2}}, \tag{21}$$

[13]The elasticity of the marginal product of agent i's effort, $\pi_{e_i} \equiv \partial\pi/\partial e_i$, with respect to agent$j$'s effort, $\mathcal{E}_{e_j}^{\pi_{e_i}}$, is equal to α_j. Therefore, if $\alpha_1 > \alpha_2$, a 1% increase in agent 1's effort increases the productivity of agent 2 by more compared to the increase in productivity of agent 1 induced by a 1% increase in agent 2's effort.

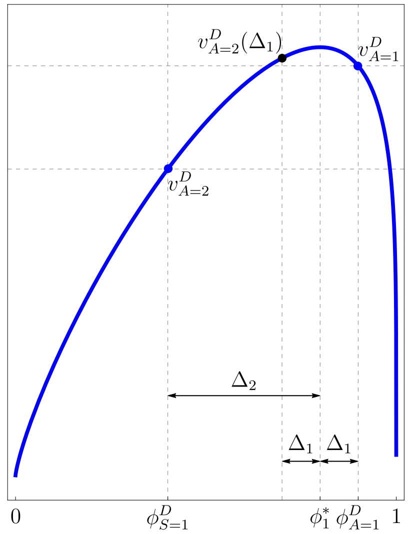

Figure 2: Optimal delegation hierarchy

This figure plots the principal's expected payoff in the delegated contracting scheme, as a function of the budget allocation to agent 1, ϕ_1, for the optimal compensation budget b^D. The solid blue dots, corresponding to $v^D_{A=i}$ for $i = 1, 2$, represent the principal's expected payoff under the optimal contracts $(b^D, \phi^D_{A=i})$. The solid black dot, corresponding to $v^D_{A=2}(\Delta_1)$, represents the principal's expected payoff under the contracts $(b^D, \phi_1 = \phi^*_1 - \Delta_1)$. Parameter values are: $\alpha_1 = 0.8$, $\alpha_2 = 0.2$.

$$\begin{aligned} v^D_{A=2} &= K(b^D)\left[(1-\phi^*_2-\Delta_2)^{\alpha_1}(\phi^*_2+\Delta_2)^{\alpha_2}\right]^{\frac{1}{2-\alpha_1-\alpha_2}} \\ &= K(b^D)\left[(\underbrace{\phi^*_1-\Delta_2}_{\phi^D_{S=1}})^{\alpha_1}(1-\phi^*_1+\Delta_2)^{\alpha_2}\right]^{\frac{1}{2-\alpha_1-\alpha_2}}, \end{aligned} \tag{22}$$

where the last equality follows from the identity $\phi^*_i = 1 - \phi^*_j$. A comparison between (21) and (22) shows two differences. The first is the sign in front of the rent extraction: when the principal delegates to agent 1, the rent extraction increases the budget allocation to agent 1, whereas it decreases it when the principal delegates to agent 2. The second difference is the extent of the rent extraction: $\Delta_1 \neq \Delta_2$ if $\alpha_1 \neq \alpha_2$. These two differences correspond to the two opposing effects characterizing the optimal delegation choice of the principal.

In order to isolate these effects, we also plot the expected payoff of the principal when she delegates to agent 2, but now we articially impose that agent 2 chooses agent 1's optimal rent extraction Δ_1,

$$v^D_{A=2}(\Delta_1) = K(b^D)\left[(\phi_1^* - \Delta_1)^{\alpha_1}(1 - \phi_1^* + \Delta_1)^{\alpha_2}\right]^{\frac{1}{2-\alpha_1-\alpha_2}}. \tag{23}$$

Given the same rent extraction Δ_1, the difference between (21) and (23) captures the difference in observability gains through effort complementarity. Under the maintained assumption that agent 1 is more skilled than agent 2 ($\alpha_1 > \alpha_2$), the plot in Figure 2 shows that $v^D_{A=1} > v^D_{A=2}$. In particular, it highlights that it is the larger rent extraction by agent 2 — compared to that of agent 1 — that makes the principal worse off. Indeed, if the two agents were to extract the same rent Δ, then the principal would prefer to delegate to agent 2, since $v^D_{A=2}(\Delta) > v^D_{A=1}(\Delta)$ for any $\Delta > 0$ (see Proof of Lemma 1 in the Appendix). For instance, when the rent extraction $\Delta_A = \Delta_1$ for both agents, the plot in Figure 2 shows that $v^D_{A=2}(\Delta_1) > v^D_{A=1}$.

5.2 Optimal Delegation

Given the optimal choice of the principal to delegate to the more skilled agent in the delegated contracting scheme, we now discuss the conditions under which she prefers delegated contracting to centralized contracting. A comparison of the compensation budgets and allocations characterizing the optimal contracts in the two contracting schemes, yields that

$$b^C < b^D, \qquad \phi_i^C < \phi^D_{A=i}, \tag{24}$$

where, given the result in Lemma 1, agent i is the more skilled of the two agents. So, compared to centralized contracting, the principal allocates a larger compensation budget when delegating, and, in that case, the more skilled agent receives a larger fraction of the budget.

The different compensation budgets and allocations across the two contracting schemes affect the principal's expected payoff in opposite ways. We first note that, in our setting, the equilibrium expected payoff of the principal – as well as the equilibrium effort levels, the probability of success, and the agents' expected payoffs – admit the same functional form across the two contracting schemes: $v^C = v(b^C, \phi_i^C)$ and $v^D = v(b^D, \phi^D_{A=i})$, where

$$v(b, \phi_i) = (1-b)\left[(\alpha_i b)^{\alpha_i}(\alpha_j b)^{\alpha_j}\phi_i^{\alpha_i}(1-\phi_i)^{\alpha_j}\right]^{\frac{1}{2-\alpha_i-\alpha_j}}. \tag{25}$$

For a given allocation ϕ_i, the function $v(b, \phi_i)$ is maximized at a level of the compensation budget equal to $(\alpha_1 + \alpha_2)/2$, which corresponds to the optimal compensation budget under second-best (public) contracts b^*. Since $b^C < b^D < b^*$, and $v(b, \phi_i)$ is increasing in b in the interval $(0, b^*]$ and decreasing otherwise, the larger budget associated with delegated contracting is beneficial to the principal. This reflects the fact that more contract observability allows the principal to credibly commit to a larger budget, which increases agents' incentives, and hence her expected profitability. Indeed, since the Agent is the one offering the contract to the Subagent, the Agent does not fear expropriation by the principal, who could, instead, (secretly) save on the contract offered to the other agent with centralized contracting. This makes the Agent's effort respond more strongly to changes in compensation, which in turn induces the principal to reduce her stake in the future output, in exchange of a higher likelihood of success of the risky project.

For a given budget b, instead, the function $v(b, \phi_i)$ is maximized for a budget share equal to $\alpha_i/(\alpha_1+\alpha_2)$, which corresponds to the optimal budget allocation under second-best (public) contracts ϕ_i^*. Since $\phi_i^* \leqslant \phi_i^C < \phi_i^D$, and $v(b, \phi_i)$ is increasing in ϕ_i in the interval $(0, \phi_i^*]$ and decreasing otherwise, the larger budget share associated with delegated contracting is detrimental to the principal. This reflects the distortion in the efficiency of effort provision induced by the Agent, who under-incentivizes the Subagent in order to extract rents. For instance, when the two agents are equally skilled ($\alpha_1 = \alpha_2$), the principal allocates the compensation budget equally to these agents when contracting with both of them directly, since their marginal productivities in generating expected output are the same. With delegated contracting, instead, the Agent allocates more than half of the budget to himself, as he finds it optimal to increase his own stake in the future output, despite the negative effect that this has on the likelihood of success of the risky project.

The principal's loss of control over the budget allocation is the price to pay in order to gain the benefits of more contract observability. Under what conditions it is worthwhile for the principal to pay this price? Can the two agents also benefit from delegation? The following proposition answers these questions.

Proposition 3. *The principal prefers delegated contracting over centralized contracting iff the Agent's skill is high enough, $\alpha_A > \bar{\alpha}_A(\alpha_S)$, where the threshold $\bar{\alpha}_A(\alpha_S)$ decreases with the Subagent's skill α_S. Delegated contracting is Pareto improving iff $\alpha_A > \bar{\alpha}_A^e(\alpha_S) > \bar{\alpha}_A(\alpha_S)$.*

Proposition 3 states that it is beneficial for the principal to delegate contracting if the agent that is relatively more skilled (i.e., the Agent) is skilled enough. Indeed, when α_A is sufficiently large, the benefit induced by more observability under delegated contracting dominates the cost of the loss of control. The intuition is as follows. First, the additional compensation budget that the principal optimally gives to the agents when choosing delegation, $b^D - b^C$, is increasing in the Agent's skill.

This is because the higher transparency of contracts makes the Agent's effort more responsive to compensation, while the high skill of the Agent makes the probability of success of the project more responsive to her effort. So, the principal benefits more from the observability of contracts when the Agent has higher skill.

Second, the distortion in effort provision due the Agent's rent extraction is also reduced when the Agent is more skilled. In particular, since the optimal budget allocation ϕ_A^D in (18) is independent of α_A, it is the increase in the second-best allocation ϕ_A^* in (B.2) that makes the rent extraction Δ_A in (19) decrease with the Agent's skill. Intuitively, when the Agent is more skilled, it is optimal from the perspective of principal as well as the Agent to heavily tilt the budget allocation towards the Agent. Therefore, in this case, the rent extraction due to the loss of control becomes less of a problem, and consequently the principal loses less from giving up control over the budget allocation.

Proposition 3 also reveals that delegation can be Pareto improving. Notice that this happens under the same condition as when the Subagent's effort increases (see corollary 2). Thus, the Subagent's utility can be higher from delegation even though his direct compensation in the case of success falls because of rent extraction. The reason is that the Agent exerts more effort under delegation, raising the Subagent's productivity sufficiently that the Subagent decides to work more despite reduced rewards in the case of success. The increase in the probability of success from both agents working more increases the subagent's expected compensation.

We illustrate these effects in Figure 3, where we consider the case $\alpha_1 > \alpha_2$. The left panel shows the principal's expected payoff in the delegated contracting scheme (solid blue line) and in the centralized scheme (dashed red line), as a function of the budget allocation to agent 1 (i.e., the Agent). The plot highlights how, fixing the allocation ϕ_1, the principal is always better off under delegated contracting. In particular, we identify the difference between the two curves at the optimal allocation under centralized contracting, $v^C(b^D) - v^C$, as the benefit of making the contract of the Subagent observable to the Agent. Moving to the right of $v^C(b^D)$ along the solid blue line captures the cost induced by the Agent's rent extraction, which in equilibrium is quantified by the difference $v^C(b^D) - v^D$. For the chosen parameters ($\alpha_1 = 0.8$ and $\alpha_2 = 0.7$), the plot shows that the benefit of delegation is larger than the cost, making $v^D > v^C$.

The shaded area in the middle panel shows the region of the plane defined by the agents' skills (subject to $\alpha_A > \alpha_S$) in which the principal prefers delegated contracting over centralized contracting. The darker shading indicates the subregion in which delegated contracting is Pareto improving. For any Subagent's skill α_S, there exists a level of the Agent's skill above which delegated contracting is chosen by the principal. This skill level is depicted by the solid line and corresponds to the threshold $\bar{\alpha}_A$. Moreover, for any Subagent's skill α_S, there exists a level of the Agent's skill above

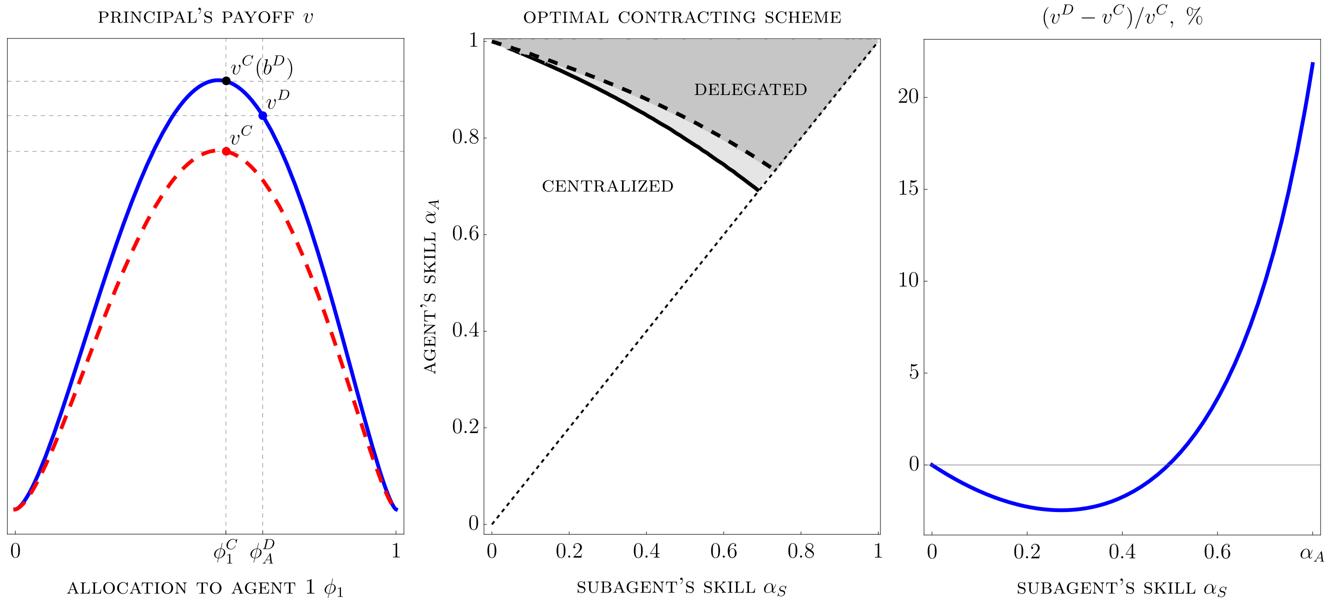

Figure 3: Optimal contracting scheme

The left panel plots the principal's expected payoff in the delegated contracting scheme (solid blue line) and in the centralized scheme (dashed red line), as a function of the budget allocation to agent 1, ϕ_1, for the optimal compensation budget b^D and b^C, respectively. The solid blue and red dots, corresponding to v^D and v^C, represent the principal's expected payoff under the optimal contracts $(b^D, \phi^D_{A=1})$ and (b^C, ϕ^C_1), respectively. The solid black dot, corresponding to $v^C(b^D)$, represents the principal's expected payoff under the contracts (b^D, ϕ^C_1). The middle panel plots the region of agents' skill (α_A, α_S) in which delegated and centralized contracting are optimal. The solid line represents the threshold $\bar{\alpha}(\alpha_S)$. The dashed line represents the threshold $\bar{\alpha}^e_A(\alpha_S)$. The dotted line delimits the relevant region $\alpha_A \geqslant \alpha_S$. The right panel plots the percentage increase in the principal's expected payoff when choosing delegated over centralized contracting, $v^D/v^C - 1$, as a function of the Subagent's skill α_S. Parameter values are: $\alpha_1 = 0.8$, $\alpha_2 = 0.7$.

which delegated contracting is Pareto improving. This skill level is depicted by the dashed line and corresponds to the threshold $\bar{\alpha}^e_A$. This confirms the above intuition that contracting with a more skilled Agent increases the benefit of delegation and decreases its cost. The thresholds $\bar{\alpha}_A$ and $\bar{\alpha}^e_A$ decrease with α_S because the Agent engages in less rent extraction when the Subagent is more skilled.

Finally, the right panel plots the percentage change in the principal's expected payoff when she chooses delegated over centralized contracting, as a function of the Subagent's skill. That the percentage change is positive and increasing in α_S when the Subagent is sufficiently skilled confirms the forces at play discussed above. The plot further reveals that the net benefit of more contract transparency can be significant when the agents are particularly skilled, making their effort complementarities particularly important. The non-monotonic behavior of $(v^D - v^C)/v^C$ in this plot reflects the fact that the Agent's rent extraction Δ_A decreases with α_S when the Subagent's skill is sufficiently low.

6 Delegation with Common Observability

In the previous section, we saw that the principal's choice between delegated and centralized contracting is governed by the trade-off between improved observability of contracts and more control over the division of the agents' compensation budget. In this section, we show that delegated contracting may also be preferred by the principal *even when the observability of contracts is the same across the two contracting schemes*. Thus improving observability is not the only reason why delegation may be preferred, even in our stripped-down set-up. We will show that when observability is imperfect, but the same across centralized and delegated contracting settings, compensation will be skewed away from the second best in either case, and that sometimes delegated contracting permits the principal to commit to a better distribution of compensation than does centralized contracting.

To highlight this effect, in this section we will allow the less skilled agent's contract (i.e., the Subagent's contract) to be *publicly observed*. In particular, suppose that the Agent can observe (and hence can condition his decision on) the contract signed by the Subagent in the centralized contracting scheme as well as the delegated contracting scheme. We denote the optimal compensation budget and allocation in the centralized scheme with one public contract by (b', ϕ'), and derive them in Online Appendix C.

Proposition 4. *When the less skilled agent's contract is public, the principal prefers delegated contracting over centralized contracting iff both the Agent and Subagent are skilled enough,* $\alpha_A > \bar{\bar{\alpha}}_A(\alpha_S)$ *and* $\alpha_S > \bar{\bar{\alpha}}_S$*. The threshold* $\bar{\bar{\alpha}}_A(\alpha_S) > \bar{\alpha}_A(\alpha_S)$ *decreases with the Subagent's skill for* $\alpha_S > \bar{\bar{\alpha}}_S$*.*

Proposition 4 reveals that, even though the two contracting schemes have the same observability of contracts, delegated contracting may still be preferable for the principal. To see the reason for this result, note first that the principal gains from the improved observability: his payoff from centralized contracting with one public contract is higher than when both contracts are private, even though the total compensation budget he chooses under this arrangement is also larger. This is because the principal would really like to commit to offering both agents higher compensation, because when each agent knows that the other is receiving strong incentive pay, the complementarity between the agents' efforts kicks in: the agents work harder for a given prize, knowing that their own effort is more effective when the other agent is working hard. The difficulty for the principal is that he is unable to commit to high compensation for agents whose contracts are private because when their pay is unobservable by the other agent, the principal prefers to privately reduce it. So it is better for the principal to have one contract observable than none. However, having only one contract observable distorts the principal's allocation of the compensation budget in favor of the agent whose

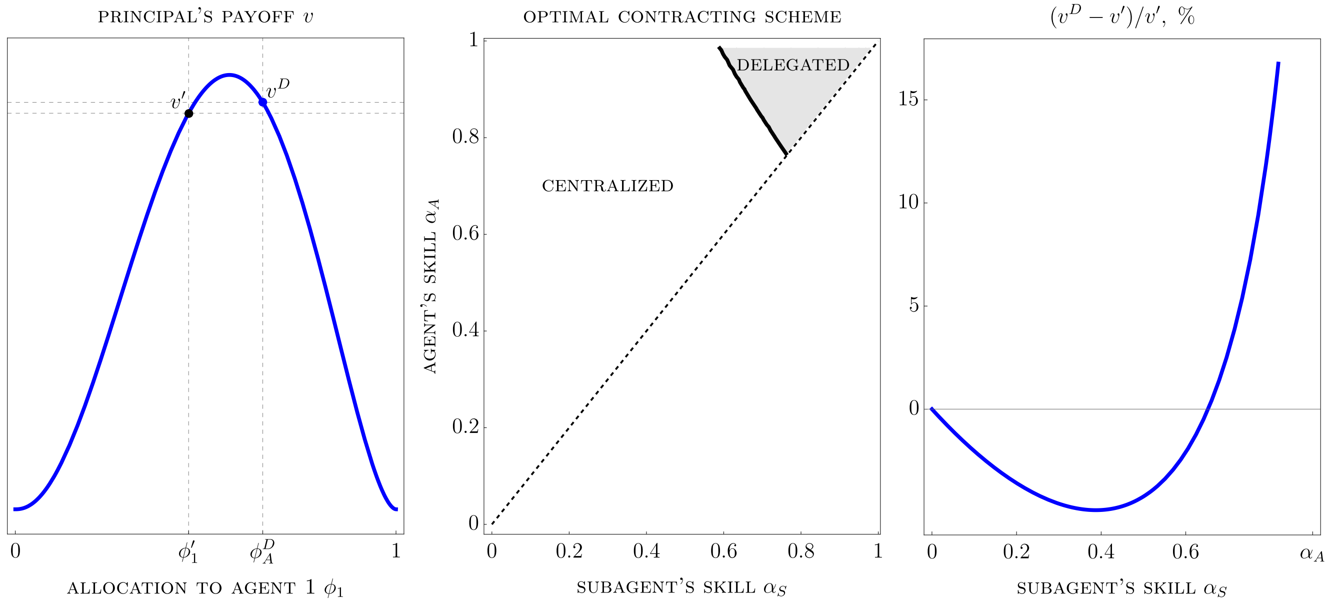

Figure 4: Optimal contracting scheme with one public contract

The left panel plots the principal's expected payoff as a function of the budget allocation to agent 1, ϕ_1, for the optimal compensation budget $b^D = b'$. The solid blue dot, corresponding to v^D, represents the principal's expected payoff under the optimal contracts $(b^D, \phi^D_{A=1})$. The solid black dot, corresponding to v', represents the principal's expected payoff under the contracts (b', ϕ_1'). The middle panel plots the region of agents' skill (α_A, α_S) in which delegated and centralized contracting are optimal. The solid black line represents the threshold $\bar{\bar{\alpha}}(\alpha_S)$. The dotted line delimits the relevant region $\alpha_A \geqslant \alpha_S$. The right panel plots the percentage increase in the principal's expected payoff when choosing delegated over centralized contracting, as a function of the Subagent's skill α_S. Parameter values are: $\alpha_1 = 0.9$, $\alpha_2 = 0.7$.

contract is public. For each dollar the principal publicly promises to pay this agent, the principal anticipates both: (i) the direct effect of more effort from the recipient of the higher pay; and (ii) the indirect effect of more effort from the other agent who observes his co-worker's higher pay, anticipates the latter's higher effort and hence puts in more effort himself. By contrast, every dollar paid to the agent whose contract is unobserved only generates the first, direct, effort response. So with centralized contracting, the principal optimally skews the distribution of the compensation budget when only one contract is observable.

This compensation distortion under centralized contracting creates room for delegation to improve matters even when observability is held fixed. Because delegation allows rent extraction by the agent, compensation is also distorted relative to the second best in this case. But if the rent extraction is not too severe, then the distortion can be smaller than the distortion associated with centralized contracting. The first panel of figure 4 illustrates one such case: compensation under centralized contracting is more distorted than compensation under delegated contracting, resulting in a lower expected payoff for the principal, $v' < v^D$. Why then, does the principal not offer the same, less distorted compensation under centralized contracting? The answer is that such an offer would not be credible: the Subagent would anticipate that the principal would secretly cut the promised compensation to the Agent because it is not optimal for him to offer such large compensation to the Agent privately. By contrast, because the Subagent in the delegated hierarchy knows that the Agent engages in rent extraction, he can be confident that the Agent is receiving a substantial portion of the compensation budget even though the actual contract that the Agent receives is his own private information. Thus the commitment to a structure where rent extraction occurs allows the principal to commit to a different, and potentially less distorted, division of the compensation budget from what he would choose in equilibrium under centralized contracting.

Under what conditions is it useful for the principal to use the delegated contract structure to commit to a different budget allocation even when one contract is observable? Proposition 4 states that it is when both agents are particularly skilled. In this case: (i) the rent extraction is low, and (ii) the complementarity between agents, and the need to incentivize the Subagent more effectively is large. The plots in Figure 4 confirm this finding.

7 Applications

Our theory applies to any setting in which teamwork is present and contractual observability is imperfect. In the following, we highlight some financial and economic applications where we believe that the issues that we address are particularly pertinent.

- *Venture capital*: In a typical venture capital (VC) investment, limited partners (the principal) contract with general partners (the Agent) who then have responsibility for contracting with entrepreneurial companies (the Subagent). Our model can be applied to this setting since normally, the LPs provide funding, while the GPs and the entrepreneurs must both make effort in order for the venture to succeed, and the contracts between these latter two are not generally very transparent to investors.[14] Our model provides a different perspective to most of the theoretical literature on venture capital, which has adopted a simplified, two-tier structure in which LPs and GPs are grouped together as a single agent (e.g., Casamatta (2003), Repullo

[14]Indeed, Gornall and Strebulaev (2020) argue that the complexity of venture firms share structure makes it difficult for investors to value their stake in venture-backed firms, i.e., to assess the incentive power of contracts between general partners and entrepreneurial firms. Note that our theory does not require that investors do not observe the entrepreneurs' contracts, only that they do not control them. Even if sophisticated investors can adequately assess how incentives are divided between the general partners and the entrepreneurs, it would be very unusual for them to stipulate what the terms of the contract between them should be: this is left to the discretion of the general partners.

and Suarez (2004), and Hori and Osano (2013)). The focus of these papers is on solving the double-sided moral hazard problem between GPs and LPs (Bhattacharyya and Lafontaine (1995)), neglecting the conflict of interest between GPs and LPs. By contrast, building on this paper, Liu (2020) considers a VC setting with a full three-tier structure in which the entrepreneur (one of the agents who must make effort) owns the project, and offers contracts to competitive investors (rather than a monopolistic principal), a consultant (who must also make effort) and/or a VC firm (which bundles investor financing and consulting). Gryglewicz and Mayer (2022) study a three-tier hierarchy where a financial intermediary (VC/PE) firm and an entrepreneurial firm make additive efforts, and the intermediary makes a take-it-or-leave it offer to investors for funding. Their focus is on the dynamic evolution of incentives and they do not study the impact of the hierarchical structure on expected output and agency rents.

- *Investment banking syndicates*: Firms usually use financial syndicates when they wish to obtain large loans, or issue equity or bonds. In the case of loan syndications, the issuer (the principal) usually organizes a competition ("a bake off") to select a lead bank (the Agent).[15] The lead bank takes charge of due diligence on the loan, but all banks perform some monitoring, the extent of which is not easily verified. The company typically contracts only with the lead banker, and this contract will specify the spread that the borrower will pay on the loan. The spread and other fees earned by other members of the syndicate (the subagents) is usually delegated to the lead banker to decide. Occasionally the issuing firm will instead choose several lead managers to contract with directly, corresponding to a more centralized form of contracting.[16]

  Similarly, underwriting a typical initial public offering (IPO) involves a set of banks, all of whom make effort in selling the issue (Corwin and Schultz (2005)).[17] But these banks are organized into a hierarchy (Pichler and Wilhelm (2001)). One bank wins the initial competition to be book runner and this bank determines the very complicated schedule of fees and rewards that will ultimately be paid to the other managers of the issue (Chen and Ritter (2000)).

- *Mutual fund families*: Generally, mutual fund investors do not contract directly with fund managers, but instead allocate their capital to a mutual fund family, which then is responsible for hiring and compensating fund managers. While the fund manager chooses investors' asset allocation, his choices are facilitated and supported by the resources provided by the fund

[15] See Esty (2001) for a case study of Chase Manhattan's syndication of one particular loan.

[16] Pichler and Wilhelm (2001) argue that the hierarchical structure makes the lead bank's effort observable, which it would not be otherwise, and hence improves incentives. Luo (2022) argues that the hierarchy of claims in investment banking syndicates can facilitate truthful cheap talk communication between its members about the likelihood of project success.

[17] Hatfield, Kominers, Lowery, and Barry (2020) find that the lead bank's need for other banks help in underwriting process may help sustain collusion in the initial "bake off" for the right to lead the syndicate.

family: a setting with complementary efforts. Our model shows that this three-tier structure has costs and benefits. The benefit is that the fund family has incentives to put in effort to providing resources, a trading platform and access to information for the manager, since it observes that he has a suitable contract which will provide him with incentives; the cost is that the fund family extracts too large a share from the investor's point of view: given the fees paid to the mutual fund family, the investor would prefer the manager to have higher-powered incentives).[18]

- *Outsourcing*: When the principal decides to contract with a *single* agent for supply of the good that *two agents* work on together, we can interpret this as outsourcing: the principal asks the team of agents to supply output and delegates employment of the second agent to the first one. Centralized contracting, by contrast, corresponds to production "in house", or vertical integration, since the principal directly contracts with all of the agents working on the project, and none of those agents see each others' contracts; all of them depend directly on the principal for compensation. The existing outsourcing literature has the assumption of incomplete contracts (Williamson (1985), Grossman and Hart (1986)) as a central tenet. It argues that when contracts are incomplete, outsourcing gives suppliers ownership of key assets, and hence greater bargaining power ex post, and enhanced incentives to produce ex ante.[19] Our theory provides an alternative view of the costs and benefits of outsourcing, based on complete contracting. In our paper, outsourcing is not needed to provide direct incentives to any particular supplier because these are contractible. Instead, we show that the privacy of contracts can drive outsourcing, because it implies that indirect incentives can be lacking for in-house production. (The existing literature assumes that all contracts, in-house or out-sourced, are public). With private contracts, headquarters might be unable to commit to providing sufficient incentives to each of two in-house agents (which could be individuals or divisions of the firm) working together. If instead, production is outsourced to one of the agents, that agent controls, and therefore observes, the incentives provided to the other. Our theory suggests that the likelihood of outsourcing will depend on the privacy of contracts, the importance of incentive provision, and the level of skill asymmetries and complementarities associated with the task that agents need to carry out. It is complementary to the existing theoretical literature since

[18] See Elton, Gruber, and Blake (2003) for a discussion of managerial incentives in the mutual fund industry.

[19] A series of papers (McLaren (2000), Grossman and Helpman (2002), Grossman and Helpman (2005), Grossman and Rossi-Hansberg (2012), and Legros and Newman (2013), highlight industry feedback mechanisms which can lead to multiple equilibria in choices about outsourcing versus vertical integration. Hold-up is less likely, and search costs are lower, in markets with more unintegrated providers. If integration involves fixed costs but reduces variable costs, then it will be driven by margins, which are in turn endogenous to the amount of integration.

asset ownership plays no role; and hence it may provide a better explanation for the outsourcing of some services.[20]

- *Compensating teams within organizations*: Recent evidence has shown that there is very little transparency of compensation contracts within organizations.[21] Although the reasons for pay secrecy are outside the scope of our paper,[22] our model shows that one consequence of the lack of pay transparency is that it may become worthwhile for firms to delegate bonus-setting to team-leaders, and to choose the most skillful employees for this role.[23] Our theory also predicts that organizations where pay schemes are more transparent can organize in flatter, more centralized, ways without compromising incentives.

While each of these applications has its own particularities, (e.g., efforts may take different forms: screening adversely-selected entrepreneurs or issuers, picking stocks and providing investment research and back-office support, working on a team project in an organization, or producing a product with multiple inputs for headquarters,...), they all have in common complementary efforts and opacity of contracts to outsiders. Therefore our theory can speak to the costs and benefits of delegating incentive contracting to effort-taking parties in each case. When contracts are private, one may often see delegated structures (which would be dominated with public contracts); or, if delegated structures exist for exogenous reasons, the delegation of control over contracting will be less harmful, and can even be beneficial to the principal, when contracts are private. Where delegation occurs, the middle man in the contracting (the Agent: the Lead Bank, the owner of the firm receiving the outsourcing contract, the general partners in the venture fund, the fund family) inevitably extracts too much rent

[20] As in the venture capital literature, almost all theoretical papers on outsourcing model firms as two-level hierarchies, where the levels must work together to produce output; the decision of whether to outsource is a question of how much control to cede to the other unit, and under which circumstances. An interesting exception is Antràs and Chor (2013), who model production as being composed of a continuum of complementary production processes. Alfaro, Bloom, Conconi, Fadinger, Legros, Newman, Sadun, and Van Reenen (2018) distinguish the integration decision from the decision to delegate or centralize decision rights. For them, integration has an option value: though integrating does not minimize expected costs, it gives firm owners authority to choose to delegate or centralize decision rights, depending on which problems arise in the future course of a relationship.

[21] According to the IWPR (2017), two thirds of private sector employees report that they are actively discouraged or could be punished for discussing their pay with other workers. Cullen and Perez-Truglia (2020) provide revealed preference evidence that individuals at a large Asian bank were unwilling to allow their peers to learn their salary.

[22] Publicizing pay can lead to the departure of key staff (Zenger (2016), BBC (2018)). Mandated pay transparency in California cities resulted in pay reductions and a 75% increase in the quit rate (Mas (2017)).

[23] For example, Rose and Sesia (2010) describes how compensation at Credit Suisse was delegated both before and after the financial crisis. Headquarters determined how much each of the bonus pool each division of the bank would be allocated. The bosses in each division would determine how bonuses were divided up within the various parts of the decision, and the leader of each team would be largely responsible for dividing bonuses among the members of his team. Interestingly, Levin (2003) gives the example of bankers in two other banks who were aggrieved when one year the bonus pool was much smaller than bankers considered that they had been led to expect based on bank performance. In the context of our model, these provide examples of possible ex post expropriation by the principal.

from the principal's point of view at the expense of those below him/her (the Subagent: non-lead banks, workers in outsourced firms, entrepreneurs, fund managers). Delegation always results in the principal paying more in compensation, but this can sometimes be beneficial for the principal as it elicits more effort. If both the agent and the subagent are highly skilled and there is strong complementarity between them, then delegation is more likely to be Pareto improving.

8 Related Literature

The problem of organizing contracting in a setting where agents have to work in a team goes back at least to Alchian and Demsetz (1972), who observe that the role of an employer is to monitor individual employees' efforts when the market only observes joint output. Holmstrom (1982) observes that instead of monitoring, the principal can write a public contract with the agents stipulating that agents will not be compensated unless the output coincides with the Pareto optimal level. This discrete drop in output resulting from a slight reduction in effort can be sufficient to allow agents to attain the first-best. However, Eswaran and Kotwal (1984) point out that Holmstrom's solution does not consider the possibility for the principal to write an unobserved side contract with one of the agents, where that agent agrees to exert a lower-than-first-best effort in exchange for a share of the output the principal receives when output is below first-best. They do not, however, solve for the optimal contract when such secret side-contracting is possible. In our paper, we explicitly take into account the opportunism problem faced by the agents and solve for the optimal contract. We also consider how creating hierarchy within the team can improve or worsen incentives.

Recent work on moral hazard in teams includes Rayo (2007), Garicano, Meirowitz, and Rayo (2017), and Edmans, Goldstein, and Zhu (2013). The first two papers look at how relational contracts interact with providing incentives in teams, whereas Edmans, Goldstein, and Zhu (2013) looks at optimal team composition when agents' effort affects not only the probability of a successful outcome, but also other agents' effort costs. Recent contributions on monitoring teams include Camboni and Porcellacchia (2022) and Halac, Kremer, and Winter (2022). All of these papers assume public contracts.

Our paper contributes to a small but growing theoretical literature on the consequences of pay transparency (Halac, Lipnowski, and Rappoport (2021), Cullen and Pakzad-Hurson (2019)).[24] Halac,

[24]There is also a literature on the benefits of transparency in organizations in general. Jehiel (2015) argues that full transparency to agents in organizations is never optimal for the principal: while giving agents more information has the benefit of allowing agents to tailor their action to the particular problem they are facing, it also makes incentive constraints more difficult to satisfy. Various papers have shown that it may also be optimal for the principal to avoid full transparency of information for himself. Prendergast (1993) notes that when the principal has prior expectations

Lipnowski, and Rappoport (2021) consider a model similar to ours in which two agents work in a team to produce a single output, and neither team-member observes the other's actual pay package. Differently to our paper, however, the principal can commit to a distribution of pay packages for each agent, so it is only the realization of an agent's actual incentive pay outcome that is private information: that is, agents view their teammates' pay packages as risky rather than subject to strategic uncertainty. They focus on unique-implementation schemes that rule out bad equilibria (Winter (2004)). They show that if the principal can commit to a randomization of pay contracts, and yet keep the outcome of randomization private, bad equilibria can be excluded with pay levels that are lower on average and less asymmetric than when the (realization of the) actual bonus contract is public information. Such strategies are not available to the principal in our model, who lacks any public commitment device on pay.

Cullen and Pakzad-Hurson (2019) investigate a very different setting: an asymmetric information bargaining problem where a principal bargains bilaterally with each of a set of prospective agents. The degree of transparency is measured by the speed at which information about individual pay agreements leaks to other bargaining pairs. They show that full pay transparency enhances the principal's bargaining power in negotiations, since the principal knows that if she yields a higher wage to one agent, all agents will learn that her reservation wage is higher—and hence the principal will be forced to pay the same high wage to all the agents with whom she has not yet concluded a deal. Hence, contrary to the result in Halac, Lipnowski, and Rappoport (2021), in equilibrium, transparency lowers pay levels and reduces pay inequality. In our paper, moving from private contracts to full transparency raises pay levels and reduces inequality.

Our assumption of effort complementarities is a specific form of positive externality that the agents impose on one another. In a general model of contracting with externalities between agents, including vertical relations, takeover battles, debt workouts, and network externalities, Segal (1999) explores the principal's incentive to deviate from an efficient trade profile when her contract offer to each agent is only privately observed. He characterizes the optimal mechanism when agents' contracts can be made contingent on other agents' messages to the principal, but does not consider how delegation may be used to solve the problem of contractual privacy.[25]

about the solution to a problem, agents will conform excessively to those expectations; Prat (2005) shows that when the agent receives information about the state of the world before choosing his action, then it can be harmful to the principal to learn about the agent's action itself, rather than just the outcome of that action; Crémer (1995) shows that when the agent has to make effort to produce output, the principal may do better when she does not acquire information about the reasons for the agent's failure. Our model is different from these since we have no uncertainty about the state of the world, only strategic uncertainty.

[25]Katz (1991) studies delegation by a game-playing principal to an agent. He shows that when a principal delegates his actions to an agent who shares the same preferences in a game, this has no impact when the contract between the principal and the agent is private. It will of course affect the game if their preferences differ, or if contract is public (e.g., Spencer and Brander (1983), Brander and Spencer (1985), Vickers (1985)). Moreover, if the contract is public

DeMarzo and Kaniel (2021) build a model of contracting with externalities featuring multiple agents with individual efforts and outputs subject to a common shock. The principal would ordinarily use relative performance evaluation in such a context (Holmstrom (1982)), but DeMarzo and Kaniel (2021)'s agents have a "keeping up with the Joneses (KUJ)" component to their preferences, meaning that they want their compensation to keep pace with their peers' compensation. Therefore, an agent that makes more effort, or a principal that offers higher pay, exerts a negative externality on other agents. In equilibrium, agents' KUJ preferences result in a less negative (or even positive) load of optimal compensation on peer output, providing one rationale for observed "payment for luck" (Bertrand and Mullainathan (2001)). As an extension, the authors also analyze the case in which contracts are private, finding that this exacerbates the externalities, agents' effort may increase beyond the first best, and principals' profits are reduced compared to the case with public contracts. This work is complementary to our own, since we analyze a setting where the externalities are positive, arise from the impact of agents' effort on other agents productivity, and only team output, not individual outputs, is observable.

The problem of opportunism between a principal and two agents has some similarities to that between an upstream monopolist and two retailers (see, e.g., Hart and Tirole (1990), McAfee and Schwartz (1994)). But a key difference is that in the principal-agent framework, the vertical foreclosure solution of integrating the principal with the agent may not be available.[26] Instead, in the principal-agent setting, a novel solution presents itself: the transparency of contracts can be improved by contracting with a co-worker can be delegated to one of the agents — a solution that is typically unavailable in the vertical integration context because of anti-trust concerns.

Like us, Aghion and Tirole (1997) study a double-sided moral hazard problem where delegation can be advantageous. Their model, however, involves only two, not three, actors, and so cannot speak to the organization of teams. It features a principal and an agent who may both make effort to obtain information about which project should be adopted. The principal can encourage the agent's effort by delegating the choice of project to the agent, even though this might involve a loss of control. The reasons for delegation in their paper, however, are quite different from ours: in their model, delegation commits the principal to making a *lower* effort, encouraging the agent to increase his own effort because efforts are substitutes. In our setting, there are two agents whose efforts are complements and the principal makes no effort contribution; he delegates to increase the transparency of one agent's contract to the other. In delegating, the principal increases the effort

and can be made contingent on the contracts written by other principals with their agents then a folk theorem obtains and a plethora of equilibrium outcomes can be supported (Katz (2006)).

[26]A further difference is that in the vertical integration framework, output, i.e., retailers' strategic variable, is verifiable, whereas in our setting, agents' efforts are not. Thus, if the principal contracts directly with both agents, the principal is stuck with the third best outcome (because neither effort nor contracts are observable).

of one agent at the possible cost of reducing the effort of the other agent (subagent). We provide conditions under which this is profitable for the principal.

Our paper is also related to the literature on delegation and hierarchies, surveyed by Mookherjee (2006) and Poitevin (2000). The latter observes that the revelation principle ensures that delegation is always weakly dominated by centralization, (e.g., Baron and Besanko (1992)), unless the centralized mechanism is undermined by (i) costly communication between one or more agents or contract complexity (e.g., Melumad, Mookherjee, and Reichelstein (1995); Melumad, Mookherjee, and Reichelstein (1997)); (ii) renegotiation by the principal due to limited commitment (e.g., Beaudry and Poitevin (1995); Baliga and Sjöström (2001)); or (iii) collusion between agents (e.g., Tirole (1986); Laffont and Martimort (1998); Ortner and Chassang (2018); Troya-Martinez and Wren-Lewis (2018)).[27] In our paper, communication is costless, but the agents do not posses any information valuable to the principal. Moreover, since the agents do not observe each others' efforts, there is no role for collusion between them. So, it is the inability of the principal to commit not to secretly renegotiate contracts bilaterally that makes delegation optimal in our model. Interestingly, the prior literature has been almost entirely concerned with delegation of tasks, and has not been much concerned with the delegation of contracting itself.

9 Conclusion

We have analyzed a moral hazard in teams problem with the realistic innovation that compensation contracts are observed only by their signatories, and not by third-parties. In this environment, principals contracting with agents working in a team face a credibility problem. Since rewards depend on the value of joint output, agents care not only about their own bonus (which they observe) but also about their teammates' effort on the project, and hence, indirectly, about their teammates' incentive pay (which they do not observe). In particular, when effort productivity is complementary, the principal cannot commit not to economize on other agents' pay, thus inducing agents to choose lower effort themselves. In this paper, we explore how these difficulties arising from contractual privacy are affected by the organizational structure of teams.

Our theory highlights a novel trade-off that arises with private contracts between an observability gain from delegation and a rent extraction cost. The principal can gain from making one of the agents

[27]Related work on hierarchies includes Qian (1994) and Rahman (2012), who investigate models in which an agent must be incentivized to monitor the effort exerted by his immediate subordinates. A very different rationale for hierarchies is set out by Garicano (2000), where heterogeneous agents have differing abilities to solve problems, and difficult problems must be passed up the hierarchy to more able agents.

"team leader," giving him a compensation budget that he can choose to share between himself and the other team members. The main benefit is that the team leader now observes all the contracts and consequently no longer fears that the principal will opportunistically offer low pay to his colleague; this improves the team leader's incentives to make effort. The cost is that the team leader selfishly retains too much of the compensation budget, paying his colleagues too little from the principal's point of view. We find that compared to centralized bonus provision, contractual delegation results in higher total compensation, but results in excessive pay inequality. It is worthwhile when both agents are sufficiently skilled (so that effort responds strongly enough to incentives) and not too heterogeneous (so pay inequality is not too severe). Moreover, it is always optimal for the principal to choose the more skilled agent as the team leader, as the ensuing rent extraction is less inefficient.

Despite the above trade-off, we also uncover that delegated contracting can sometimes have an advantage over centralized contracting *even with observability held constant across the two settings*. The reason for this surprising result is that as long as observability remains imperfect in both settings, the distribution of compensation is skewed relative to the second best in both cases, but the direction of the skew differs. With delegation, compensation will be skewed towards the agent with the power to subcontract; whereas with centralized contracting, it will be skewed towards the agent with the public contract. When agents are not too different, and the returns to increasing effort are high, the skew that results from delegation is better for the principal. The principal cannot duplicate the outcome of delegation using centralized contracts even though observability is the same, because without complete observability, promises to increase the bonus of the agent with the unobservable contract are merely cheap talk. Therefore, delegation provides a new way for the principal to commit to higher pay for the team leader, despite his pay being unobserved by his teammates.

Our model reveals a key difficulty of providing incentives in teams in a world where contracts are mostly private information. Our framework can also be useful in addressing other questions about the design of organizations, hierarchies, and incentive structures. What happens to the optimality of delegation versus centralization as the number of agents on the team increases, for example? How should the number of levels of the hierarchy, and the number of agents in each level, be determined if three or more agents must work together? How does the flatness or steepness of the optimal hierarchy vary with the skill level of the agents, or with their asymmetry? We hope to address some of these questions in future research.

Finally, our theory takes as given the difficulty in verifying other agents' contracts, or equivalently, the problems with making compensation public. We take this difficulty as a fact about the world. Since, in our model, the principal would be better off if compensation contracts were made public, it is important to understand the real world problems that firms and institutions endure as a result of being obliged to make compensation contracts public, and the costs that prevent others from doing so. We conjecture that envy of other agents' contracts is one such cost.[28] It would be interesting to explore in future work, theoretically and experimentally, the trade-offs that arise between making the compensation of agents with unequal talents transparent, in order to induce greater effort when efforts are complementary, and keeping them opaque in order to efficiently match incentives to skills without inducing envy.

Appendix

A Proofs

Proof of Proposition 1. In the centralized contracting scheme with private contracts, agent i's maximization problem is

$$e_i(\phi_i b, \hat{e}_j) = \arg\max_{e_i} \quad \phi_i b \; e_i^{\alpha_i} \hat{e}_j^{\alpha_j} - \frac{e_i^2}{2}. \tag{A.1}$$

The first order condition with respect to e_i is

$$\phi_i b \; \alpha_i e_i^{\alpha_i - 1} \hat{e}_j^{\alpha_j} - e_i = 0, \tag{A.2}$$

and the second order condition is

$$\phi_i b \; \alpha_i(\alpha_i - 1) e_i^{\alpha_i - 2} \hat{e}_j^{\alpha_j} - 1 < 0, \tag{A.3}$$

since $\alpha_i < 1$. Therefore, solving for e_i from (A.2), we obtain (7).

In order to induce positive effort from each agent, we consider (and later verify that) $b > 0$ and $\phi_i \in (0, 1)$. Substituting (7) into (8) for $i = 1, 2$, the principal's maximization problem becomes

$$(b^C, \phi^C) = \arg\max_{b, \phi_i} \quad (1-b) \; \alpha_i^{\frac{\alpha_i}{2-\alpha_i}} \alpha_j^{\frac{\alpha_j}{2-\alpha_j}} \hat{e}_i^{\frac{\alpha_i \alpha_j}{2-\alpha_j}} \hat{e}_j^{\frac{\alpha_i \alpha_j}{2-\alpha_i}} \phi_i^{\frac{\alpha_i}{2-\alpha_i}} (1-\phi_i)^{\frac{\alpha_j}{2-\alpha_j}} b^{\frac{2\alpha_i + 2\alpha_j - 2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}}.$$

The first order condition with respect to b is

$$\alpha_i^{\frac{\alpha_i}{2-\alpha_i}} \alpha_j^{\frac{\alpha_j}{2-\alpha_j}} \hat{e}_i^{\frac{\alpha_i \alpha_j}{2-\alpha_j}} \hat{e}_j^{\frac{\alpha_i \alpha_j}{2-\alpha_i}} \phi_i^{\frac{\alpha_i}{2-\alpha_i}} (1-\phi_i)^{\frac{\alpha_j}{2-\alpha_j}} \left(-b^{\frac{2\alpha_i + 2\alpha_j - 2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} + (1-b) \frac{2\alpha_i + 2\alpha_j - 2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)} b^{\frac{2\alpha_i + 2\alpha_j - 2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)} - 1} \right) = 0.$$

[28] Perez-Truglia (2020) documents that when Norwegian income data become more easily available, lower-paid Norwegians' happiness and life satisfaction was reduced relative to their higher-paid peers. Card, Mas, Moretti, and Saez (2012) document that pay transparency reduces job satisfaction and increases the probability of departure for lower-paid workers, while Obloj and Zenger (2017) suggest that allowing bonus comparisons reduces employee productivity because of envy. Cullen and Perez-Truglia (2019) distinguish horizontal transparency (knowledge of one's peers' salary) from vertical transparency (knowledge of one's boss's salary), which have different effects on effort.

Since ϕ, α_i, and α_j are all bounded in $(0,1)$, the first order condition can be reduced to

$$-b^{\frac{2\alpha_i+2\alpha_j-2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} + (1-b)\frac{2\alpha_i+2\alpha_j-2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)} b^{\frac{2\alpha_i+2\alpha_j-2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}-1} = 0,$$

yielding

$$b^C = \frac{2\alpha_i+2\alpha_j-2\alpha_i\alpha_j}{4-\alpha_i\alpha_j}. \tag{A.4}$$

The second order condition, evaluated at $b = b^C$, is

$$\alpha_i^{\frac{\alpha_i}{2-\alpha_i}} \alpha_j^{\frac{\alpha_j}{2-\alpha_j}} \hat{e}_i^{\frac{\alpha_i\alpha_j}{2-\alpha_j}} \hat{e}_j^{\frac{\alpha_i\alpha_j}{2-\alpha_i}} \phi^{\frac{\alpha_i}{2-\alpha_i}} (1-\phi)^{\frac{\alpha_j}{2-\alpha_j}} \frac{2\alpha_i+2\alpha_j-2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)} b^{\frac{2\alpha_i+2\alpha_j-2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}-2}$$
$$\times \left[-2b + (1-b)\left(\frac{2\alpha_i+2\alpha_j-2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)} - 1\right)\right] < 0,$$

since $-2b^C + (1-b^C)\left(\frac{2\alpha_i+2\alpha_j-2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)} - 1\right) = -1$. Hence, b^C maximizes the principal's objective function. Similarly, the first order condition with respect to ϕ is

$$(1-b)\alpha_i^{\frac{\alpha_i}{2-\alpha_i}} \alpha_j^{\frac{\alpha_j}{2-\alpha_j}} \hat{e}_i^{\frac{\alpha_i\alpha_j}{2-\alpha_j}} \hat{e}_j^{\frac{\alpha_i\alpha_j}{2-\alpha_i}} b^{\frac{2\alpha_i+2\alpha_j-2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \left(\frac{\alpha_i}{2-\alpha_i}\phi^{\frac{\alpha_i}{2-\alpha_i}-1}(1-\phi)^{\frac{\alpha_j}{2-\alpha_j}} - \phi^{\frac{\alpha_i}{2-\alpha_i}} \frac{\alpha_j}{2-\alpha_j}(1-\phi)^{\frac{\alpha_j}{2-\alpha_j}-1}\right) = 0.$$

Since $b = 1$ is not an optimal choice for the principal, and α_i, and α_j are bounded in $(0,1)$, the first order condition can be reduced to

$$\frac{\alpha_i}{2-\alpha_i}\phi^{\frac{\alpha_i}{2-\alpha_i}-1}(1-\phi)^{\frac{\alpha_j}{2-\alpha_j}} - \phi^{\frac{\alpha_i}{2-\alpha_i}} \frac{\alpha_j}{2-\alpha_j}(1-\phi)^{\frac{\alpha_j}{2-\alpha_j}-1} = 0,$$

yielding

$$\phi^C = \frac{2\alpha_i-\alpha_i\alpha_j}{2\alpha_i+2\alpha_j-2\alpha_i\alpha_j}. \tag{A.5}$$

The second order condition is

$$(1-b)\alpha_i^{\frac{\alpha_i}{2-\alpha_i}} \alpha_j^{\frac{\alpha_j}{2-\alpha_j}} \hat{e}_i^{\frac{\alpha_i\alpha_j}{2-\alpha_j}} \hat{e}_j^{\frac{\alpha_i\alpha_j}{2-\alpha_i}} b^{\frac{2\alpha_i+2\alpha_j-2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \phi^{\frac{\alpha_i}{2-\alpha_i}-2}(1-\phi)^{\frac{\alpha_j}{2-\alpha_j}-2} \times$$
$$\left(\frac{\alpha_i}{2-\alpha_i}\Big(\frac{\alpha_i}{2-\alpha_i}-1\Big)(1-\phi)^2 - 2\frac{\alpha_i}{2-\alpha_i}\phi\frac{\alpha_j}{2-\alpha_j}(1-\phi) + \phi^2\frac{\alpha_j}{2-\alpha_j}\Big(\frac{\alpha_j}{2-\alpha_j}-1\Big)\right) < 0,$$

since α_i and $\alpha_j \in (0,1)$ imply $\frac{\alpha_i}{2-\alpha_i} < 1$ and $\frac{\alpha_j}{2-\alpha_j} < 1$. Hence, ϕ^C maximizes the principal's objective function. Since α_i and $\alpha_j \in (0,1)$, b^C and $\phi^C \in (0,1)$.

Agent i's equilibrium compensation is $\phi^C b^C = \frac{2\alpha_i-\alpha_i\alpha_j}{4-\alpha_i\alpha_i} \in (0,1)$, while agent j's equilibrium compensation is $(1-\phi^C)b^C = \frac{2\alpha_j-\alpha_i\alpha_j}{4-\alpha_i\alpha_i} \in (0,1)$. The equilibrium condition requires that

$$\hat{e}_i = e_i^C, \tag{A.6}$$
$$\hat{e}_j = e_j^C. \tag{A.7}$$

Substituting (A.6) and (A.7) into the first order conditions of each agent's effort (A.2), we obtain

$$\begin{cases} e_i^C = \alpha_i^{\frac{1}{2-\alpha_i}} \left(e_j^C\right)^{\frac{\alpha_j}{2-\alpha_i}} \left(\phi^C b^C\right)^{\frac{1}{2-\alpha_i}}, \\ e_j^C = \alpha_j^{\frac{1}{2-\alpha_j}} \left(e_i^C\right)^{\frac{\alpha_i}{2-\alpha_j}} \left((1-\phi^C) b^C\right)^{\frac{1}{2-\alpha_j}}. \end{cases}$$

Solving the system of equations in (e_i^C, e_j^C), we obtain the equilibrium effort levels of the two agents:

$$e_i^C = \alpha_i^{\frac{2-\alpha_j}{2(2-\alpha_i-\alpha_j)}} \alpha_j^{\frac{\alpha_j}{2(2-\alpha_i-\alpha_j)}} \left(\phi^C\right)^{\frac{2-\alpha_j}{2(2-\alpha_i-\alpha_j)}} \left(1-\phi^C\right)^{\frac{\alpha_j}{2(2-\alpha_i-\alpha_j)}} \left(b^C\right)^{\frac{1}{2-\alpha_i-\alpha_j}}, \tag{A.8}$$

$$e_j^C = \alpha_i^{\frac{\alpha_i}{2(2-\alpha_i-\alpha_j)}} \alpha_j^{\frac{2-\alpha_i}{2(2-\alpha_i-\alpha_j)}} \left(\phi^C\right)^{\frac{\alpha_i}{2(2-\alpha_i-\alpha_j)}} \left(1-\phi^C\right)^{\frac{2-\alpha_i}{2(2-\alpha_i-\alpha_j)}} \left(b^C\right)^{\frac{1}{2-\alpha_i-\alpha_j}}. \tag{A.9}$$

Since b^C, ϕ^C, α_i, and α_j are all bounded in $(0,1)$, e_i^C and $e_j^C \in (0,1)$. Substituting (A.8) and (A.9) into (2), we obtain the equilibrium probability of success under centralized contracting

$$\pi^C = \alpha_i^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} \alpha_j^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} \left(\phi^C\right)^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} \left(1-\phi^C\right)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} \left(b^C\right)^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}}.$$

The principal's expected payoff, $v^C = (1-b^C)\pi^C$, is strictly positive since b^C and $\pi^C \in (0,1)$. So, implementing the risky project under the centralized contracting scheme is profitable (in expectation) for the principal. The expected payoffs of agent i and agent j are equal to

$$u_i^C = \left(1 - \frac{\alpha_i}{2}\right) \phi^C b^C \pi^C,$$
$$u_j^C = \left(1 - \frac{\alpha_j}{2}\right) (1-\phi^C) b^C \pi^C,$$

respectively. Since α_i, α_j, ϕ^C, b^C, and π^C are $\in (0,1)$, the agents' expected payoffs are strictly positive. So, both agents' participation constraints are satisfied.

The optimal compensation budget, the budget allocation, as well as the agents' total compensations, have the following properties:

$$\frac{\partial b^C}{\partial \alpha_i} = \frac{2(2-\alpha_j)}{(4-\alpha_i\alpha_j)^2} > 0, \qquad \frac{\partial b^C}{\partial \alpha_j} = \frac{2(2-\alpha_i)}{(4-\alpha_i\alpha_j)^2} > 0,$$

$$\frac{\partial \phi^C}{\partial \alpha_i} = \frac{\alpha_j(2-\alpha_j)}{2(\alpha_i+\alpha_j-\alpha_i\alpha_j)^2} > 0, \qquad \frac{\partial \phi^C}{\partial \alpha_j} = -\frac{\alpha_i(2-\alpha_i)}{2(\alpha_i+\alpha_j-\alpha_i\alpha_j)^2} < 0,$$

$$\frac{\partial \phi^C b^C}{\partial \alpha_i} = \frac{4(2-\alpha_j)}{(4-\alpha_i\alpha_j)^2} > 0, \qquad \frac{\partial \phi^C b^C}{\partial \alpha_j} = -\frac{2\alpha_i(2-\alpha_i)}{(4-\alpha_i\alpha_j)^2} < 0,$$

$$\frac{\partial \left(1-\phi^C\right) b^C}{\partial \alpha_i} = -\frac{2\alpha_j(2-\alpha_j)}{(4-\alpha_i\alpha_j)^2} < 0, \qquad \frac{\partial \left(1-\phi^C\right) b^C}{\partial \alpha_j} = \frac{4(2-\alpha_i)}{(4-\alpha_i\alpha_j)^2} > 0.$$

□

Proof of Corollary 1. The optimal private contracts in the centralized contracting scheme are given in Proposition 1 and are equal to

$$b^C = \frac{2(\alpha_i+\alpha_j-\alpha_i\alpha_j)}{4-\alpha_i\alpha_j}, \qquad \phi_i^C = \frac{2\alpha_i - \alpha_i\alpha_j}{2(\alpha_i+\alpha_j-\alpha_i\alpha_j)}.$$

The optimal public contracts in the centralized contracting scheme, instead, are given in Proposition B.1 in the Online Appendix B and are equal to

$$b^* = \frac{\alpha_i + \alpha_j}{2}, \qquad \phi_i^* = \frac{\alpha_i}{\alpha_i + \alpha_j}.$$

We compare these optimal contracts along the following dimensions:

(i) Compensation budget, b: $b^C < b^*$ since α_i and $\alpha_j \in (0,1)$.

(ii) Budget allocation to agent i, ϕ_i: $\phi_i^C > \phi_i^*$ since $0 < \alpha_j < \alpha_i < 1$.

(iii) Agent i's compensation, $\phi_i b$:

$$\phi_i^C b^C = \frac{2\alpha_i - \alpha_i\alpha_j}{4 - \alpha_i\alpha_j}, \qquad \phi_i^* b^* = \frac{\alpha_i}{2}$$

implies that $\phi_i^C b^C < \phi_i^* b^*$ since α_i and $\alpha_j \in (0,1)$.

(iv) Agent j's compensation $\phi_j b$:

$$\phi_j^C b^C = (1 - \phi_i^C) b^C = \frac{2\alpha_j - \alpha_i\alpha_j}{4 - \alpha_i\alpha_j}, \qquad \phi_j^* b^* = (1 - \phi_i^*) b^* = \frac{\alpha_j}{2}$$

implies that $\phi_j^C b^C < \phi_j^* b^*$ since α_i and $\alpha_j \in (0,1)$.

□

Proof of Proposition 2. In the delegated contracting scheme with private contracts, the Subagent's maximization problem is the same as that in the centralized contracting scheme. Therefore, the Subagent's optimal effort level is determined as

$$e_S((1-\phi)b, \hat{e}_A) = \alpha_S^{\frac{1}{2-\alpha_S}} \hat{e}_A^{\frac{\alpha_A}{2-\alpha_S}} ((1-\phi)b)^{\frac{1}{2-\alpha_S}}. \tag{A.10}$$

The Agent's maximization problem with respect to his effort level is given by

$$e_A(b, \phi, \hat{e}_A) = \arg\max_{e_A} \ \phi b \ e_A^{\alpha_A} e_S((1-\phi)b, \hat{e}_A)^{\alpha_S} - \frac{e_A^2}{2}.$$

The first order condition with respect to e_A is

$$\phi b \ \alpha_A e_A^{\alpha_A - 1} e_S((1-\phi)b, \hat{e}_A)^{\alpha_S} - e_A = 0, \tag{A.11}$$

and the second order condition is

$$\phi b \ \alpha_A(\alpha_A - 1) e_A^{\alpha_A - 2} e_S((1-\phi)b, \hat{e}_A)^{\alpha_S} - 1 < 0,$$

since $\alpha_A < 1$. Solving (A.11) for the effort level e_A, we obtain

$$e_A(b, \phi, \hat{e}_A) = \alpha_A{}^{\frac{1}{2-\alpha_A}} \alpha_S{}^{\frac{\alpha_S}{(2-\alpha_A)(2-\alpha_S)}} \hat{e}_A^{\frac{\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}} \phi^{\frac{1}{2-\alpha_A}} (1-\phi)^{\frac{\alpha_S}{(2-\alpha_A)(2-\alpha_S)}} b^{\frac{2}{(2-\alpha_A)(2-\alpha_S)}}, \tag{A.12}$$

which corresponds to (14). Given his optimal effort choice, the Agent's maximization problem with respect to the budget allocation is given by

$$\phi_A^D = \arg\max_{\phi} \quad \phi b\ e_A(b,\phi,\hat{e}_A)^{\alpha_A} e_S((1-\phi)b,\hat{e}_A)^{\alpha_S} - \frac{e_A(b,\phi,\hat{e}_A)^2}{2},$$

$$= \arg\max_{\phi} \quad \left(1-\frac{\alpha_A}{2}\right)\alpha_A^{\frac{\alpha_A}{2-\alpha_A}}\alpha_S^{\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}\hat{e}_A^{\frac{2\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}\phi^{\frac{2}{2-\alpha_A}}(1-\phi)^{\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}b^{\frac{4}{(2-\alpha_A)(2-\alpha_S)}}.$$

The first order condition with respect to ϕ is

$$\left(1-\frac{\alpha_A}{2}\right)\alpha_A^{\frac{\alpha_A}{2-\alpha_A}}\alpha_S^{\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}\hat{e}_A^{\frac{2\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}b^{\frac{4}{(2-\alpha_A)(2-\alpha_S)}}$$
$$\times\left(\frac{2}{2-\alpha_A}\phi^{\frac{2}{2-\alpha_A}-1}(1-\phi)^{\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}-\phi^{\frac{2}{2-\alpha_A}}\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}(1-\phi)^{\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}-1}\right)=0.$$

Since, as we show later, the Agent's equilibrium effort choice is bounded in $(0,1)$, the first order condition can be reduced to

$$\frac{2}{2-\alpha_A}(1-\phi)-\phi\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}=0,$$

yielding

$$\phi_A^D = 1-\frac{\alpha_S}{2}. \tag{A.13}$$

The second order condition, evaluated at $\phi=\phi_A^D$ is

$$\left(1-\frac{\alpha_A}{2}\right)\alpha_A^{\frac{\alpha_A}{2-\alpha_A}}\alpha_S^{\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}\hat{e}_A^{\frac{2\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}b^{\frac{4}{(2-\alpha_A)(2-\alpha_S)}}\phi^{\frac{2}{2-\alpha_A}-2}(1-\phi)^{\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}-2}\frac{2}{2-\alpha_A}$$
$$\times\left[\left(\frac{2}{2-\alpha_A}-1\right)(1-\phi)^2-2\phi\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}(1-\phi)+\phi^2\frac{\alpha_S}{2-\alpha_S}\left(\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}-1\right)\right]<0,$$

since

$$\left(\frac{2}{2-\alpha_A}-1\right)(1-\phi_A^D)^2-2\phi_A^D\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}(1-\phi_A^D)+(\phi_A^D)^2\frac{\alpha_S}{2-\alpha_S}\left(\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}-1\right)=-\frac{\alpha_S}{2}.$$

Hence, ϕ_A^D maximizes the Agent's objective function.

The principal's maximization problem is given by

$$b^D = \arg\max_{b} \quad (1-b)\ e_A(b,\phi_A^D,\hat{e}_A)^{\alpha_A} e_S((1-\phi_A^D)b,\hat{e}_A)^{\alpha_S},$$

$$= \arg\max_{b} \quad (1-b)\ \alpha_A^{\frac{\alpha_A}{2-\alpha_A}}\alpha_S^{\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}\hat{e}_A^{\frac{2\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}(\phi_A^D)^{\frac{\alpha_A}{2-\alpha_A}}(1-\phi_A^D)^{\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}b^{\frac{2\alpha_A+2\alpha_S-\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}.$$

The first order condition with respect to b is

$$\alpha_A^{\frac{\alpha_A}{2-\alpha_A}}\alpha_S^{\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}\hat{e}_A^{\frac{2\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}(\phi_A^D)^{\frac{\alpha_A}{2-\alpha_A}}(1-\phi_A^D)^{\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}$$
$$\times\left(-b^{\frac{2\alpha_A+2\alpha_S-\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}}+(1-b)\frac{2\alpha_A+2\alpha_S-\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}b^{\frac{2\alpha_A+2\alpha_S-\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}-1}\right)=0.$$

Since ϕ_A^D, α_A, and α_S are all bounded in $(0,1)$, the first order condition can be reduced to

$$-b^{\frac{2\alpha_A+2\alpha_S-\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}} + (1-b)\frac{2\alpha_A+2\alpha_S-\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)} b^{\frac{2\alpha_A+2\alpha_S-\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}-1} = 0,$$

yielding

$$b^D = \frac{2\alpha_A+2\alpha_S-\alpha_A\alpha_S}{4}. \tag{A.14}$$

The second order condition, evaluated at $b = b^D$, is

$$\alpha_A^{\frac{\alpha_A}{2-\alpha_A}} \alpha_S^{\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}} \hat{e}_A^{\frac{2\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}} (\phi_A^D)^{\frac{\alpha_A}{2-\alpha_A}} (1-\phi_A^D)^{\frac{2\alpha_S}{(2-\alpha_A)(2-\alpha_S)}} \frac{2\alpha_A+2\alpha_S-\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)} b^{\frac{2\alpha_A+2\alpha_S-\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}-2}$$
$$\times\left[-2b + (1-b)\left(\frac{2\alpha_A+2\alpha_S-\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)} - 1\right)\right] < 0,$$

since $-2b^D + (1-b^D)\left(\frac{2\alpha_A+2\alpha_S-\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)} - 1\right) = -1$. Hence, b^D maximizes the principal's objective function. Since α_A and $\alpha_S \in (0,1)$, b^D and $\phi_A^D \in (0,1)$.

The Agent's equilibrium compensation is $\phi_A^D b^D = \left(1-\frac{\alpha_S}{2}\right)\frac{2\alpha_A+2\alpha_S-\alpha_A\alpha_S}{4} \in (0,1)$, while the Subagent's equilibrium compensation is $(1-\phi_A^D)b^D = \frac{\alpha_S}{2}\frac{2\alpha_A+2\alpha_S-\alpha_A\alpha_S}{4} \in (0,1)$. The equilibrium condition requires that

$$\hat{e}_A = e_A^D. \tag{A.15}$$

Substituting (A.15) into the first order condition of the Agent's effort (A.11) and the first order condition of the Subagent's effort (A.10), we obtain

$$\begin{cases} e_A^D = \alpha_A^{\frac{1}{2-\alpha_A}} \alpha_S^{\frac{\alpha_S}{(2-\alpha_A)(2-\alpha_S)}} (e_A{}^D)^{\frac{\alpha_A\alpha_S}{(2-\alpha_A)(2-\alpha_S)}} (\phi_A^D)^{\frac{1}{2-\alpha_A}} (1-\phi_A^D)^{\frac{\alpha_S}{(2-\alpha_A)(2-\alpha_S)}} (b^D)^{\frac{2}{(2-\alpha_A)(2-\alpha_S)}}, \\ e_S^D = \alpha_S^{\frac{1}{2-\alpha_S}} (e_A{}^D)^{\frac{\alpha_A}{2-\alpha_S}} ((1-\phi_A^D)b^D)^{\frac{1}{2-\alpha_S}}. \end{cases}$$

Solving the system of equations in (e_A^D, e_S^D), we obtain the equilibrium effort levels of the two agents:

$$e_A^D = \alpha_A^{\frac{2-\alpha_S}{2(2-\alpha_A-\alpha_S)}} \alpha_S^{\frac{\alpha_S}{2(2-\alpha_A-\alpha_S)}} (\phi_A^D)^{\frac{2-\alpha_S}{2(2-\alpha_A-\alpha_S)}} (1-\phi_A^D)^{\frac{\alpha_S}{2(2-\alpha_A-\alpha_S)}} (b^D)^{\frac{1}{2-\alpha_A-\alpha_S}}, \tag{A.16}$$

$$e_S^D = \alpha_A^{\frac{\alpha_A}{2(2-\alpha_A-\alpha_S)}} \alpha_S^{\frac{2-\alpha_A}{2(2-\alpha_A-\alpha_S)}} (\phi_A^D)^{\frac{\alpha_A}{2(2-\alpha_A-\alpha_S)}} (1-\phi_A^D)^{\frac{2-\alpha_A}{2(2-\alpha_A-\alpha_S)}} (b^D)^{\frac{1}{2-\alpha_A-\alpha_S}}. \tag{A.17}$$

Since b^D, ϕ_A^D, α_A, and α_S are all bounded in $(0,1)$, e_A^D and $e_S^D \in (0,1)$. Substituting (A.16) and (A.17) into (2), we obtain the equilibrium probability of success under delegated contracting

$$\pi^D = \alpha_A^{\frac{\alpha_A}{2-\alpha_A-\alpha_S}} \alpha_S^{\frac{\alpha_S}{2-\alpha_A-\alpha_S}} (\phi_A^D)^{\frac{\alpha_A}{2-\alpha_A-\alpha_S}} (1-\phi_A^D)^{\frac{\alpha_S}{2-\alpha_A-\alpha_S}} (b^D)^{\frac{\alpha_A+\alpha_S}{2-\alpha_A-\alpha_S}},$$

The principal's expected payoff, $v^D = (1-b^D)\pi^D$, is equal to

$$v^D = \left[(1-b^D)\alpha_A^{\frac{\alpha_A}{2-\alpha_A-\alpha_S}} \alpha_S^{\frac{\alpha_S}{2-\alpha_A-\alpha_S}} (b^D)^{\frac{\alpha_A+\alpha_S}{2-\alpha_A-\alpha_S}}\right] \left[(\phi_A^D)^{\alpha_A}(1-\phi_A^D)^{\alpha_S}\right]^{\frac{1}{2-\alpha_A-\alpha_S}} > 0, \tag{A.18}$$

since $b^D \in (0,1)$. So, implementing the risky project under the centralized contracting scheme is profitable (in expectation) for the principal. The expected payoffs of the Agent and the Subagent are equal to

$$\begin{aligned} u_A^D &= \left(1 - \frac{\alpha_A}{2}\right) \phi_A^D b^D \pi^D, \\ u_S^D &= \left(1 - \frac{\alpha_S}{2}\right) (1 - \phi_A^D) b^D \pi^D, \end{aligned}$$

respectively. Since α_A, α_S, ϕ_A^D, b^D, and π^D are $\in (0,1)$, the agent's expected payoffs are strictly positive. So, both agents' participation constraints are satisfied.

The optimal compensation budget, the budget allocation, as well as the agents' total compensations, have the following properties:

$$\begin{aligned} \frac{\partial b^D}{\partial \alpha_A} &= \frac{2-\alpha_S}{4} > 0, & \frac{\partial b^D}{\partial \alpha_S} &= \frac{2-\alpha_A}{4} > 0, \\ \frac{\partial \phi_A^D}{\partial \alpha_A} &= 0, & \frac{\partial \phi_A^D}{\partial \alpha_S} &= -\frac{1}{2} < 0, \\ \frac{\partial \phi_A^D b^D}{\partial \alpha_A} &= \frac{(2-\alpha_S)^2}{8} > 0, & \frac{\partial \phi_A^D b^D}{\partial \alpha_S} &= \frac{\alpha_A\alpha_S - 2\alpha_A - 2\alpha_S + 2}{4} \gtreqless 0, \\ \frac{\partial \big(1-\phi_A^D\big) b^D}{\partial \alpha_A} &= \frac{\alpha_S(2-\alpha_S)}{8} > 0, & \frac{\partial \big(1-\phi_A^D\big) b^D}{\partial \alpha_S} &= \frac{\alpha_A(1-\alpha_S)+2\alpha_S}{4} > 0. \end{aligned}$$

□

Proof of Corollary 2. Without loss of generality, we assume that

$$0 < \alpha_j \leq \alpha_i < 1.$$

Given the optimal contracts in Proposition 1 and Proposition 2, we compare the two contracting schemes with respect to the following equilibrium quantities:

(i) Compensation budget, b: $b^C < b^D$ since α_i, $\alpha_j \in (0,1)$.

(ii) Budget allocation to the Agent (i.e., agent i), ϕ_i: $\phi_i^C < \phi_{A=i}^D$ since α_i, $\alpha_j \in (0,1)$.

(iii) The Agent's compensation, $\phi_i b$:

$$\phi_i^C b^C = \frac{2\alpha_i - \alpha_i\alpha_j}{4 - \alpha_i\alpha_j}, \qquad \phi_{A=i}^D b^D = \frac{(2-\alpha_j)(2\alpha_i + 2\alpha_j - \alpha_i\alpha_j)}{8}.$$

Define $f(x,y) \equiv x^2y - 2xy - 2x^2 - 4x + 8$ where $0 \leq y \leq x \leq 1$. Taking the first order partial derivative with respect to y, we get $f_y(x,y) = x^2 - 2x = (1-x)^2 - 1 < 0$, for any $0 < x \leq 1$. Therefore, $f(x,y)$ is decreasing in $y \in (0,x]$. Hence, $f(x,y) > f(x,x) = x^3 - 4x^2 - 4x + 8$ for $0 \leq y < x \leq 1$. Define $g(x) \equiv x^3 - 4x^2 - 4x + 8$ for $x \in [0,1]$. Taking the first order derivative, we get $g_x(x) = 3x^2 - 8x - 4 = 3(x - \frac{4}{3})^2 - \frac{28}{3} < 0$, for any $x \in [0,1]$. Therefore, $g(x)$ is decreasing in $[0,1]$ and $g(x) > g(1) = 1$ for any $x \in (0,1)$. $g(x) > 0$ implies that $f(x,y) > 0$ for any $0 < y < x < 1$. Rearranging $f(\alpha_i, \alpha_j) > 0$, we obtain that $\frac{2\alpha_i - \alpha_i\alpha_j}{4-\alpha_i\alpha_j} < \frac{(2-\alpha_j)(2\alpha_i+2\alpha_j-\alpha_i\alpha_j)}{8}$. Hence,

$$\phi_i^C b^C < \phi_{A=i}^D b^D.$$

(iv) The Subagent's compensation, $\phi_j b$:

$$\phi_j^C b^C = (1-\phi_i^C)b^C = \frac{2\alpha_j - \alpha_i\alpha_j}{4-\alpha_i\alpha_j}, \qquad \phi_{S=j}^D b^D = (1-\phi_{A=i}^D)b^D = \frac{\alpha_j(2\alpha_i + 2\alpha_j - \alpha_i\alpha_j)}{8}.$$

Define $f(x,y) \equiv x^2y^2 - 2x^2y - 2xy^2 - 4xy + 16x + 8y - 16$ where $0 \le y \le x \le 1$. Taking the first order partial derivative with respect to y, we obtain $f_y(x,y) = 2x^2y - 2x^2 - 4xy - 4x + 8$. Taking the second order partial derivative with respect to y, we obtain $f_{yy}(x,y) = 2x^2 - 4x = 2(x-1)^2 - 2 < 0$ for any $0 < x \le 1$. Therefore, $f_y(x,y)$ is decreasing in $y \in [0,x]$ and $f_y(x,y) > f_y(x,x)$ where $y \in [0,x)$. Define $g(x) \equiv f_y(x,x) = 2x^3 - 6x^2 - 4x + 8$ where $0 \le x \le 1$. Taking the first order derivate, we obtain $g_x(x) = 6x^2 - 12x - 4 = 6(x-1)^2 - 10 < 0$ for any $x \in [0,1]$. Therefore, $g(x)$ is decreasing in $x \in [0,1]$. Hence, $g(x) > g(1) = 0$ where $0 \le x < 1$. This implies that $f_y(x,y) > 0$ for any $y \in [0,x)$. Therefore, $f(x,y)$ is increasing in $y \in [0,x)$. Hence, $f(x,0) < f(x,y) < f(x,x)$ for any $y \in (0,x)$, where $f(x,0) = 16(x-1) < 0$ for any $x \in (0,1)$. Define $h(x) \equiv f(x,x) = x^4 - 4x^3 - 4x^2 + 24x - 16$ where $x \in [0,1]$. Taking the first order derivative, we obtain $h_x(x) = 4x^3 - 12x^2 - 8x + 24$. Taking the second order derivative, we obtain $h_{xx}(x) = 12x^2 - 24x - 8 = 12(x-1)^2 - 20 < 0$ $\forall x \in [0,1]$. Therefore, $h_x(x)$ is decreasing in $x \in [0,1]$, and $h_x(x) > h_x(1) = 8 > 0$. Hence, $h(x)$ is increasing in $x \in [0,1]$. Since, $f(1,1) = h(1) = 1 > 0$, combined with $f(x,0) < 0$ for any $x \in (0,1]$, it follows that $f(x,y) > 0$ for any $0 < y \le x < 1$ as long as x is close enough to 1. As a result, there exists a unique $\bar{\alpha}_i^c(\alpha_j) \in (0,1)$ such that

$$\begin{cases} (1-\phi_i^C)b^C \ge (1-\phi_i^D)b^D & \text{for} \quad \alpha_i \le \bar{\alpha}_i^c(\alpha_j) \\ (1-\phi_i^C)b^C < (1-\phi_i^D)b^D & \text{for} \quad \alpha_i > \bar{\alpha}_i^c(\alpha_j). \end{cases}$$

Next, consider the implicit function $f(x,y) = 0$ where $0 \le y \le x \le 1$. From the above derivation, $f_y(x,y) > 0$ for any $0 < y \le x < 1$. Taking the first order partial derivative with respect to x, we obtain $f_x(x,y) = 2xy^2 - 4xy - 2y^2 - 4y + 16$. Taking the second order partial derivative with respect to x, we get $f_{xx}(x,y) = 2y^2 - 4y = 2(y-1)^2 - 2 < 0$ for any $0 < y < 1$. Hence, $f_x(x,y)$ is decreasing in $x \in (y,1)$, and $f_x(x,y) > f_x(1,y) = -8y + 16 > 0$ for any $0 < y < 1$. Therefore, $f_x(x,y) > 0$ for any $0 < y \le x < 1$. Using the Implicit Function Theorem, $\frac{dx}{dy} = -\frac{f_y(x,y)}{f_x(x,y)} < 0$, for any $0 < y < x < 1$. As a result,

$$\frac{\partial \bar{\alpha}_i^c(\alpha_j)}{\partial \alpha_j} < 0.$$

(v) The Agent's effort level, e_i:

$$\frac{e_i^C}{e_i^D} = \frac{(\phi^C)^{\frac{2-\alpha_j}{2(2-\alpha_i-\alpha_j)}}(1-\phi^C)^{\frac{\alpha_j}{2(2-\alpha_i-\alpha_j)}}(b^C)^{\frac{1}{2-\alpha_i-\alpha_j}}}{(\phi^D)^{\frac{2-\alpha_j}{2(2-\alpha_i-\alpha_j)}}(1-\phi^D)^{\frac{\alpha_j}{2(2-\alpha_i-\alpha_j)}}(b^D)^{\frac{1}{2-\alpha_i-\alpha_j}}},$$

implying that

$$\left(\frac{e_i^C}{e_i^D}\right)^{2-\alpha_i-\alpha_j} = \frac{\left(\frac{\alpha_i}{2}\right)^{1-\frac{\alpha_j}{2}}\left(1-\frac{\alpha_i}{2}\right)^{\frac{\alpha_j}{2}}}{\left(1-\frac{\alpha_i}{2}\frac{\alpha_j}{2}\right)\left(\frac{\alpha_i}{2}+\frac{\alpha_j}{2}-\frac{\alpha_i}{2}\frac{\alpha_j}{2}\right)}. \tag{A.19}$$

Define $f(x,y) \equiv x^{1-y}(1-x)^y/[(1-xy)(x+y-xy)]$ where $0 < y \le x < \frac{1}{2}$. Taking the first order partial derivative with respect to x, we obtain

$$f_x(x,y) = \frac{(1-y)y(-x^3 + x^2(y+1) - x(y+2) + 1)}{(1-x)^{1-y}x^y(1-xy)^2(x+y-xy)^2}.$$

Define $g(x,y) \equiv -x^3 + x^2(y+1) - x(y+2) + 1$ where $0 \le y \le x \le \frac{1}{2}$. Taking the first order partial derivative with respect to x, we obtain $g_x(x,y) = -3x^2+2x(y+1)-y-2 = -3(x-\frac{1}{3})^2-(1-2x)y-\frac{5}{3} < 0$ for any $0 \le y \le x \le \frac{1}{2}$. Therefore, $g(x,y)$ is decreasing in $x \in [y, \frac{1}{2}]$ and $g(x,y) > g(\frac{1}{2}, y) = \frac{1}{8}(1-2y) > 0$ for any $0 < y \le x < \frac{1}{2}$. Hence,

$$f_x(x,y) = \left[\frac{(1-y)y}{(1-x)^{1-y}x^y(1-xy)^2(x+y-xy)^2}\right] g(x,y) > 0$$

for any $0 < y \le x < \frac{1}{2}$. It follows that $f(x,y)$ is increasing in $x \in [y, \frac{1}{2})$ and $f(x,y) < f(\frac{1}{2}, y) = \frac{2}{-(y-\frac{1}{2})^2+\frac{9}{4}} < \frac{2}{-(y-\frac{1}{2})^2+\frac{9}{4}}\Big|_{y=0} = 1$ for any $0 < y \le x < \frac{1}{2}$. As a result,

$$e_i^C < e_i^D.$$

(vi) The Subagent's effort level, e_j:

$$\frac{e_j^C}{e_j^D} = \frac{(\phi^C)^{\frac{\alpha_i}{2(2-\alpha_i-\alpha_j)}}(1-\phi^C)^{\frac{2-\alpha_i}{2(2-\alpha_i-\alpha_j)}}(b^C)^{\frac{1}{2-\alpha_i-\alpha_j}}}{(\phi^D)^{\frac{\alpha_i}{2(2-\alpha_i-\alpha_j)}}(1-\phi^D)^{\frac{2-\alpha_i}{2(2-\alpha_i-\alpha_j)}}(b^D)^{\frac{1}{2-\alpha_i-\alpha_j}}}, \tag{A.20}$$

implying that

$$\left(\frac{e_j^C}{e_j^D}\right)^{2-\alpha_i-\alpha_j} = \frac{\left(\frac{\alpha_i}{2}\right)^{\frac{\alpha_i}{2}}\left(1-\frac{\alpha_i}{2}\right)^{1-\frac{\alpha_i}{2}}}{\left(1-\frac{\alpha_i}{2}\frac{\alpha_j}{2}\right)\left(\frac{\alpha_i}{2}+\frac{\alpha_j}{2}-\frac{\alpha_i}{2}\frac{\alpha_j}{2}\right)}. \tag{A.21}$$

Define $f(x,y) \equiv x^x(1-x)^{1-x}/[(1-xy)(x+y-xy)]$ where $0 < y \le x \le \frac{1}{2}$. Taking the first order partial derivative with respect to x, we obtain

$$f_x(x,y) = \frac{x^x(1-x)^{1-x}}{(1-xy)(x+y-xy)}\left(\frac{y}{1-xy} - \frac{1-y}{x+y-xy} + \log\frac{x}{1-x}\right).$$

Define $g(x,y) \equiv \frac{y}{1-xy} - \frac{1-y}{x+y-xy} + \log\frac{x}{1-x}$ where $0 < y \le x \le \frac{1}{2}$. Taking the first order partial derivative with respect to x, we obtain $g_x(x,y) = \frac{1}{x(1-x)} + \frac{y^2}{(1-xy)^2} + \frac{(1-y)^2}{(x+y-xy)^2} > 0$ for any $0 < y \le x \le \frac{1}{2}$. Therefore, $g(x,y)$ is increasing in $x \in [y, \frac{1}{2}]$, and $g(x,y) < g(\frac{1}{2}, y) = \frac{4(1-2y)}{(y-\frac{1}{2})^2-\frac{9}{4}} < 0$ for any $y \in (0, \frac{1}{2})$. Hence,

$$f_x(x,y) = \left[\frac{x^x(1-x)^{1-x}}{(1-xy)(x+y-xy)}\right] g(x,y) < 0$$

for any $0 < y \le x < \frac{1}{2}$. It follows that $f(x,y)$ is decreasing in $x \in [y, \frac{1}{2}]$, and $f(x,y) > f(\frac{1}{2}, y)$ for any $0 < y \le x < \frac{1}{2}$, since $f(\frac{1}{2}, y) = \frac{2}{(2-y)(1+y)} < 1$ for any $0 < y \le x < \frac{1}{2}$. Continuity of $f(x,y)$ implies that $\lim_{x\to\frac{1}{2}^-} f(x,y) = f(\frac{1}{2},y) < 1$ for any $y \in (0,x]$. Therefore, $f(x,y) < 1$ for any $0 < y \le x < \frac{1}{2}$ as long as x is close enough to $\frac{1}{2}$. As a result, there exists a unique $\bar{\alpha}_i^e(\alpha_j) \in (0,1)$ such that

$$\begin{cases} e_j^C \ge e_j^D & \text{for} \quad \alpha_i \le \bar{\alpha}_i^e(\alpha_j) \\ e_j^C < e_j^D & \text{for} \quad \alpha_i > \bar{\alpha}_i^e(\alpha_j). \end{cases}$$

Next, consider an implicit function $f(x,y) = 1$ where $0 < y \le x \le \frac{1}{2}$. From the above derivation, $f_x(x,y) < 0$ for any $0 < y \le x < \frac{1}{2}$. Taking the first order partial derivative with respect to y, we obtain

$$f_y(x,y) = -\frac{x^x(1-x)^{1-x}}{(1-xy)^2(x+y-xy)^2}\left[x^2(2y-1) - x(2y+1) + 1\right].$$

Define $h(x,y) \equiv x^2(2y-1) - x(2y+1) + 1$ where $0 < y \le x \le \frac{1}{2}$. Taking the first order partial derivative with respect to x, we obtain $h_x(x,y) = 2x(2y-1) - 2y - 1$. Taking the second order derivative with respect to x, we obtain $h_{xx}(x,y) = 2(2y-1) < 0$ for any $0 < y \le x < \frac{1}{2}$. Therefore, $h_x(x,y)$ is decreasing in $x \in [y, \frac{1}{2})$, and $h_x(x,y) \le h_x(y,y) = (1-2y)^2 - 2 < 0$ for any $0 < y \le x \le \frac{1}{2}$. Hence, $h(x,y)$ is decreasing in $x \in [y, \frac{1}{2}]$ and $h(x,y) > h(\frac{1}{2},y) = \frac{1}{4}(1-2y) > 0$ for any $0 < y \le x < \frac{1}{2}$. It follows that,

$$f_y(x,y) = -\frac{x^x(1-x)^{1-x}}{(1-xy)^2(x+y-xy)^2}h(x,y) < 0$$

for any $0 < y \le x < \frac{1}{2}$. Using the *Implicit Function Theorem*, $\frac{dx}{dy} = -\frac{f_y(x,y)}{f_x(x,y)} < 0$ for any $\forall 0 < y \le x < 1$. As a result,

$$\frac{\partial \bar{\alpha}_i^e(\alpha_j)}{\partial \alpha_j} < 0.$$

(vii) The probability of success, π:

$$\frac{\pi^C}{\pi^D} = \frac{(\phi^C)^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}}(1-\phi^C)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}}(b^C)^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}}}{(\phi^D)^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}}(1-\phi^D)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}}(b^D)^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}}},$$

implying that

$$\left(\frac{\pi^C}{\pi^D}\right)^{1-\frac{\alpha_i}{2}-\frac{\alpha_j}{2}} = \frac{\left(\frac{\alpha_i}{2}\right)^{\frac{\alpha_i}{2}}\left(1-\frac{\alpha_i}{2}\right)^{\frac{\alpha_j}{2}}}{\left(1-\frac{\alpha_i}{2}\frac{\alpha_j}{2}\right)^{\frac{\alpha_i}{2}+\frac{\alpha_j}{2}}\left(\frac{\alpha_i}{2}+\frac{\alpha_j}{2}-\frac{\alpha_i}{2}\frac{\alpha_j}{2}\right)^{\frac{\alpha_i}{2}+\frac{\alpha_j}{2}}}.$$

Define $f(x,y) \equiv x^x(1-x)^y/[(1-xy)^{x+y}(x+y-xy)^{x+y}]$ where $0 < y \le x \le \frac{1}{2}$. Taking the first order partial derivative with respect to x, we obtain

$$f_x(x,y) = \frac{x^x(1-x)^y}{(1-xy)^{x+y}(x+y-xy)^{x+y}}\left(-\frac{y}{1-x}+\frac{y(x+y)}{1-xy}-\frac{(1-y)(x+y)}{x+y-xy}+\log\frac{x}{(1-xy)(x+y-xy)}+1\right).$$

Define $g(x,y) \equiv -\frac{y}{1-x} + \frac{y(x+y)}{1-xy} - \frac{(1-y)(x+y)}{x+y-xy} + \log\frac{x}{(1-xy)(x+y-xy)} + 1$ where $0 < y \le x \le \frac{1}{2}$, and define $h(x,y) \equiv 1 - x - x(x+y-xy)$ where $0 < y \le x \le \frac{1}{2}$. Taking the first order partial derivative with respect to x, we obtain $h_x(x,y) = -1 - y - 2x(1-y) < 0$ for any $0 < y \le x \le \frac{1}{2}$. Therefore, $h(x,y)$

is decreasing in $x \in [y, \frac{1}{2}]$, and $h(x,y) \geq h(\frac{1}{2}, y) = \frac{1}{4}(1-y) > 0$ for any $0 < y \leq x \leq \frac{1}{2}$. The fact that $h(x,y) = 1 - x - x(x+y-xy) > 0$ implies that $\frac{x}{(1-xy)(x+y-xy)} < 1$, which further implies that $\log \frac{x}{(1-xy)(x+y-xy)} < 0$ for any $0 < y \leq x \leq \frac{1}{2}$. Hence,

$$g(x,y) < -\frac{y}{1-x} + \frac{y(x+y)}{1-xy} - \frac{(1-y)(x+y)}{x+y-xy} + 1 = \frac{y\left(x^3y - x^3 + x^2y^2 - 2x^2y + x^2 - 2xy^2 + 2xy - x + y^2\right)}{(1-x)(1-xy)(x+y-xy)}.$$

Define $k(x,y) \equiv x^3y - x^3 + x^2y^2 - 2x^2y + x^2 - 2xy^2 + 2xy - x + y^2$ where $0 \leq y \leq x \leq \frac{1}{2}$. Taking the first order partial derivative with respect to x, we obtain $k_x(x,y) = -\frac{1}{3}(1-y)(3x+y-1)^2 - \frac{1}{3}(y^3 + 3y^2 + 2 - 3y) < 0$ where $0 \leq y \leq x \leq \frac{1}{2}$. Therefore, $k(x,y)$ is decreasing in $x \in [y, \frac{1}{2}]$, and $k(x,y) \leq k(y,y) = -(1-y)^2 y(1-2y) < 0$ for any $0 < y \leq x \leq \frac{1}{2}$. Hence, $g(x,y) < \frac{y}{(1-x)(1-xy)(x+y-xy)} k(x,y) < 0$ for any $0 < y \leq x \leq \frac{1}{2}$. It follows that

$$f_x(x,y) = \left[\frac{x^x(1-x)^y}{(1-xy)^{x+y}(x+y-xy)^{x+y}}\right] g(x,y) < 0$$

for any $0 < y \leq x \leq \frac{1}{2}$. Thus, $f(x,y)$ is decreasing in $x \in [y, \frac{1}{2}]$, and $f(x,y) > f(\frac{1}{2}, y)$ for any $0 < y \leq x < \frac{1}{2}$, since $f(\frac{1}{2}, y) = 1/\left(\frac{9}{8} - \frac{1}{8}(1-2y)^2\right)^{y+\frac{1}{2}} < 1$ for any $0 < y \leq x < \frac{1}{2}$. Continuity of $f(x,y)$ implies that $\lim_{x \to \frac{1}{2}^-} f(x,y) = f(\frac{1}{2}, y) < 1$ for any $y \in (0, x]$. Therefore, $f(x,y) < 1$ for any $0 < y \leq x < \frac{1}{2}$ as long as x is close enough to $\frac{1}{2}$. As a result, there exists a unique $\bar{\alpha}_i^\pi(\alpha_j) \in (0,1)$ such that

$$\begin{cases} \pi^C \geq \pi^D & \text{for} \quad \alpha_i \leq \bar{\alpha}_i^\pi(\alpha_j) \\ \pi^C < \pi^D & \text{for} \quad \alpha_i > \bar{\alpha}_i^\pi(\alpha_j). \end{cases}$$

Next, consider the implicit function $f(x,y) = 1$ where $0 < y \leq x \leq \frac{1}{2}$. When $y = x$, the solution to $f(x,y) = 1$ is $\{x = y = 0.2892\}$. When, instead, y is close to zero, say $y = \bar{y} = 10^{-6}$, the solution to $f(x,y) = 1$ is $\{x = \bar{x} \equiv 0.3856, y = \bar{y}\}$. Since, given the above derivation, $f_x(x,y) < 0$ for any $0 < y \leq x \leq \frac{1}{2}$, we consider $0 < x \leq \bar{x}$. Taking the first order partial derivative of $f(x,y)$ with respect to y, we obtain

$$f_y(x,y) = -\frac{(1-x)(x+y)}{x+y-xy} + \frac{x(x+y)}{1-xy} + \log \frac{1-x}{(1-xy)(x+y-xy)}.$$

Taking the second order partial derivative with respect to y, we obtain

$$f_{yy}(x,y) = \frac{x}{1-xy} - \frac{(1-x)x^2}{(x+y-xy)^2} - \frac{1-x}{x+y-xy} + \frac{x(1+x^2)}{(1-xy)^2}.$$

Taking the third order partial derivative with respect to y, we obtain

$$f_{yyy}(x,y) = \frac{2x^2(x^2+1)}{(1-xy)^3} + \frac{x^2}{(1-xy)^2} + \frac{2(1-x)^2x^2}{(x+y-xy)^3} + \frac{(1-x)^2}{(x+y-xy)^2} > 0$$

for any $0 < y \leq x \leq \bar{x}$. Therefore, $f_{yy}(x,y)$ is increasing in $y \in [0,x]$ and $f_{yy}(x,y) < f_{yy}(x,x) = \frac{2(x^5 - 6x^3 + 6x^2 + x - 1)}{(2-x)^2 x(1-x^2)^2}$. Define $g(x) \equiv x^5 - 6x^3 + 6x^2 + x - 1$ where $0 < x \leq \bar{x}$. Taking the first order derivative, we obtain $g_x(x) = 5x^4 - 18x^2 + 12x + 1 = -(1-x)(5x^3 + 5x^2 - 13x - 1)$. Define $h(x) \equiv 5x^3 + 5x^2 - 13x - 1$ where $0 \leq x \leq \bar{x}$. Taking the first order derivative, we get $h_x(x) =$

$15x^2 + 10x - 13 = 15(x + \frac{1}{3})^2 - \frac{44}{3} < 0$ for any $0 \le x \le \bar{x}$. Therefore, $h(x)$ is decreasing in $x \in [0, \bar{x}]$ and $h(x) < h(0) = -1 < 0$ for any $0 < x \le \bar{x}$. It follows that $g_x(x) > 0$ for any $0 < x \le \bar{x}$, and hence $g(x)$ is increasing in $x \in (0, \bar{x}]$, and $g(x) \le g(\bar{x}) = -0.0578 < 0$ for any $0 < x \le \bar{x}$. As a consequence, $f_{yy}(x, y) < f_{yy}(x, x) = \frac{2}{(2-x)^2 x(1-x^2)^2} g(x) < 0$ for any $0 < y \le x \le \bar{x}$. Hence, $f_y(x, y)$ is decreasing in $y \in (0, x]$, and $f_y(x, y) < f_y(\bar{x}, \bar{y}) = -4.9187 \times 10^{-6} < 0$. Since $f_x(x, y) < 0$ for any $0 < y \le x \le \bar{x}$, using the *Implicit Function Theorem*, $\frac{dx}{dy} = -\frac{f_y(x,y)}{f_x(x,y)} < 0$ for any $0 < y \le x \le \bar{x}$. As a result, for $0 < \alpha_j \le \bar{\alpha}_i^e(\alpha_j) \le 2 \times \bar{x} = 0.7712$,

$$\frac{\partial \bar{\alpha}_i^\pi(\alpha_j)}{\partial \alpha_j} < 0.$$

(viii) The Agent's expected payoff, u_i:

$$\frac{u_i^C}{u_i^D} = \frac{(1-\alpha_i/2)\phi^C b^C \pi^C}{(1-\alpha_i/2)\phi^D b^D \pi^D},$$

implying that

$$\left(\frac{u_i^C}{u_i^D}\right)^{2-\alpha_i-\alpha_j} = \frac{\left(\frac{\alpha_i}{2}\right)^{1-\frac{\alpha_j}{2}} \left(1-\frac{\alpha_i}{2}\right)^{\frac{\alpha_j}{2}}}{\left(1-\frac{\alpha_i}{2}\frac{\alpha_j}{2}\right)\left(\frac{\alpha_i}{2}+\frac{\alpha_j}{2}-\frac{\alpha_i}{2}\frac{\alpha_j}{2}\right)}. \tag{A.22}$$

Since the RHS in (A.22) is the same as the RHS in (A.19), it follows that

$$u_i{}^C < u_i{}^D. \tag{A.23}$$

(ix) The Subagent's expected payoff, u_j:

$$\frac{u_j^C}{u_j^D} = \frac{(1-\alpha_j/2)(1-\phi^C) b^C \pi^C}{(1-\alpha_j/2)(1-\phi^D) b^D \pi^D},$$

implying that

$$\left(\frac{u_j^C}{u_j^D}\right)^{2-\alpha_i-\alpha_j} = \frac{\left(\frac{\alpha_i}{2}\right)^{\frac{\alpha_i}{2}} \left(1-\frac{\alpha_i}{2}\right)^{1-\frac{\alpha_i}{2}}}{\left(1-\frac{\alpha_i}{2}\frac{\alpha_j}{2}\right)\left(\frac{\alpha_i}{2}+\frac{\alpha_j}{2}-\frac{\alpha_i}{2}\frac{\alpha_j}{2}\right)} \tag{A.24}$$

Since the RHS in (A.24) is the same as the RHS in (A.21), it follows that

$$\begin{cases} u_j^C \ge u_j^D & \text{for} \quad \alpha_i \le \bar{\alpha}_i^e(\alpha_j) \\ u_j^C < u_j^D & \text{for} \quad \alpha_i > \bar{\alpha}_i^e(\alpha_j). \end{cases} \tag{A.25}$$

We finally prove that

$$\frac{1}{2} < \bar{\alpha}_i^\pi(\alpha_j) < \bar{\alpha}_i(\alpha_j) < \bar{\alpha}_i^e(\alpha_j) < \bar{\alpha}_i^c(\alpha_j) < 1.$$

We proceed in five steps, by showing sequentially that, for any $\alpha_j \in (0, 1)$: (1) $\frac{1}{2} < \bar{\alpha}_i^\pi(\alpha_j)$; (2) $\bar{\alpha}_i^\pi(\alpha_j) < \bar{\alpha}_i(\alpha_j)$; (3) $\bar{\alpha}_i(\alpha_j) < \bar{\alpha}_i^e(\alpha_j)$; (4) $\bar{\alpha}_i^e(\alpha_j) < \bar{\alpha}_i^c(\alpha_j)$; and (5) $\bar{\alpha}_i^c(\alpha_j) < 1$.

(1) Since in part (vii) of this proof we show that $\bar{\alpha}_i^{\pi}(\alpha_j)$ is decreasing in α_j, the lowest value of $\bar{\alpha}_i^{\pi}(\alpha_j)$ is $\bar{\alpha}_i^{\pi}(\alpha_i) = 0.5784 > \frac{1}{2}$.

(2) Since $(1-b^C) > (1-b^D)$ for any α_i and $\alpha_j \in (0,1)$, and since $\pi^C < \pi^D$ for any $\alpha_i > \bar{\alpha}_{\pi}(\alpha_j)$, it follows that $v^C = (1-b^C)\pi^C < v^D = (1-b^D)\pi^D$ for any $\alpha_i > \bar{\alpha}_i(\alpha_j) > \alpha_i^{\pi}(\alpha_j)$ and $\alpha_j < \alpha_i$.

(3) In part (ix) of this proof we show that the unique threshold of α_i above which $u_j^C < u_j^D$ is the same threshold above which $e_j^C < e_j^D$. Since $u_j = (1-\alpha_j/2)(1-\phi)b\pi$ and $v = (1-b)\pi$, we express v as a function of u_j:

$$v = \frac{(1-b)u_j}{\left(1-\frac{\alpha_j}{2}\right)(1-\phi)b}.$$

Assume by contradiction that $\bar{\alpha}_i > \bar{\alpha}_i^e$, and consider $\bar{\alpha}_i^e \leq \alpha_i < \bar{\alpha}_i$, implying that $u_j^C \leq u_j^D$ and $v^C > v^D$. Since

$$\frac{v^C}{v^D} = \left(\frac{\frac{\alpha_i}{2} + \frac{\alpha_j}{2} - \frac{\alpha_i}{2}\frac{\alpha_j}{2}}{1-\frac{\alpha_i}{2}}\right)\frac{u_j^C}{u_j^D},$$

$u_j^C \leq u_j^D$ and $v^C > v^D$ imply that

$$\frac{\frac{\alpha_i}{2} + \frac{\alpha_j}{2} - \frac{\alpha_i}{2}\frac{\alpha_j}{2}}{1-\frac{\alpha_i}{2}} > 1. \tag{A.26}$$

Given (A.29), (A.26) implies that

$$\left(\frac{v^C}{v^D}\right)^{1-\frac{\alpha_i}{2}-\frac{\alpha_j}{2}} < \left(\frac{1}{1-\frac{\alpha_i}{2}\frac{1-\alpha_i}{1-\frac{\alpha_i}{2}}}\right)\left(\frac{\frac{\alpha_i}{2}}{1-\frac{\alpha_i}{2}}\right)^{\frac{\alpha_i}{2}},$$

Define $f(x) \equiv \frac{1}{1-x\frac{1-2x}{1-x}}(\frac{x}{1-x})^x$ where $x \in [0, \frac{1}{2}]$. Taking the first order derivative, we obtain

$$f_x(x) = \frac{(1-x)\left(\frac{x}{1-x}\right)^x}{(2(x-1)x+1)^2}\left(2-4x+(2(x-1)x+1)\log\frac{x}{1-x}\right).$$

Define $g(x) \equiv 2-4x+\big(2(x-1)x+1\big)\log\frac{x}{1-x}$ where $x \in (0, \frac{1}{2}]$. Taking the first order derivative, we obtain $g_x(x) = \frac{1}{x(1-x)} - 2(1-2x)\log\frac{x}{1-x} - 6$. Taking the second order derivative, we obtain $g_{xx}(x) = \frac{-4x^3+6x^2-1}{(1-x)^2x^2} + 4\log\frac{x}{1-x} < 0$ for any $x \in (0, \frac{1}{2})$. Therefore, $g_x(x)$ is decreasing in $x \in (0, \frac{1}{2})$. Since $\lim_{x\to 0^+} g_x(x) > 0$ and $g_x(\frac{1}{2}) = -2 < 0$, using the *Mean Value Theorem*, it follows that there exist a unique $x^\star \in (0, \frac{1}{2})$ such that $g_x(x) \geq 0$ for any $x \in (0, x^\star]$ and $g_x(x) < 0$ for any $x \in (x^\star, \frac{1}{2})$, where $x^\star = 0.2694$. Hence, $g(x)$ is increasing in $x \in (0, x^\star]$ and decreasing in $x \in (x^\star, \frac{1}{2})$. Since $\lim\_x \to 0^+ g(x) < 0$, $g(x^\star) = 0.3175 > 0$, and $g(\frac{1}{2}) = 0$, using again the *Mean Value Theorem*, it follows that there exist a unique $x^{\star\star} \in (0, x^\star)$ such that $g(x) \leq 0$ for any $x \in (0, x^{\star\star}]$ and $g(x) > 0$ for any $x \in (x^{\star\star}, \frac{1}{2})$, where $x^{\star\star} = 0.1284$. Since $f_x(x) = g(x)(1-x)\left(\frac{x}{1-x}\right)^x/[2(x-1)x+1]^2$, it follows that $f_x(x) \leq 0$ for any $x \in (0, x^{\star\star}]$ and $f_x(x) > 0$ for any $x \in (x^{\star\star}, \frac{1}{2})$. Therefore, $f(x)$ is decreasing in $x \in (0, x^{\star\star}]$ and increasing in $x \in (x^{\star\star}, \frac{1}{2})$. Since $f(0) = 1$, $f(x^{\star\star}) = 0.8781 < 1$, and $f(\frac{1}{2}) = 1$, it follows that $f(x) < 1$ for any $x \in (0, \frac{1}{2})$. This implies that $v^C < v^D$, which is a contradiction. As a result, we can conclude that $\bar{\alpha}_i(\alpha_j) < \bar{\alpha}_i^e(\alpha_j)$.

(4) Given the ratio e_j^C / e_j^D in (A.20), it follows that

$$\frac{(1-\phi^C)b^C}{(1-\phi^D)b^D} = \left(\frac{e_j^C}{e_j^D}\right)^{\frac{2(2-\alpha_i-\alpha_j)}{\alpha_j}} \left(\frac{\phi^D b^D}{\phi^C b^C}\right)^{\frac{2-\alpha_j}{\alpha_j}}. \tag{A.27}$$

Since in part (iii) of this proof we show that $\phi^C b^C < \phi^D b^D$, it follows that the second term on the RHS of (A.27) id always larger than 1. Therefore, there must exists a value of $\alpha_i > \bar{\alpha}_i^e(\alpha_j)$ but close enough to $\bar{\alpha}_i^e(\alpha_j)$ such that $e_j^D > e_j^C$, but $(1-\phi^D)b^D < (1-\phi^C)b^C$. When, instead, $\alpha_i > \bar{\alpha}_i^c(\alpha_j)$, implying that $(1-\phi^D)b^D > (1-\phi^C)b^C$, it must be that $e_j^D > e_j^C$, implying that $\alpha_i > \bar{\alpha}_i^e(\alpha_j)$. As a result, we can conclude that $\bar{\alpha}_i^e(\alpha_j) < \bar{\alpha}_i^c(\alpha_j)$.

(5) Part (vi) of this proof implies that $\bar{\alpha}_i^c(\alpha_j) < 1$.

□

Proof of Lemma 1. Without loss of generality, we assume that

$$0 < \alpha_2 < \alpha_1 < 1.$$

We denote by $v_{A=i}^D$ the principal's expected payoff in the delegated contracting scheme where she delegates contracting to agent i (i.e., agent i is the Agent). Given the optimal contracts in Proposition 2, we take the ratio of the principal's expected payoff $v_{A=i}^D$ for the two cases $A = 1$ and $A = 2$:

$$\frac{v_{A=1}^D}{v_{A=2}^D} = \frac{\left(1-\frac{2\alpha_1+2\alpha_2-\alpha_1\alpha_2}{4}\right)\alpha_1^{\frac{\alpha_1}{2-\alpha_1-\alpha_2}}\alpha_2^{\frac{\alpha_2}{2-\alpha_1-\alpha_2}}\left(1-\frac{\alpha_2}{2}\right)^{\frac{\alpha_1}{2-\alpha_1-\alpha_2}}\left(\frac{\alpha_2}{2}\right)^{\frac{\alpha_2}{2-\alpha_1-\alpha_2}}\left(\frac{2\alpha_1+2\alpha_2-\alpha_1\alpha_2}{4}\right)^{\frac{\alpha_1+\alpha_2}{2-\alpha_1-\alpha_2}}}{\left(1-\frac{2\alpha_1+2\alpha_2-\alpha_1\alpha_2}{4}\right)\alpha_1^{\frac{\alpha_1}{2-\alpha_1-\alpha_2}}\alpha_2^{\frac{\alpha_2}{2-\alpha_1-\alpha_2}}\left(1-\frac{\alpha_1}{2}\right)^{\frac{\alpha_2}{2-\alpha_1-\alpha_2}}\left(\frac{\alpha_1}{2}\right)^{\frac{\alpha_1}{2-\alpha_1-\alpha_2}}\left(\frac{2\alpha_1+2\alpha_2-\alpha_1\alpha_2}{4}\right)^{\frac{\alpha_1+\alpha_2}{2-\alpha_1-\alpha_2}}},$$

$$= \frac{\left(1-\frac{\alpha_2}{2}\right)^{\frac{\alpha_1}{2-\alpha_1-\alpha_2}}\left(\frac{\alpha_2}{2}\right)^{\frac{\alpha_2}{2-\alpha_1-\alpha_2}}}{\left(1-\frac{\alpha_1}{2}\right)^{\frac{\alpha_2}{2-\alpha_1-\alpha_2}}\left(\frac{\alpha_1}{2}\right)^{\frac{\alpha_1}{2-\alpha_1-\alpha_2}}} = \left(\frac{1-\frac{\alpha_2}{2}}{\frac{\alpha_1}{2}}\right)^{\frac{\frac{\alpha_1}{2}}{1-\frac{\alpha_1}{2}-\frac{\alpha_2}{2}}}\left(\frac{\frac{\alpha_2}{2}}{1-\frac{\alpha_1}{2}}\right)^{\frac{\frac{\alpha_2}{2}}{1-\frac{\alpha_1}{2}-\frac{\alpha_2}{2}}},$$

implying that

$$\left(\frac{v_{A=1}^D}{v_{A=2}^D}\right)^{1-\frac{\alpha_1}{2}-\frac{\alpha_2}{2}} = \left(\frac{1-\frac{\alpha_2}{2}}{\frac{\alpha_1}{2}}\right)^{\frac{\alpha_1}{2}}\left(\frac{\frac{\alpha_2}{2}}{1-\frac{\alpha_1}{2}}\right)^{\frac{\alpha_2}{2}}. \tag{A.28}$$

The following steps allows us to prove that the RHS of (A.28) is always higher than 1 for $\alpha_1 > \alpha_2$.

(i) Define $f(x, y) \equiv \left(\frac{1-y}{x}\right)^x \left(\frac{y}{1-x}\right)^y$ where $0 < y < x < \frac{1}{2}$. Hence, $\lim_{x \to y^+} f(x, y) = 1$. Taking the first order derivative with respect to x, we obtain

$$f_x(x, y) = \frac{\partial f(x, y)}{\partial x} = \left(\frac{1-y}{x}\right)^x \left(\frac{y}{1-x}\right)^y \left[\log\left(\frac{1-y}{x}\right) - 1 + \frac{y}{1-x}\right].$$

Define $g(y) \equiv f_x(y,y) = \log\left(\frac{1-y}{y}\right) - 1 + \frac{y}{1-y}$ where $y \in (0, \frac{1}{2}]$. Taking the first order derivative, we obtain $g_y(y) = -\frac{1-2y}{y(1-y)^2} < 0$, for any $y \in (0, \frac{1}{2})$. Therefore, $g(y)$ is decreasing in $(0, \frac{1}{2}]$. Hence, $g(y) > g(\frac{1}{2}) = 0$, for any $y \in (0, \frac{1}{2})$. This implies that $\lim_{x \to y^+} f_x(x,y) > 0$.

(ii) Define $h(y) \equiv f(\frac{1}{2}, y) = \left(\frac{1-y}{\frac{1}{2}}\right)^{\frac{1}{2}} \left(\frac{y}{1-\frac{1}{2}}\right)^y = \sqrt{2}\sqrt{1-y}(2y)^y$ where $y \in (0, \frac{1}{2}]$. Taking the first order derivative, we obtain

$$h_y(y) = \frac{\sqrt{2}y^y 2^{y-1}}{\sqrt{1-y}} \left[2(1-y)\left(\log(2y)+1\right) - 1\right].$$

Define $k(y) \equiv 2(1-y)\left(\log(2y)+1\right) - 1$. Taking the first order derivative, we obtain $k_y(y) = \frac{2}{y} - 2\log(2y) - 4 > 0$ for any $y \in (0, \frac{1}{2})$. Therefore, $k(y)$ is increasing in $(0, \frac{1}{2}]$. Hence, $k(y) < k(\frac{1}{2}) = 0$. This implies that $h_y(y) < 0 \; \forall y \in (0, \frac{1}{2})$. Therefore, $h(y)$ is decreasing in $(0, \frac{1}{2}]$. Hence, $h(y) > h(\frac{1}{2}) = 1$ $\forall y \in (0, \frac{1}{2})$. This implies that $\lim_{x \to \frac{1}{2}^-} f(x,y) > 1$.

(iii) Let us consider the second order derivative of $f(x,y)$ with respect to x:

$$f_{xx}(x,y) = \left(\frac{1-y}{x}\right)^x \left(\frac{y}{1-x}\right)^y \left[\frac{2y}{1-x}\left(\log\left(\frac{1-y}{x}\right) - 1\right) + \frac{(y+1)y}{(1-x)^2} + \left(\log\left(\frac{1-y}{x}\right) - 1\right)^2 - \frac{1}{x}\right].$$

Define $g(x,y) \equiv \frac{2y}{1-x}\left(\log\left(\frac{1-y}{x}\right) - 1\right) + \frac{(y+1)y}{(1-x)^2} + \left(\log\left(\frac{1-y}{x}\right) - 1\right)^2 - \frac{1}{x}$ where $0 \le y < x \le \frac{1}{2}$. Taking the first order derivative with respect to y, we obtain

$$g_y(x,y) = \frac{2x^2 + 2(1-x)(x-y)\log(\frac{1-y}{x}) + 1 - 2x + y(1-2y)}{(1-x)^2(1-y)} > 0$$

for any $0 \le y < x \le \frac{1}{2}$. Taking the first order derivative with respect to x, we obtain

$$g_x(x,y) = \frac{-2x^4 + 5x^3 + x^2(2y^2 + 4y - 3) - x(2y+1) + 1 - 2(1-x)x(x^2 - x(y+2) + 1)\log(\frac{1-y}{x})}{(1-x)^3x^2}.$$

Define $q(x) \equiv g_x(x,0) = (2x + 1 - 2x\log(1/x))/x^2$ where $x \in (0, \frac{1}{2}]$. Define $m(x) \equiv x\log(\frac{1}{x})$ where $x \in (0, \frac{1}{2}]$. Taking the second order derivative, we obtain $m_{xx}(x) = -\frac{1}{x} < 0$ for any $x \in (0, \frac{1}{2}]$. Therefore, there must exist a unique maximal value of $m(x)$, which is equal to 0.3679. Hence, $q(x) > (2x + 1 - 2 \times 0.3679)/x^2 > 0$ for any $x \in (0, \frac{1}{2}]$. Since we have proved that $g_y(x,y) > 0$, this implies that $g_x(x,y) > g_x(x,0) = g(x) > 0$ for any $0 < y < x < \frac{1}{2}$. As a result, $g(x,y) < g(\frac{1}{2}, \frac{1}{2}) = 0$, which further implies that $f_{xx}(x,y) < 0$. So, $f(x,y)$ is concave.

(iv) Since $\lim_{x \to y^+} f(x,y) = 1$, $\lim_{x \to y^+} f_x(x,y) > 0$, $\lim_{x \to \frac{1}{2}^-} f(x,y) > 1$, and $f(x,y)$ is concave for any $0 < y < x < \frac{1}{2}$, it follows that $f(x,y) > 1$ for all $0 < y < x < \frac{1}{2}$. Hence,

$$v^D_{A=1} > v^D_{A=2}, \quad \forall \quad 0 < \alpha_2 < \alpha_1 < 1.$$

We next show that $v^D_{A=2} > v^D_{A=1}$ when $\phi^D_{A=i} = \phi^*_i + \Delta$, for any rent extraction Δ where $0 < \Delta < \frac{\alpha_j}{\alpha_i + \alpha_j}$. The allocation ϕ^*_i is the second-best allocation obtained in the centralized contracting scheme with public contract, as derived in the Online Appendix B, and is equal to $\alpha_i/(\alpha_1 + \alpha_2)$.

Taking the ratio of the principal's expected payoff $v^D_{A=i}(\Delta)$ for the two cases $A = 1$ and $A = 2$, we obtain

$$\frac{v^D_{A=1}(\Delta)}{v^D_{A=2}(\Delta)} = \frac{\left(\phi^* + \Delta\right)^{\frac{\alpha_1}{2-\alpha_1-\alpha_2}} \left(1 - \phi^* - \Delta\right)^{\frac{\alpha_2}{2-\alpha_1-\alpha_2}}}{\left(\phi^* - \Delta\right)^{\frac{\alpha_1}{2-\alpha_1-\alpha_2}} \left(1 - \phi^* + \Delta\right)^{\frac{\alpha_2}{2-\alpha_1-\alpha_2}}}$$
$$= \left(\frac{\alpha_1 + \Delta(\alpha_1 + \alpha_2)}{\alpha_1 - \Delta(\alpha_1 + \alpha_2)}\right)^{\frac{\alpha_1}{2-\alpha_1-\alpha_2}} \left(\frac{\alpha_2 - \Delta(\alpha_1 + \alpha_2)}{\alpha_2 + \Delta(\alpha_1 + \alpha_2)}\right)^{\frac{\alpha_2}{2-\alpha_1-\alpha_2}},$$

implying that

$$\left(\frac{v^D_{A=1}(\Delta)}{v^D_{A=2}(\Delta)}\right)^{2-\alpha_1-\alpha_2} = \left(\frac{\alpha_1 + \Delta(\alpha_1 + \alpha_2)}{\alpha_1 - \Delta(\alpha_1 + \alpha_2)}\right)^{\alpha_1} \left(\frac{\alpha_2 - \Delta(\alpha_1 + \alpha_2)}{\alpha_2 + \Delta(\alpha_1 + \alpha_2)}\right)^{\alpha_2}.$$

Let us define $f(x, y, z) \equiv \left(\frac{x+z(x+y)}{x-z(x+y)}\right)^x \left(\frac{y-z(x+y)}{y+z(x+y)}\right)^y$ where $0 < y < x < 1$ and $0 \le z < \frac{y}{x+y}$. Taking the first order partial derivative with respect to z, we obtrain

$$\frac{\partial f(x, y, z)}{\partial z} = -\frac{2z^2(x - y)(x + y)^4 \left(\frac{x+z(x+y)}{x-z(x+y)}\right)^x \left(\frac{y-z(x+y)}{y+z(x+y)}\right)^y}{\big(x + z(x + y)\big)\big(x - z(x + y)\big)\big(y + z(x + y)\big)\big(y - z(x + y)\big)} < 0$$

for any $0 < y < x < 1$ and $0 < z < \frac{y}{x+y}$. Therefore, $f(x, y, z)$ is decreasing in $z \in [0, \frac{y}{x+y})$, and $f(x, y, z) < f(x, y, 0) = 1$ $\forall z \in (0, \frac{y}{x+y})$. As a result,

$$v^D_{A=2}(\Delta) > v^D_{A=1}(\Delta)$$

for any $0 < \alpha_2 < \alpha_1 < 1$ and $0 < \Delta < \alpha_2/(\alpha_1 + \alpha_2)$. □

Proof of Proposition 3. Without loss of generality, we assume that

$$0 < \alpha_j \le \alpha_i < 1.$$

Taking the ratio of the principal's expected payoff in the two contracting schemes,

$$\frac{v^C}{v^D} = \frac{(1 - b^C)(\phi^C)^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} (1 - \phi^C)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} (b^C)^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}}}{(1 - b^D)(\phi^D)^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} (1 - \phi^D)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} (b^D)^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}}},$$
$$= \left(\frac{1}{1 - \frac{\alpha_i}{2}\frac{\alpha_j}{2}}\right)^{\frac{1}{1-\frac{\alpha_i}{2}-\frac{\alpha_j}{2}}} \left(\frac{\frac{\alpha_i}{2}}{\frac{\alpha_i}{2} + \frac{\alpha_j}{2} - \frac{\alpha_i}{2}\frac{\alpha_j}{2}}\right)^{\frac{\frac{\alpha_i}{2}}{1-\frac{\alpha_i}{2}-\frac{\alpha_j}{2}}} \left(\frac{1 - \frac{\alpha_i}{2}}{\frac{\alpha_i}{2} + \frac{\alpha_j}{2} - \frac{\alpha_i}{2}\frac{\alpha_j}{2}}\right)^{\frac{\frac{\alpha_j}{2}}{1-\frac{\alpha_i}{2}-\frac{\alpha_j}{2}}},$$

implies that

$$\left(\frac{v^C}{v^D}\right)^{1-\frac{\alpha_i}{2}-\frac{\alpha_2}{2}} = \left(\frac{1}{1 - \frac{\alpha_i}{2}\frac{\alpha_j}{2}}\right) \left(\frac{\frac{\alpha_i}{2}}{\frac{\alpha_i}{2} + \frac{\alpha_j}{2} - \frac{\alpha_i}{2}\frac{\alpha_j}{2}}\right)^{\frac{\alpha_i}{2}} \left(\frac{1 - \frac{\alpha_i}{2}}{\frac{\alpha_i}{2} + \frac{\alpha_j}{2} - \frac{\alpha_i}{2}\frac{\alpha_j}{2}}\right)^{\frac{\alpha_j}{2}}. \tag{A.29}$$

Define $f(x,y) \equiv \left(\frac{1}{1-xy}\right)\left(\frac{x}{x+y-xy}\right)^x\left(\frac{1-x}{x+y-xy}\right)^y$ where $0<y<x\leq\frac{1}{2}$. Taking the first order derivative with respect to x, we obtain

$$f_x(x,y) = -\frac{\left(\frac{x}{x+y-xy}\right)^x\left(\frac{1-x}{x+y-xy}\right)^y}{(1-x)(x+y-xy)(1-xy)^2}\left[y\big(x^2(1-2y)-y(1-2x)\big)+(1-x)(1-xy)(x+y-xy)\log\left(\tfrac{x+y-xy}{x}\right)\right],$$

where the ratio $\frac{(\frac{x}{x+y-xy})^x(\frac{1-x}{x+y-xy})^y}{(1-x)(x+y-xy)(1-xy)^2}>0$, since $0<y<x<\frac{1}{2}$. Define $g(x,y)\equiv y\left(x^2(1-2y)-y(1-2x)\right)+(1-x)(1-xy)(x+y-xy)\log\left(\frac{x+y-xy}{x}\right)$ where $0<y<x<\frac{1}{2}$. The domain $0<y<x<\frac{1}{2}$ implies that

$$\begin{aligned} g(x,y) &> y\left(x^2(1-2y)-x(1-2y)\right)+(1-x)(1-2y)(x+y-xy)\log\left(\frac{x+y-xy}{x}\right)\\ &> (1-x)(1-2y)\left[-xy+(x+y-xy)\log\left(\frac{x+y-xy}{x}\right)\right] \end{aligned}$$

Define $h(x,y)\equiv -xy+(x+y-xy)\log(\frac{x+y-xy}{x})$ where $0\leq y<x\leq\frac{1}{2}$. Taking the first order derivative with respect to x, we obtain $h_x(x,y)=[x(1-y)\log(\frac{x+y-xy}{x})-(x+1)y]/x$. Taking the second order derivative with respect to x, we obtain $h_{xx}(x,y)=\frac{y^2}{x^2(x+y-xy)}>0$ for any $0\leq y<x\leq\frac{1}{2}$. Therefore, $h_x(x,y)$ is increasing in $x\in(y,\frac{1}{2}]$. It follows that $h_x(x,y)<h_x(\frac{1}{2},y)=(1-y)\log(1+y)-3y$ for any $0\leq y<x<\frac{1}{2}$. Define $k(y)\equiv(1-y)\log(1+y)-3y$ where $y\in[0,\frac{1}{2})$. Taking the first order derivative, we get $k_y(y)=-\frac{2(2y+1)}{y+1}-\log(1+y)<0$ for any $y\in[0,\frac{1}{2})$. Therefore, $k(y)$ is decreasing in $y\in[0,\frac{1}{2})$. Hence, $k(y)<k(0)=0$ for any $y\in(0,\frac{1}{2})$. This implies that $h_x(x,y)<g(y)<0$ for any $0<y<x\leq\frac{1}{2}$. Therefore, $h(x,y)$ is decreasing in $x\in(y,\frac{1}{2}]$. As a consequence, $f(x,y)>f(\frac{1}{2},y)=\frac{1}{2}\left(-y+(1+y)\log(1+y)\right)$ for any $x\in(y,\frac{1}{2})$. Define $q(y)\equiv -y+(1+y)\log(1+y)$ where $y\in[0,\frac{1}{2})$. Taking the first order derivative, we obtain $q_y(y)=\log(1+y)>0$ for any $y\in[0,\frac{1}{2})$. Therefore, $q(y)$ is increasing in $[0,\frac{1}{2})$. Hence, $h(x,y)=\frac{1}{2}g(y)>\frac{1}{2}g(0)=0$ for any $y\in(0,\frac{1}{2})$. This implies that $g(x,y)>(1-x)(1-2y)h(x,y)>0$ for any $0<y<x<\frac{1}{2}$, which further implies that

$$f_x(x,y)=-\frac{(\frac{x}{x+y-xy})^x(\frac{1-x}{x+y-xy})^y}{(1-x)(x+y-xy)(1-xy)^2}g(x,y)<0$$

for any $0<y<x\leq\frac{1}{2}$. Therefore, $f(x,y)$ is decreasing in $x\in(y,\frac{1}{2}]$. Hence, $f(x,y)>f(\frac{1}{2},y)=\frac{2}{2-y}(\frac{1}{1+y})^{y+\frac{1}{2}}$ for any $x\in(y,\frac{1}{2})$. Define $w(y)\equiv\frac{2}{2-y}(\frac{1}{1+y})^{y+\frac{1}{2}}$ where $y\in[0,x)$. Taking the first order derivative, we obtain $w_y(y)=-\left[2\left(\frac{9}{4}-(y-\frac{1}{2})^2\right)\log(y+1)+y(1-2y)\right]/\left[(2-y)^2(1+y)^{y+1.5}\right]<0$ for any $y\in[0,x)$. Therefore, $w(y)$ is decreasing in $[0,x)$. Hence, $w(y)<w(0)=1$ for any $y\in(0,\frac{1}{2})$. This implies that $f(\frac{1}{2},y)<1$ for any $y\in(0,\frac{1}{2})$. Continuity of $f(x,y)$ implies that $\lim_{x\to\frac{1}{2}^-}f(x,y)=f(\frac{1}{2},y)<1$ for any $y\in(0,x)$. Therefore, $f(x,y)<1$ for any $0<y<x<\frac{1}{2}$ as long as x is close enough to $\frac{1}{2}$. As a result, there exists a unique $\bar{\alpha}_i\in(0,1)$ such that

$$\begin{cases} v^C\geq v^D & \text{for} \quad \alpha_i\leq\bar{\alpha}_i\\ v^C<v^D & \text{for} \quad \alpha_i>\bar{\alpha}_i. \end{cases}$$

Next, consider the implicit function $f(x,y)=1$ where $0\leq y\leq x\leq\frac{1}{2}$. Take the logarithm on both sides of the implicit function, we get that $F(x,y)\equiv\log(f(x,y))=0$, where

$$F(x,y)=-\log(1-xy)+x\log(x)-x\log(x+y-xy)+y\log(1-x)-y\log(x+y-xy).$$

Taking the first order derivative of $F(x,y)$ with respect to x, we obtain

$$\begin{aligned}F_x(x,y) &= -\frac{1}{(1-x)(1-xy)(x+y-xy)}\left[y\Big(x^2(1-2y)-y(1-2x)\Big)+(1-x)(1-xy)(x+y-xy)\log\Big(\tfrac{x+y-xy}{x}\Big)\right]\\ &= -\frac{1}{(1-x)(1-xy)(x+y-xy)}g(x,y).\end{aligned}$$

Since in the previous step we have shown that $g(x,y)>0$ for any $0<y<x<\frac{1}{2}$, it follows that $F_x(x,y)<0$ for any $0<y<x<\frac{1}{2}$. Taking the first, the second and the third order derivatives of $F(x,y)$ with respect to y, we obtain

$$\begin{aligned}F_y(x,y) &= \frac{x}{1-xy}-\frac{(1-x)(x+y)}{x+y-xy}+\log(\frac{1-x}{x+y-xy}),\\ F_{yy}(x,y) &= \frac{x^2}{(1-xy)^2}-\frac{(1-x)(x+y-xy+x^2)}{(x+y-xy)^2},\\ F_{yyy}(x,y) &= \frac{2x^3}{(1-xy)^3}+\frac{(1-x)^2[(1-x)y+x(2x+1)]}{(x+y-xy)^3}>0,\end{aligned}$$

respectively, for any $0<y<x<\frac{1}{2}$. Therefore, $F_{yy}(x,y)$ is increasing in $y\in[0,x]$. Hence, $F_{yy}(x,y)<F_{yy}(x,x)=\frac{3x^5-6x^4+4x^2+2x-2}{(2-x)^2x(1-x^2)^2}$ for any $x\in(0,\frac{1}{2}]$. Define $G(x)\equiv 3x^5-6x^4+4x^2+2x-2$ where $x\in[0,\frac{1}{2}]$. Taking the first, the second and the third order derivatives of $G(x)$ with respect to x, we obtain

$$\begin{aligned}G_x(x) &= 15x^4-24x^3+8x+2,\\ G_{xx}(x) &= 60x^3-72x^2+8,\\ G_{xxx}(x) &= 36x(5x-4)<0,\end{aligned}$$

respectively, for any $x\in[0,\frac{1}{2}]$. Therefore, $G_{xx}(x)$ is decreasing in $[0,\frac{1}{2}]$, and $G_{xx}(x)>G_{xx}(0)=8>0$ for any $x\in(0,\frac{1}{2}]$. This implies that $G_x(x)$ is increasing in $[0,\frac{1}{2}]$, and that $G_x(x)>G_x(0)=2>0$ for any $x\in(0,\frac{1}{2}]$. As a consequence, $G(x)$ is increasing in $[0,\frac{1}{2}]$, and $G(x)<G(\frac{1}{2})=-0.28125<0$. Therefore, it follows that $F_{yy}(x,y)<0$ for any $0<y<x<\frac{1}{2}$, which implies that $F_y(x,y)$ is decreasing in $y\in[0,x]$. Since $\{x=y=0.3454\}$ and $\{x=\frac{1}{2},y=0\}$ are both solutions to $F(x,y)=0$, and since $F_y(0.3454,0.3454)=-0.2633$ and $F_y(\frac{1}{2},0)=0$, it must be that $F_y(x,y)<0$ for any $0<y<x<\frac{1}{2}$. Using the *Implicit Function Theorem*, $\frac{dx}{dy}=-\frac{F_y(x,y)}{F_x(x,y)}<0$, for any $0<y<x<\frac{1}{2}$, where $F(x,y)=0$. As a result,

$$\frac{\partial\bar{\alpha}_i(\alpha_j)}{\partial\alpha_j}<0.$$

Corollary 2 shows that the Agent always benefits from delegation (equation (A.23)), while the Subagent benefits from it if and only if $\alpha_A>\bar{\alpha}_i^e(\alpha_S)$ (equation (A.25)). Since $\bar{\alpha}_i^e(\alpha_S)>\bar{\alpha}_i(\alpha_S)$, it follows that delegation is Pareto improving if and only if $\alpha_A>\bar{\alpha}_i^e(\alpha_S)$. □

Proof of Proposition 4. Without loss of generality, we assume that

$$0<\alpha_j\le\alpha_i<1.$$

The optimal contracts in the delegated contracting scheme and the optimal contracts in the centralized contracting scheme when agent j's contract is public (i.e., when agent i observes agent j's contract but not vice versa) are given in Proposition 2 and in Proposition C.1 (Online Appendix C), respectively, and are equal to

$$b^D = b' = \frac{2\alpha_i + 2\alpha_j - \alpha_i\alpha_j}{4}, \qquad \phi^D_{A=i} = 1 - \frac{\alpha_j}{2}, \qquad \phi'_i = \frac{2\alpha_i - \alpha_i\alpha_j}{2\alpha_i + 2\alpha_j - \alpha_i\alpha_j},$$

Taking the ratio of the principal's expected payoff in the two contracting schemes, we obtain

$$\begin{aligned}\frac{v'}{v^D} &= \frac{(1-b')(\phi'_i)^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}}(1-\phi'_i)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}}(b')^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}}}{(1-b^D)(\phi^D_{a=i})^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}}(1-\phi^D_{a=i})^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}}(b^D)^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}}},\\ &= \left(\frac{\frac{\alpha_i}{2}}{\frac{\alpha_i}{2}+\frac{\alpha_j}{2}-\frac{\alpha_i}{2}\frac{\alpha_j}{2}}\right)^{\frac{\frac{\alpha_i}{2}}{1-\frac{\alpha_i}{2}-\frac{\alpha_j}{2}}}\left(\frac{1}{\frac{\alpha_i}{2}+\frac{\alpha_j}{2}-\frac{\alpha_i}{2}\frac{\alpha_j}{2}}\right)^{\frac{\frac{\alpha_j}{2}}{1-\frac{\alpha_i}{2}-\frac{\alpha_j}{2}}},\end{aligned}$$

implying that

$$\left(\frac{v'}{v^D}\right)^{1-\frac{\alpha_i}{2}-\frac{\alpha_2}{2}} = \left(\frac{\frac{\alpha_i}{2}}{\frac{\alpha_i}{2}+\frac{\alpha_j}{2}-\frac{\alpha_i}{2}\frac{\alpha_j}{2}}\right)^{\frac{\alpha_i}{2}}\left(\frac{1}{\frac{\alpha_i}{2}+\frac{\alpha_j}{2}-\frac{\alpha_i}{2}\frac{\alpha_j}{2}}\right)^{\frac{\alpha_j}{2}}.$$

Define $f(x,y) \equiv (\frac{x}{x+y-xy})^x(\frac{1}{x+y-xy})^y$ where $0 < y \le x \le \frac{1}{2}$. Taking the first order partial derivative with respect to x, we obtain

$$f_x(x,y) = \left(\frac{x}{x+y-xy}\right)^x\left(\frac{1}{x+y-xy}\right)^y\frac{1}{x+y-xy}\left(y^2+(x+y-xy)\log\left(\frac{x}{x+y-xy}\right)\right).$$

Define $g(x,y) = y^2 + (x+y-xy)\log(x/(x+y-xy))$ where $0 < y \le x \le \frac{1}{2}$. Taking the first order partial derivative with respect to x, we obtain $g_x(x,y) = (1-y)\log(x/(x+y-xy)) + \frac{y}{x}$. Taking the second order partial derivative with respect to x, we obtain $g_{xx}(x,y) = -y^2/[x^2(x+y-xy)] < 0$ for any $0 < y \le x \le \frac{1}{2}$. Therefore, $g_x(x,y)$ is decreasing in $x \in [y, \frac{1}{2}]$, and $g_x(x,y) > g_x(\frac{1}{2}, y) = 2y + (1-y)\log(1+y)^{-1}$. Define $k(y) \equiv y + (1-y)\log(1+y)^{-1}$ where $y \in (0, \frac{1}{2}]$. Taking the first order derivative, we obtain $k_y(y) = \frac{1+3y}{1+y} - \log(1+y)^{-1} > 0$ for any $y \in (0, \frac{1}{2}]$, and $k(y) > \lim_{y\to0^+} k(y) = 0$ for any $y \in (0, \frac{1}{2}]$. Hence, $g_x(x,y) > k(y) > 0$ for any $0 < y \le x \le \frac{1}{2}$. Therefore, $g(x,y)$ is increasing in $x \in [y, \frac{1}{2}]$, and $g(x,y) \le g(\frac{1}{2}, y) = y^2 + \frac{1}{2}(1+y)\log(1+y)^{-1}$ for any $y \in (0, \frac{1}{2}]$. Define $w(y) \equiv y^2 + \frac{1}{2}(1+y)\log(1+y)^{-1}$ where $0 < y \le \frac{1}{2}$. Taking the first order derivative, we obtain $w_y(y) = \frac{1}{2}(4y - \log(y+1) - 1)$. Taking the second order derivative, we get $w_{yy}(y) = 2 - \frac{1}{2(1+y)} > 0$ for any $y \in (0, \frac{1}{2}]$. Therefore, $w_y(y)$ is increasing in $y \in (0, \frac{1}{2}]$. Since $\lim_{y\to0^+} w_y(y) = -0.5 < 0$ and $w_y(\frac{1}{2}) = 0.2973 > 0$, there exists a unique $y^\star = 0.3193 \in (0, \frac{1}{2}]$ such that $w_y(y) \le 0$ whenever $y \in (0, y^\star]$ and $w_y(y) > 0$ whenever $y \in (y^\star, \frac{1}{2})$. Hence, $w(y)$ is decreasing in $y \in (0, y^\star]$ and increasing in $y \in (y^\star, \frac{1}{2}]$. Since $\lim_{y\to0^+} w(y) = 0$ and $w(\frac{1}{2}) = -0.0541 < 0$, it follows that $w(y) < 0$ for any $y \in (0, \frac{1}{2}]$. Hence, $g(x,y) \le w(y) < 0$ for any $y \in (0, \frac{1}{2}]$. As a consequenc, $f_x(x,y) < 0$ for any $0 < y \le x \le \frac{1}{2}$.

Since $f(x,y) > f(\frac{1}{2}, y) = 2^y\left(\frac{1}{1+y}\right)^{y+\frac{1}{2}}$ for any $0 < y \le x \le \frac{1}{2}$, define $h(y) \equiv 2^y(\frac{1}{y+1})^{y+\frac{1}{2}}$ where $0 \le y \le \frac{1}{2}$. Take the first order derivative, we obtain

$$h_y(y) = 2^y\left(\frac{1}{y+1}\right)^{y+\frac{3}{2}}\left(-y-\frac{1}{2}+(y+1)\log\frac{2}{y+1}\right).$$

Define $m(y) \equiv -y - \frac{1}{2} + (y+1)\log\frac{2}{y+1}$ where $0 \le y \le \frac{1}{2}$. Taking the first order derivative, we obtain $m_y(y) = \log\frac{2}{1+y} - 2 < 0$ for any $y \in [0, \frac{1}{2}]$. Therefore, $m(y)$ is decreasing in $y \in [0, \frac{1}{2}]$. Since $m(0) = 0.1931 > 0$ and $m(\frac{1}{2}) = -0.5685 < 0$, there exists a unique $y^{\star\star} = 0.1406 \in (0, \frac{1}{2}]$ such that $m(y) \ge 0$ for $y \in [0, y^{\star\star}]$ and $m(y) < 0$ for $y \in (y^{\star\star}, \frac{1}{2}]$. This implies that $h_y(y) \ge 0$ for $y \in [0, y^{\star\star}]$ and $h_y(y) < 0$ for $y \in (y^{\star\star}, \frac{1}{2}]$. Hence, $h(y)$ is increasing in $y \in [0, y^{\star\star}]$ and decreasing in $y \in (y^{\star\star}, \frac{1}{2}]$. Since $h(0) = 1$, $h(y^{\star\star}) = 1.0133 > 1$, and $h(\frac{1}{2}) = 0.9429 < 1$, it follows that there exists a unique $y^{\star\star\star} = 0.2891 \in (y^{\star\star}, \frac{1}{2}]$ such that $h(y) \ge 1$ for $y \in [y^{\star\star}, y^{\star\star\star}]$ and $h(y) < 1$ for $y \in (y^{\star\star\star}, \frac{1}{2}]$. This implies that $f(x, y) > 1$ for $0 < y \le y^{\star\star\star}$. So, for any $y \in (0, y^{\star\star\star}]$ there does not exist a value of $x \in [y, \frac{1}{2})$ such that $f(x, y) \le 1$. However, $f(\frac{1}{2}, y) < 1$ for any $y \in (y^{\star\star\star}, \frac{1}{2})$. Since $f_x(x, y) < 0$ for any $0 < y \le x \le \frac{1}{2}$, continuity of $f(x, y)$ implies that $\lim_{x \to \frac{1}{2}^-} f(x, y) = f(\frac{1}{2}, y) < 1$ for any $y \in (y^{\star\star\star}, x]$. Therefore,

$$\left(\frac{x}{x+y-xy}\right)^x \left(\frac{1}{x+y-xy}\right)^y < 1$$

for any $y^{\star\star\star} < y \le x < \frac{1}{2}$ as long as x is close enough to $\frac{1}{2}$. As a result, given $\bar{\bar{\alpha}}_j \equiv 2 \times y^{\star\star\star} = 0.5638$, there exists a unique $\bar{\bar{\alpha}}_i(\alpha_j) \in (\bar{\bar{\alpha}}_j, 1)$ such that

$$\begin{cases} v' \le v^D & \text{for} \quad \alpha_j > \bar{\bar{\alpha}}_j \wedge \alpha_i > \bar{\bar{\alpha}}_i(\alpha_j) \\ v' > v^D & \text{otherwise.} \end{cases}$$

Next, consider the implicit function $f(x, y) = 1$ where $y^{\star\star\star} \le y \le x \le \frac{1}{2}$. Taking the first order partial derivative with respect to y, we obtain

$$f_y(x, y) = \frac{-(1-x)(x+y) + (x+y-xy)\log\left(\frac{1}{x+y-xy}\right)}{x+y-xy}.$$

Define $n(x, y) \equiv -(1-x)(x+y) + (x+y-xy)\log\frac{1}{x+y-xy}$ where $y^{\star\star\star} \le y \le x \le \frac{1}{2}$. Taking the first order partial derivative with respect to y, we obtain $n_y(x, y) = -(1-x)(2 - \log(x+y-xy)^{-1})$. Since $y^{\star\star\star} \le y \le x \le \frac{1}{2}$, it follows that $x+y-xy \ge x+y-xy\big|_{x=y=y^{\star\star\star}} = 0.4843$, and $\log(x+y-xy)^{-1} \le 0.7251$. This implies that $n_y(x, y) < 0$ for any $y \in [0.2819, x]$. Therefore, $n(x, y)$ is decreasing in $y \in [y^{\star\star\star}, x]$ and $n(x, y) \le n(x, y^{\star\star\star}) = -(1-x)(x+y^{\star\star\star}) + (x+(1-x)y^{\star\star\star})\log(x+(1-x)y^{\star\star\star})^{-1}$. Define $q(x) \equiv -(1-x)(x+y^{\star\star\star}) + (x+(1-x)y^{\star\star\star})\log(x+(1-x)y^{\star\star\star})^{-1}$ where $y^{\star\star\star} \le x \le \frac{1}{2}$. Taking the first order derivative, we obtain $q_x(x) = -2((1-y^{\star\star\star}) - x) + (1-y^{\star\star\star})\log(x+(1-x)y^{\star\star\star})^{-1}$. Taking the second order derivative, we obtain $q_{xx}(x) = 2 - (1-y^{\star\star\star})^2/(x+(1-x)y^{\star\star\star}) = 2 - 0.7181/(x+0.3926) > 0$ for any $x \in [y^{\star\star\star}, \frac{1}{2}]$. Therefore, $q_x(x)$ is increasing in $x \in [y^{\star\star\star}, \frac{1}{2}]$ and $q_x(x) \le q_x(\frac{1}{2}) = -0.1168 < 0$ for any $x \in [y^{\star\star\star}, \frac{1}{2}]$. It follows that $q(x)$ is decreasing in $x \in [y^{\star\star\star}, \frac{1}{2}]$ and that $q(x) \le q(y^{\star\star\star}) = -0.0537 < 0$. This implies that $n(x, y) < 0$ for any $y^{\star\star\star} \le y \le x \le \frac{1}{2}$, which further implies that $f_y(x, y) = n(x, y)/(x+y-xy) < 0$ for any $y^{\star\star\star} \le y \le x \le \frac{1}{2}$. Since $f_x(x, y) < 0$ for any $0 < y \le x < \frac{1}{2}$, using the *Implicit Function Theorem*, $\frac{dx}{dy} = -\frac{f_y(x,y)}{f_x(x,y)} < 0$ for any $y^{\star\star\star} < y \le x < 1$. As a result, for any $\alpha_j > \bar{\bar{\alpha}}_j$,

$$\frac{\partial \bar{\bar{\alpha}}_i(\alpha_j)}{\partial \alpha_j} < 0.$$

We next compare the threshold $\bar{\bar{\alpha}}_i(\alpha_j)$ with the threshold $\bar{\alpha}_i(\alpha_j)$, for any $\alpha_j \in (\bar{\bar{\alpha}}_j, 1)$. To this purpose, we first compare the principal's expected payoff in the centralized contracting scheme with one public contract with that in the centralized contracting scheme with two private contracts. Taking the ratio of v' and v^C, we obtain

$$\frac{v'}{v^C} = \frac{(1-b')\alpha_i^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}}\alpha_j^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}}\left(\phi_i'\right)^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}}\left(1-\phi_i'\right)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}}\left(b'\right)^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}}}{(1-b^C)\alpha_i^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}}\alpha_j^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}}\left(\phi_i^C\right)^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}}\left(1-\phi_i^C\right)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}}\left(b^C\right)^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}}},$$
$$= \left(\frac{1-\frac{\alpha_i}{2}\frac{\alpha_j}{2}}{1}\right)^{\frac{1-\frac{\alpha_j}{2}}{1-\frac{\alpha_i}{2}-\frac{\alpha_j}{2}}}\left(\frac{1-\frac{\alpha_i}{2}\frac{\alpha_j}{2}}{1-\frac{\alpha_i}{2}}\right)^{\frac{\frac{\alpha_j}{2}}{1-\frac{\alpha_i}{2}-\frac{\alpha_j}{2}}},$$

implying that

$$\left(\frac{v'}{v^C}\right)^{1-\frac{\alpha_i}{2}-\frac{\alpha_j}{2}} = \frac{1-\frac{\alpha_i}{2}\frac{\alpha_j}{2}}{\left(1-\frac{\alpha_i}{2}\right)^{\frac{\alpha_j}{2}}}.$$

Define $F(x,y) \equiv (1-xy)/(1-x)^y$ where $0 \le y \le x < \frac{1}{2}$. Taking the first order partial derivative with respect to x, we obtain $F_x(x,y) = xy(1-y)/(1-x)^{y+1} > 0$ for any $x \in [y, \frac{1}{2})$. Therefore, $F(x,y)$ is increasing in $x \in [y, \frac{1}{2})$ and $F(x,y) \ge F(y,y) = (1+y)(1-y)^{1-y}$ for any $x \in [y, \frac{1}{2})$. Define $G(y) \equiv F(y,y)$ where $0 \le y < \frac{1}{2}$. Taking the first order derivative with respect to y, we obtain $G_y(y) = -(1-y)^{1-y}\left(y + (1+y)\log(1-y)\right)$. Define $H(y) \equiv y + (1+y)\log(1-y)$ where $0 \le y < \frac{1}{2}$. Taking the first order derivative, we obtain $H_y(y) = \log(1-y) - \frac{2y}{1-y} < 0$ for any $y \in [0, \frac{1}{2})$. Therefore, $H(y)$ is decreasing in $y \in [0, \frac{1}{2})$ and $H(y) < H(0) = 0$ for any $y \in (0, \frac{1}{2})$. Hence, $G_y(y) > 0$ for any $y \in (0, \frac{1}{2})$, implying that $G(y)$ is increasing in $y \in [0, \frac{1}{2})$, and that $G(y) > G(0) = 1$ for any $y \in (0, \frac{1}{2})$. It follows that $F(x,y) \ge F(y,y) > G(0) = 1$ for any $0 < y \le x < \frac{1}{2}$. As a result, $v' > v^C$. Consequently, the lowest value of α_i that makes $v^D \ge v'$ need to be larger than the lowest value that makes $v^D \ge v^C$, that is $\bar{\bar{\alpha}}_i(\alpha_j) > \bar{\alpha}_i(\alpha_j)$, for any $\alpha_j > \bar{\bar{\alpha}}_j$. □

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Online Appendix
"Providing Incentives with Private Contracts"

by Andrea M. Buffa, Qing Liu and Lucy White

In this online appendix, we derive the optimal contracts in a centralized contracting scheme when both contracts (Section B) or only one of the two contracts (Section C) are public. As for the case with private contracts, we look for an equilibrium with strictly positive effort choices. Table I summarizes the optimal contracts for the different contracting schemes considered and discussed in this paper.

B Centralized Contracting with Two Public Contracts

We consider a centralized contracting scheme where contracts are public. We refer to the optimal contracts in this scheme as *second-best*, and we denote the corresponding compensation budget and allocation by (b^*, ϕ^*).

Proposition B.1. *When contracts are public, the optimal compensation budget and allocation with centralized contracting are respectively equal to*

$$b^* = \frac{\alpha_i + \alpha_j}{2}, \tag{B.1}$$

$$\phi_i^* = \frac{\alpha_i}{\alpha_i + \alpha_j}. \tag{B.2}$$

Proof. Agent i's and agent j's maximization problems in stage two are

$$\begin{cases} e_i(b,\phi) = \arg\max\limits_{e_i} \ \ \phi b \ {e_i}^{\alpha_i} e_j(b,\phi)^{\alpha_j} - \frac{e_i^2}{2}, \\ e_j(b,\phi) = \arg\max\limits_{e_j} \ \ (1-\phi) b \ e_i(b,\phi)^{\alpha_i} {e_j}^{\alpha_j} - \frac{e_j^2}{2}. \end{cases}$$

The first order conditions are

$$\begin{cases} \phi b \ \alpha_i {e_i}^{\alpha_i - 1} e_j(b,\phi)^{\alpha_j} - e_i = 0, \\ (1-\phi) b \ e_i(b,\phi)^{\alpha_i} \alpha_j {e_j}^{\alpha_j - 1} - e_j = 0, \end{cases}$$

and the second order conditions are

$$\begin{cases} \phi b \ \alpha_i(\alpha_i - 1) {e_i}^{\alpha_i - 2} e_j(b,\phi)^{\alpha_j} - 1 < 0, \\ (1-\phi) b \ e_i(b,\phi)^{\alpha_i} \alpha_j (\alpha_j - 1) {e_j}^{\alpha_j - 2} - 1 < 0, \end{cases}$$

since α_i and $\alpha_j \in (0,1)$. The first order conditions imply that

$$\begin{cases} \phi b \ \alpha_i e_i(b,\phi)^{\alpha_i - 1} e_j(b,\phi)^{\alpha_j} = e_i(b,\phi), \\ (1-\phi) b \ e_i(b,\phi)^{\alpha_i} \alpha_j e_j(b,\phi)^{\alpha_j - 1} = e_j(b,\phi). \end{cases}$$

Solving the above system of equation in (e_i, e_j), we obtain

$$\begin{cases} e_i(b,\phi) = \alpha_i^{\frac{2-\alpha_j}{2(2-\alpha_i-\alpha_j)}} \alpha_j^{\frac{\alpha_j}{2(2-\alpha_i-\alpha_j)}} \phi^{\frac{2-\alpha_j}{2(2-\alpha_i-\alpha_j)}} (1-\phi)^{\frac{\alpha_j}{2(2-\alpha_i-\alpha_j)}} b^{\frac{1}{2-\alpha_i-\alpha_j}}, \\ e_j(b,\phi) = \alpha_i^{\frac{\alpha_i}{2(2-\alpha_i-\alpha_j)}} \alpha_j^{\frac{2-\alpha_i}{2(2-\alpha_i-\alpha_j)}} \phi^{\frac{\alpha_i}{2(2-\alpha_i-\alpha_j)}} (1-\phi)^{\frac{2-\alpha_i}{2(2-\alpha_i-\alpha_j)}} b^{\frac{1}{2-\alpha_i-\alpha_j}}. \end{cases}$$

In order to induce a strictly positive probability of success of the risky project, both agents need to exert effort in equilibrium, which requires $b > 0$ and $\phi \in (0, 1)$.

The principal's maximization problem in stage one becomes

$$\begin{aligned} (b^*, \phi^*) &= \arg\max_{b,\,\phi} \;\; (1-b)\; e_i(b,\phi)^{\alpha_i} e_j(b,\phi)^{\alpha_j}, \\ &= \arg\max_{b,\,\phi} \;\; (1-b)\; \alpha_i^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} \alpha_j^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} \phi^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} (1-\phi)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} b^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}}. \end{aligned}$$

The first order conditions are

$$\begin{cases} \alpha_i^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} \alpha_j^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} \phi^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} (1-\phi)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} \left(-b^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}} + (1-b)\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j} b^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}-1} \right) = 0, \\ (1-b)\; \alpha_i^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} \alpha_j^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} b^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}} \left(\frac{\alpha_i}{2-\alpha_i-\alpha_j} \phi^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}-1} (1-\phi)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} - \phi^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} \frac{\alpha_j}{2-\alpha_i-\alpha_j} (1-\phi)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}-1} \right) = 0. \end{cases}$$

$b = 1$ is not a solution to the first equation as $\phi \in (0, 1)$. Therefore, b, ϕ, α_i, and α_j are all bounded in $(0, 1)$. The first order conditions can be reduced to

$$\begin{cases} -b^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}} + (1-b)\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j} b^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}-1} = 0, \\ \frac{\alpha_i}{2-\alpha_i-\alpha_j} \phi^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}-1} (1-\phi)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} - \phi^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} \frac{\alpha_j}{2-\alpha_i-\alpha_j} (1-\phi)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}-1} = 0. \end{cases}$$

Solving the above system of equation in (b, ϕ), we obtain

$$b^* = \frac{\alpha_i + \alpha_j}{2}, \qquad \phi^* = \frac{\alpha_i}{\alpha_i + \alpha_j}.$$

The second order conditions are

$$\begin{cases} \alpha_i^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} \alpha_j^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} \phi^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} (1-\phi)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} \frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j} b^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}-2} \left(-2b + (1-b)\left(\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j} - 1\right)\right) < 0, \\ (1-b)\; \alpha_i^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} \alpha_j^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} b^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}} \phi^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} (1-\phi)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} \\ \quad \times \left(\frac{\alpha_i}{2-\alpha_i-\alpha_j}\left(\frac{\alpha_i}{2-\alpha_i-\alpha_j} - 1\right)(1-\phi)^2 - 2\frac{\alpha_i}{2-\alpha_i-\alpha_j}\phi\frac{\alpha_j}{2-\alpha_i-\alpha_j}(1-\phi) + \phi^2 \frac{\alpha_j}{2-\alpha_i-\alpha_j}\left(\frac{\alpha_j}{2-\alpha_i-\alpha_j} - 1\right)\right) < 0, \end{cases}$$

since $-2b + (1-b)\big(\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j} - 1\big) = -1$ for $b = b^*$, $\frac{\alpha_i}{2-\alpha_i-\alpha_j}\big(\frac{\alpha_i}{2-\alpha_i-\alpha_j} - 1\big)(1-\phi)^2 - 2\frac{\alpha_i}{2-\alpha_i-\alpha_j}\phi\frac{\alpha_j}{2-\alpha_i-\alpha_j}(1-\phi) + \phi^2\frac{\alpha_j}{2-\alpha_i-\alpha_j}\big(\frac{\alpha_j}{2-\alpha_i-\alpha_j} - 1\big) = -\frac{\alpha_i\alpha_j}{(\alpha_i+\alpha_j)(2-\alpha_i-\alpha_j)} < 0$ for $\phi = \phi^*$, and α_i and $\alpha_j \in (0, 1)$. Hence, b^* and ϕ^* maximize the principal's objective function.

Since b^*, ϕ^*, α_i, and α_j are all bounded in $(0,1)$, e_i^* and $e_j^* \in (0,1)$. The equilibrium probability of success of the risky project is equal to

$$\pi^* = \alpha_i^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} \alpha_j^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} \left(\phi^*\right)^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} \left(1-\phi^*\right)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} \left(b^*\right)^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}},$$

which is also $\in (0,1)$. As a consequence, the expected payoff for the principal, $v^* = (1-b^*)\pi^*$, is strictly positive. The same holds for the expected payoff of the two agents,

$$u_i^* = \left(1-\frac{\alpha_i}{2}\right)\phi^* b^* \pi^* > 0,$$
$$u_j^* = \left(1-\frac{\alpha_j}{2}\right)(1-\phi^*)\, b^* \pi^* > 0.$$

□

C Centralized Contracting with One Public Contract

We consider a centralized contracting scheme where only one contract is public. In particular, we assume that the contract offered to agent i is private, while the contract offered to agent j is public. Therefore, agent i can observe agent j's contract but not vice versa. We denote the compensation budget and allocation characterizing the optimal contracts in this scheme by (b', ϕ').

Proposition C.1. *When only the contract of agent j is public, the optimal compensation budget and allocation with centralized contracting are respectively equal to*

$$b' = \frac{2\alpha_i + 2\alpha_j - \alpha_i\alpha_j}{4}, \tag{C.1}$$

$$\phi_i' = \frac{2\alpha_i - \alpha_i\alpha_j}{2\alpha_i + 2\alpha_j - \alpha_i\alpha_j}. \tag{C.2}$$

Proof. Since the contract observability in this scheme is the same as that characterizing the delegated contracting scheme (one agent observes the contract of the other agent, but not vice versa), agent j's and agent i's optimal effort levels are equal to the effort level of the Subagent in (A.10) and of the Agent in (A.12), respectively:

$$e_j((1-\phi)b, \hat{e}_i) = \alpha_j^{\frac{1}{2-\alpha_j}} \hat{e}_i^{\frac{\alpha_i}{2-\alpha_j}} ((1-\phi)b)^{\frac{1}{2-\alpha_j}}, \tag{C.3}$$

$$e_i(b, \phi, \hat{e}_i) = \alpha_i^{\frac{1}{2-\alpha_i}} \alpha_j^{\frac{\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \hat{e}_i^{\frac{\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \phi^{\frac{1}{2-\alpha_i}} (1-\phi)^{\frac{\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} b^{\frac{2}{(2-\alpha_i)(2-\alpha_j)}}. \tag{C.4}$$

In order to induce positive effort from each agent, we consider (and later verify that) $b > 0$ and $\phi \in (0,1)$.

The principal's maximization problem is

$$(b', \phi') = \arg\max_{b,\,\phi} \;\; (1-b)\; e_i(b, \phi, \hat{e}_i)^{\alpha_i} e_j((1-\phi)b, \hat{e}_i)^{\alpha_j},$$
$$= \arg\max_{b,\,\phi} \;\; (1-b)\; \alpha_i^{\frac{\alpha_i}{2-\alpha_i}} \alpha_j^{\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \hat{e}_i^{\frac{2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \phi^{\frac{\alpha_i}{2-\alpha_i}} (1-\phi)^{\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} b^{\frac{2\alpha_i+2\alpha_j-\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}}.$$

The first order conditions are

$$\begin{cases} \alpha_i^{\frac{\alpha_i}{2-\alpha_i}} \alpha_j^{\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \hat{e}_i^{\frac{2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \phi^{\frac{\alpha_i}{2-\alpha_i}} (1-\phi)^{\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \\ \qquad \times \left(-b^{\frac{2\alpha_i+2\alpha_j-\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} + (1-b)\frac{2\alpha_i+2\alpha_j-\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)} b^{\frac{2\alpha_i+2\alpha_j-\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}-1} \right) = 0, \\ (1-b)\; \alpha_i^{\frac{\alpha_i}{2-\alpha_i}} \alpha_j^{\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \hat{e}_i^{\frac{2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} b^{\frac{2\alpha_i+2\alpha_j-\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \\ \qquad \times \left(\frac{\alpha_i}{2-\alpha_i} \phi^{\frac{\alpha_i}{2-\alpha_i}-1} (1-\phi)^{\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} - \phi^{\frac{\alpha_i}{2-\alpha_i}} \frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)} (1-\phi)^{\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}-1} \right) = 0. \end{cases}$$

$b = 1$ is not a solution to the first equation as $\phi \in (0,1)$. Therefore, b, ϕ, α_i, and α_j are all bounded in $(0,1)$. The first order conditions can be reduced to

$$\begin{cases} -b^{\frac{2\alpha_i+2\alpha_j-\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} + (1-b)\frac{2\alpha_i+2\alpha_j-\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)} b^{\frac{2\alpha_i+2\alpha_j-\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}-1} = 0, \\ \frac{\alpha_i}{2-\alpha_i} \phi^{\frac{\alpha_i}{2-\alpha_i}-1} (1-\phi)^{\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} - \phi^{\frac{\alpha_i}{2-\alpha_i}} \frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)} (1-\phi)^{\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}-1} = 0. \end{cases}$$

Solving the above system of equation in (b, ϕ), we obtain

$$b' = \frac{2\alpha_i + 2\alpha_j - \alpha_i\alpha_j}{4}, \qquad \phi' = \frac{2\alpha_i - \alpha_i\alpha_j}{2\alpha_i + 2\alpha_j - \alpha_i\alpha_j}.$$

The second order conditions are

$$\begin{cases} \alpha_i^{\frac{\alpha_i}{2-\alpha_i}} \alpha_j^{\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \hat{e}_i^{\frac{2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \phi^{\frac{\alpha_i}{2-\alpha_i}} (1-\phi)^{\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \frac{2\alpha_i+2\alpha_j-\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)} b^{\frac{2\alpha_i+2\alpha_j-\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}-2} \\ \qquad \times \left(-2b + (1-b)\left(\frac{2\alpha_i+2\alpha_j-\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)} - 1 \right) \right) < 0, \\ (1-b)\; \alpha_i^{\frac{\alpha_i}{2-\alpha_i}} \alpha_j^{\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \hat{e}_i^{\frac{2\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} b^{\frac{2\alpha_i+2\alpha_j-\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} \phi^{\frac{\alpha_i}{2-\alpha_i}-2} (1-\phi)^{\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}-2} \\ \quad \times \left(\frac{\alpha_i}{2-\alpha_i}\left(\frac{\alpha_i}{2-\alpha_i} - 1\right)(1-\phi)^2 - 2\frac{\alpha_i}{2-\alpha_i}\phi\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}(1-\phi) + \phi^2 \frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}\left(\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)} - 1\right) \right) < 0, \end{cases}$$

since $-2b + (1-b)\left(\frac{2\alpha_i+2\alpha_j-\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)} - 1\right) = -1$ for $b = b'$ and $\frac{\alpha_i}{2-\alpha_i}\left(\frac{\alpha_i}{2-\alpha_i} - 1\right)(1-\phi)^2 - 2\frac{\alpha_i}{2-\alpha_i}\phi\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}(1-\phi) + \phi^2\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)}\left(\frac{2\alpha_j}{(2-\alpha_i)(2-\alpha_j)} - 1\right) = -\frac{2\alpha_i\alpha_j}{(2-\alpha_i)(2\alpha_i+2\alpha_j-\alpha_i\alpha_j)} < 0$ for $\phi = \phi'$. Hence, b' and ϕ' maximize the principal's objective function.

Agent i's equilibrium compensation is $\phi' b' = \alpha_i(2-\alpha_j)/4 \in (0,1)$, while agent j's equilibrium compensation is $(1-\phi')b' = \frac{\alpha_j}{2} \in (0,1)$. The equilibrium condition requires that

$$\hat{e}_i = e_i'. \qquad \text{(C.5)}$$

Substituting (C.5) into the first order condition of agent j's effort (C.3) and the first order condition of the agent i's effort (C.4), we obtain

$$\begin{cases} e_i' = {\alpha_i}^{\frac{1}{2-\alpha_i}} {\alpha_j}^{\frac{\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} ({e_i}')^{\frac{\alpha_i\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} (\phi')^{\frac{1}{2-\alpha_i}} (1-\phi')^{\frac{\alpha_j}{(2-\alpha_i)(2-\alpha_j)}} (b')^{\frac{2}{(2-\alpha_i)(2-\alpha_j)}}, \\ e_j' = {\alpha_j}^{\frac{1}{2-\alpha_j}} ({e_i}')^{\frac{\alpha_i}{2-\alpha_j}} ((1-\phi')b')^{\frac{1}{2-\alpha_j}}. \end{cases}$$

Solving the above system of equations in (e_i', e_j'), we obtain

$$e_i' = {\alpha_i}^{\frac{2-\alpha_j}{2(2-\alpha_i-\alpha_j)}} {\alpha_j}^{\frac{\alpha_j}{2(2-\alpha_i-\alpha_j)}} \left(\phi'\right)^{\frac{2-\alpha_j}{2(2-\alpha_i-\alpha_j)}} \left(1-\phi'\right)^{\frac{\alpha_j}{2(2-\alpha_i-\alpha_j)}} \left(b'\right)^{\frac{1}{2-\alpha_i-\alpha_j}},$$
$$e_j' = {\alpha_i}^{\frac{\alpha_i}{2(2-\alpha_i-\alpha_j)}} {\alpha_j}^{\frac{2-\alpha_i}{2(2-\alpha_i-\alpha_j)}} \left(\phi'\right)^{\frac{\alpha_i}{2(2-\alpha_i-\alpha_j)}} \left(1-\phi'\right)^{\frac{2-\alpha_i}{2(2-\alpha_i-\alpha_j)}} \left(b'\right)^{\frac{1}{2-\alpha_i-\alpha_j}}.$$

Since b', ϕ', α_i, and α_j are all bounded in $(0,1)$, e_i' and $e_j' \in (0,1)$. The equilibrium probability of success of the risky project is equal to

$$\pi' = {\alpha_i}^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} {\alpha_j}^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} \left(\phi'\right)^{\frac{\alpha_i}{2-\alpha_i-\alpha_j}} \left(1-\phi'\right)^{\frac{\alpha_j}{2-\alpha_i-\alpha_j}} \left(b'\right)^{\frac{\alpha_i+\alpha_j}{2-\alpha_i-\alpha_j}},$$

which is also $\in (0,1)$. As a consequence, the expected payoff for the principal, $v' = (1-b')\pi'$, is strictly positive. The same holds for the expected payoff of the two agents,

$$u_i' = \left(1 - \frac{\alpha_i}{2}\right) \phi' b' \pi' > 0,$$
$$u_j' = \left(1 - \frac{\alpha_j}{2}\right) (1-\phi') b' \pi' > 0.$$

□

Table I: Contracting Schemes, Observability and Optimal Contracts

This table presents a summary of the optimal contracts characterizing different contracting schemes under different observability assumptions. We maintain the assumption $\alpha_i \geqslant \alpha_j$.

Contracting Scheme	Contract Observed	b	ϕ_i	ϕ_j	$\phi_i b$	$\phi_j b$
			PANEL A: PRIVATE CONTRACTS			
Centralized (C)	0	$\frac{2(\alpha_i+\alpha_j-\alpha_i\alpha_j)}{4-\alpha_i\alpha_j}$	$\frac{\alpha_i(2-\alpha_j)}{2(\alpha_i+\alpha_j-\alpha_i\alpha_j)}$	$\frac{\alpha_j(2-\alpha_i)}{2(\alpha_i+\alpha_j-\alpha_i\alpha_j)}$	$\frac{\alpha_i(2-\alpha_j)}{4-\alpha_i\alpha_j}$	$\frac{\alpha_j(2-\alpha_i)}{4-\alpha_i\alpha_j}$
Delegated (D)	j	$\frac{2(\alpha_i+\alpha_j)-\alpha_i\alpha_j}{4}$	$\frac{2-\alpha_j}{2}$	$\frac{\alpha_j}{2}$	$\frac{(2-\alpha_j)(2(\alpha_i+\alpha_j)-\alpha_i\alpha_j)}{8}$	$\frac{\alpha_j(2(\alpha_i+\alpha_j)-\alpha_i\alpha_j)}{8}$
			PANEL B: PUBLIC CONTRACTS			
Centralized (*)	i, j	$\frac{\alpha_i+\alpha_j}{2}$	$\frac{\alpha_i}{\alpha_i+\alpha_j}$	$\frac{\alpha_j}{\alpha_i+\alpha_j}$	$\frac{\alpha_i}{2}$	$\frac{\alpha_j}{2}$
Centralized ($'$)	j	$\frac{2(\alpha_i+\alpha_j)-\alpha_i\alpha_j}{4}$	$\frac{\alpha_i(2-\alpha_j)}{2(\alpha_i+\alpha_j)-\alpha_i\alpha_j}$	$\frac{2\alpha_j}{2(\alpha_i+\alpha_j)-\alpha_i\alpha_j}$	$\frac{\alpha_i(2-\alpha_j)}{4}$	$\frac{\alpha_j}{2}$
			PANEL C: COMPARISON ACROSS CONTRACTING SCHEMES			
			Compensation Budget: $b^C < b^D = b' < b^*$			
			Budget Allocation: $\phi_i' < \phi_i^* \leqslant \phi_i^C < \phi_i^D$			