Renegotiation in Debt Chains

by

Vincent Glode
Christian Opp

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Vincent Glode[†] Christian C. Opp[‡]

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Abstract

We develop a tractable model of strategic debt renegotiation in which businesses are sequentially interconnected through their liabilities. This financing structure, which we refer to as a *debt chain*, gives rise to externalities as a lender's willingness to provide concessions to his privately-informed borrower depends on how this lender's own liabilities are expected to be renegotiated. Our analysis reveals how targeted government subsidies and debt reductions as well as incentives for early renegotiation following large economic shocks such as COVID-19 or a financial crisis can prevent default waves.

Keywords: Debt Renegotiation, Credit, Bargaining Power, Default Waves.
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[†]Wharton School - University of Pennsylvania. Email: vglode@wharton.upenn.edu.

[‡]University of Rochester and NBER. Email: opp@rochester.edu.

1 Introduction

The COVID-19 pandemic has imposed unprecedented hardships on businesses worldwide, prompting governments and private parties to search for interventions that could prevent large-scale default waves. A key challenge to designing effective measures in this context is the fact that businesses are sequentially interconnected through their liabilities, a financing structure we refer to as a *debt chain*. For example, a small business might have a large account payable owed to its inventory supplier. The inventory supplier, in turn, has a loan from a local credit union, which also has financial obligations to a large national bank. Perhaps this large national bank is partly financed with bonds held by a pension fund that owes retirement benefits to workers, etc.

When a large economic shock affects such businesses, private debt renegotiations are a prevalent response. As Roberts and Sufi (2009) document, changes in borrowers' financial conditions are a key reason for why over 90% of long-term debt contracts end up being renegotiated prior to their maturity. However, complicating the renegotiation process in a debt chain is the fact that firms are heterogeneously exposed to these shocks and typically have private information about their individual financial conditions. Furthermore, bargaining between a borrower and his lender is generically bilateral, giving rise to the possibility that an agent's bargaining power impedes the efficiency of not only one credit relationship but that of the whole chain.

To analyze the effectiveness of private and public interventions in such an environment, we develop a tractable model of strategic renegotiation in a debt chain. Our model features agents endowed with private information about their ability to repay the liabilities they owe to their lenders and with bargaining power they can use when strategically renegotiating with their borrowers (see Chava and Roberts 2008, Roberts and Sufi 2009, Adelino, Gerardi, and Willen 2013, Roberts 2015, for related empirical evidence). Our analysis reveals how private renegotiation decisions are generically interrelated in a debt chain: a lender's willingness to provide concessions to his borrower depends on his own liabilities and how they are expected to be renegotiated (see Murfin 2012, Chodorow-Reich and Falato 2020, for related empirical evidence). The more a lender's own liabilities are expected to be reduced, the greater are his incentives to renegotiate down those of his borrower. On the other hand, a lender who is deeply indebted himself typically finds it suboptimal to reduce how much his borrower owes him — while the probability of being paid is higher after making concessions, the payment collected in case of no default is lower. Whereas a tough renegotiation strategy may be privately optimal, it not only increases the potential for costly default in the specific bilateral credit relationship but also creates negative externalities to renegotiation efforts elsewhere in the chain. If a lender such as the local credit union in the example above takes a tough stance with its borrower (i.e., the inventory supplier), so will the inventory supplier with its own borrower (i.e., the small business). Furthermore, if the large national bank expects the credit union to take this tough stance with the inventory supplier, thereby accruing significant credit risk, it might reduce the bank's incentives to renegotiate the credit union's liabilities to a default-free level. As a result, an unaccommodating renegotiation in a particular credit relationship can trigger tough renegotiations and increased default probabilities everywhere else in the chain.

Accounting for the endogenous responses of all debt-chain members, we analyze how targeted government policies affect renegotiation outcomes throughout the chain. First, we show that providing subsidies to "downstream" borrowers like the small business (whose debt payments are expected to flow up the chain) can be particularly effective in eliminating default waves. Such subsidies generally have to cover only a fraction of the potential shortfall the targeted borrower is facing, since the lender also has private incentives to reduce the debt to a default-free level. That is, private renegotiation is an important factor determining the magnitudes of government subsidies needed to avoid default. Moreover, providing subsidies to a borrower like the small business also strengthens "upstream" lenders' incentives to renegotiate their borrowers' debt to default-free levels. By boosting the maximum debt payment the small business can pay without defaulting on its inventory supplier, a subsidy to the small business can first push the inventory supplier, then the local credit union, and then the large national bank to more efficiently renegotiate with their respective borrowers. As a result, awarding a subsidy to a downstream borrower can be highly effective in preventing default waves, compared to giving the same subsidy to an upstream borrower, all thanks to the recursivity of debt-chain members' optimal renegotiation decisions.

Second, we show how government interventions affecting bargaining power in the private renegotiation process can help prevent default waves. In particular, preventing an upstream lender from being able to choose his renegotiation strategy and instead mandating him to reduce his borrower's debt can incentivize downstream agents to voluntarily renegotiate the debt owed to them to levels that avoid default. For example, reducing how much the local credit union owes to the large national bank may first push the credit union, and then the inventory supplier to more efficiently renegotiate with their respective borrowers. If poorly designed, this type of intervention can, however, backfire as it may significantly reduce how much the bank's bondholders would collect from efficiently renegotiating the bank's debt. Such intervention may thereby result in the pension fund that holds the bank's bonds toughening its renegotiation strategy with the bank and increasing the default risk in the chain.

A key friction impeding efficient renegotiation in our environment is agents' private information. Absent the resulting information asymmetries, each lender would know his borrower's financial condition and would have no incentive to ask for more than the borrower can actually pay. As a result, inefficiencies associated with default would be avoided throughout the chain. In contrast, in the presence of private information each lender faces a generic tradeoff when renegotiating with his borrower. On the one hand, significantly lowering how much a borrower owes increases the probability of repayment and reduces the probability of having to seize his assets and perhaps losing a good business partner in the future. On the other hand, not providing concessions implies a higher amount being collected if the borrower happens to be able to make his payment. The uncertainty the lender faces about the borrower's financial condition as well as the expected renegotiation outcomes elsewhere in the chain significantly affect the tradeoff associated with a lender's renegotiation decision. How much the inventory supplier knows about the small business' ability to pay its debt and whether it expects to have its own loan renegotiated by the local credit union determine the optimal renegotiation strategy regarding the small business' liabilities.

Finally, as a third policy implication, our analysis reveals how the timing of the renegotiation process, relative to the arrival of private information prior to debt payment dates, is an important determinant of inefficiencies. If, at the time of renegotiation, an agent has *imperfect* private information about the conditions he will face at a future payment date, he can anticipate to collect an information rent from the offer his lender will make. The reason for this is that when the agent needs to make the ultimate default decision (i.e., at a later date when the payment is due), he will be able to incorporate new information that arrived since the renegotiation occurred. The presence of this information rent, in turn, implies a higher equity value at the time when the agent renegotiates with his own borrower. This higher equity value implies greater skin in the game, which weakens incentives to make tough renegotiation offers. An implication of this mechanism is that government policies facilitating *early* renegotiation following a large shock tend to be desirable as they tend to lead to more efficient outcomes.[1]

Literature review. Our paper sheds light on debt renegotiation decisions and how they are impacted by the network of debt agreements in the economy. There is an existing literature on renegotiation that abstracts from network effects. Bolton and Scharfstein (1996) show how creditor dispersion can impede the efficient renegotiation of debt (see also Zhong 2020, for a dynamic analysis). Garleanu and Zwiebel (2009) analyze the design and renegotiation of debt covenants, showing that adverse selection problems lead to the allocation of greater ex-ante decision rights to the creditor. Our model considers renegotiation in a debt chain in which agents are generally both lenders and borrowers, leading to externalities from renegotiation decisions that are a main focus of our paper.

Our paper is related to models of sequential strategic interactions in financial markets. Di Maggio and Tahbaz-Salehi (2015) study sequential lending relationships, but unlike us, they are interested in the use of collateral in origination decisions, rather than in debt renegotiation. They show how the allocation of collateral affects an intermediation chain's ability to shepherd liquidity towards a good investment opportunity. Glode and Opp (2016) and Glode, Opp, and Zhang (2019) study an environment subject to asymmetric information and market power problems to examine how sequentially trading an asset through moderately-informed intermediaries can facilitate efficient trade in over-the-counter markets. In contrast, we consider the renegotiation of lending relationships and a setting where agents are privately informed about their own financial conditions, rather than about the value of a single asset traded through the chain.

[1] More generally, the benefits of early renegotiation uncovered by our analysis shed light on the fact that renegotiation indeed tends to occur early in the life of a loan (see Roberts and Sufi 2009).

Our paper also relates to the theoretical literature studying the impact of debt and limited liability on firm decisions. This literature shows how outstanding debt can affect firms' incentives to invest (see Myers 1977), take risks (see Jensen and Meckling 1976), charge high prices for their products (see Brander and Lewis 1986), and negotiate with stakeholders like unionized workers (see Perotti and Spier 1993, Matsa 2010). In our model of debt chains, most agents are both lenders and borrowers and we show how an agent's outstanding debt as a borrower, which depends on renegotiations with his lender, weakens that agent's willingness as a lender to renegotiate his borrower's liabilities. By providing concessions to a struggling borrower, a lender makes the distribution of payment outcomes more concentrated and reduces the probability that his borrower will default. Due to limited liability, the benefits of these concessions are, however, not fully internalized by a lender who risks defaulting on his own liabilities. Moreover, optimal renegotiation decisions and equilibrium default risk are generically interrelated across debt-chain members, due to the presence of externalities. Our policy analysis allows to identify which debt-chain member(s) should be targeted by government interventions to maximize the total benefits throughout the chain.

Finally, our analysis complements insights from the existing literature on cascades and contagion in financial networks, which abstracts from the strategic renegotiation of debt contracts under asymmetric information. Allen and Gale (2000), Elliott, Golub, and Jackson (2014), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) study different channels through which small economic shocks can spread and expand through networks of firms connected by financial obligations. Allen, Babus, and Carletti (2012) study the interaction between asset commonality and funding maturity in generating this type of contagion. Babus and Hu (2017) study how agents' incentives to default on their financial obligations can be weakened by a star network, in which a central intermediary monitors everyone's trading history. Taschereau-Dumouchel (2020) studies how firms' failure to produce inputs can lead to cascades of firm shutdowns. Our paper contributes to this literature by analyzing the optimal renegotiation strategies of lenders/borrowers who are part of a debt chain. In an environment that features the asymmetric information and bargaining power problems inherent in most bilateral renegotiations, our analysis accounts for agents' strategic responses to each other and to government policies. In particular, it shows how upstream renegotiation outcomes affect the optimal renegotiation decisions of downstream agents, contrasting with a typical default cascade which would propagate from the final borrower's balance sheet to that of the initial lender (i.e., from downstream to upstream).

Roadmap. In the next section, we introduce our model environment. In Section 3, we analyze the equilibrium renegotiation strategies of debt-chain members and the interconnectedness of these strategies. In Section 4, we analyze how various government policies affect renegotiation outcomes throughout the whole chain. In Section 5, we discuss the robustness of our model's key insights, and the last section concludes.

2 The Environment

In this section, we introduce our model of renegotiation in debt chains.

Agents and asset endowments. We consider an environment with $N \geq 3$ agents. At date $t = 1$, each agent j owns an endowment asset that takes a random value v_j at date $t = 2$. The cumulative distribution function (CDF) of v_j, based on public information available at date $t = 1$, is denoted by $F_j(v_j)$. We denote by $\underline{v}_j$ and $\overline{v}_j$ the upper and lower bound of the support of v_j, respectively. As of date $t = 1$, the asset values v_j are independently distributed across agents, reflecting the notion that agents face heterogeneous financial conditions, while still allowing for an aggregate shock that hit before date $t = 1$ and shaped the distributions $F_j(v_j)$ (see Section 5 for a related discussion).

Existing debt obligations. Capturing the central notion of a debt chain, the N agents are linked through existing debt obligations. In particular, at date $t = 1$, each agent $j \geq 2$ owes agent $(j-1)$ a payment equal to $\overline{d}_j$ that is due at date $t = 2$. We consider a setting where the initial face values $\overline{d}_j$ were chosen at a prior date (e.g., at an unmodeled date $t = 0$) based on the information available at that time. Our paper's focus on the renegotiation of existing debt contracts (rather than the initial security design problem) is motivated by the relevance of such phenomena after an economy is hit by large negative shocks such as the current worldwide pandemic, which was essentially unanticipated prior to the end of 2019.

Debt contract settlement and default costs. If at date $t = 2$ an agent j defaults on the payment of his (potentially renegotiated) face value d_j, the lending agent $(j-1)$ attempts to collect the remaining assets that agent j owns, which generally consist of the endowment asset worth v_j at $t = 2$ and the funds agent j collects from agent $(j+1)$. Default is associated with deadweight losses that are proportional to the value of the assets, following a common specification in the corporate finance literature. Specifically, if agent j defaults, agent $(j-1)$ can collect a fraction $(1-\rho)$ of agent j's assets, where the parameter $\rho \in [0,1]$ captures the default costs. As a convention, debt contracts are settled at date $t = 2$ starting with agent N's liability, then agent $(N-1)$'s liability, up until agent 2's liability that is owed to agent 1. This specification increases the tractability of our model by ensuring that each agent j observes the realized value of his debt claim to agent $(j+1)$ (that is, the face value payment or the recovery value) before deciding on whether to default himself on what he owes to agent $(j-1)$.[2]

Private information. At date $t = 1$, each agent obtains a private signal s_j that is informative about the future realization of his own asset value v_j. We denote by $F_j(v_j|s_j)$ the conditional CDF of v_j as perceived by agent j. Introducing this private signal allows us to identify relevant differences in the implications that incomplete vs. private information have for the efficiency of the strategic renegotiation process. When dates $t = 1$ (renegotiation) and $t = 2$ (payment due date) are very close to each other, agent j is likely to have close to perfect private information about his endowment asset value at date $t = 2$. In contrast, when renegotiation ($t = 1$) occurs a long time before the actual payment is due ($t = 2$), then, at date $t = 1$, agent j himself is likely to face significant uncertainty (incomplete information) about this value.

[2] If contracts were settled in the reverse order, agents could not rely on payments from the debt claims they own to fulfill their liabilities. In this case, a firm could consider issuing additional securities to bridge a temporary short-fall caused by the delayed settlement of the debt claim it owns. However, such issuance would generally involve a security design decision and associated signaling concerns, significantly complicating the model environment.

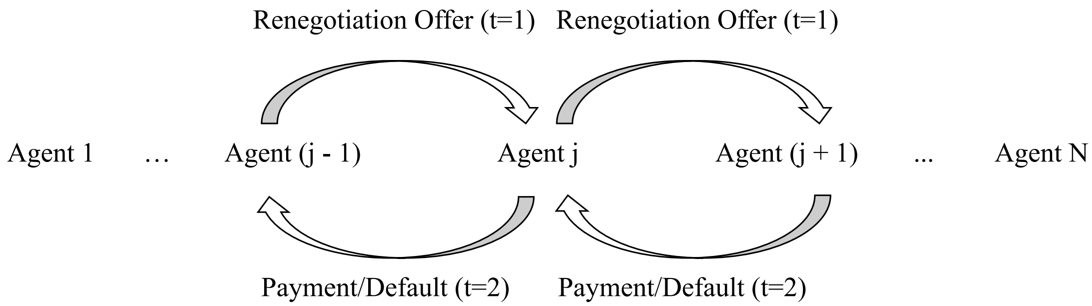

Figure 1: The figure illustrates the flow of renegotiation offers, payments, and default decisions in a debt chain.

Renegotiation. At date $t = 1$, agents can renegotiate their debt contracts. Specifically, agent $(j-1)$ chooses whether and by how much to make a concession to agent j by lowering the face value of the debt contract to $d_j \leq \overline{d}_j$. Formally, agent $(j-1)$ proposes the new face value with a take-it-or-leave-it offer to agent j. It is a dominant strategy for agent j to accept a new face value as long as it is weakly lower than the initial face value $\overline{d}_j$. However, at date $t = 2$, agent j will optimally use his limited liability and potentially default on this renegotiated face value. Renegotiation offers and outcomes are not publicly observable at date $t = 1$. Figure 1 gives an overview of the setup by illustrating the flow of renegotiation offers, payments, and default decisions in a debt chain.

Timeline. To summarize, the timeline of the model is as follows.

- Date $t = 1$: Renegotiation
  - (i) Each agent j obtains a signal s_j that is informative about his future endowment asset value v_j.
  - (ii) Each agent $j = 1, ..., (N-1)$ simultaneously makes a take-it-or-leave-it offer to his debtor $(j+1)$, specifying a new face value d_{j+1}.
  - (iii) Each agent $j = 2, ..., N$ decides whether to accept the newly proposed face value.
- Date $t = 2$: Payment

(i) Each agent j observes the asset value v_j.

(ii) Debt contracts are settled sequentially, starting with contract d_N, then contract d_{N-1}, and so on.

3 Equilibrium Renegotiation and Default

In this section, we first characterize agents' optimization problems as borrowers and lenders. We then derive explicit conditions for default-free equilibrium outcomes in debt chains, considering both discrete and continuous distributions.

3.1 Renegotiation and Equity Values

An agent's equity value depends on where he is located in the debt chain and on other agents' renegotiation strategies.

Agent N. Agent N is special in that he does not hold a claim against any other agent in the chain. At date $t = 1$, it is a dominant strategy for agent N to accept any renegotiation offer below the pre-existing face value, $d_N \leq \overline{d}_N$. At date $t = 2$, agent N knows the endowment asset value v_N and, using his limited liability, optimally pays the face value d_N as long as:

$$d_N \leq v_N. \tag{1}$$

Otherwise, agent N defaults.

Agent (j – 1). Our analysis will primarily focus on the conditions under which efficient renegotiation occurs, that is, the conditions under which default-free, subgame-perfect Nash equilibria exist.[3] These conditions will reflect agents incentives to deviate to strategies that trigger defaults among debt-chain members. Apart from our interest in efficient renegotiation, this focus greatly increases

[3]In the general setting we consider, it is in principle possible that multiple equilibria exist when specific distributional assumptions and parameter values are considered. However, for the distributional assumptions and parameterizations we consider as examples in this paper, such multiplicity does not exist. Complementarities are likely more relevant when debt connections are circular, which however is less relevant for the economic applications that motivate our analysis.

the tractability of our analysis, which features $(N-1)$ rounds of strategic debt renegotiation with asymmetric information. However, we discuss in Section 5 how the main insights derived from this baseline analysis also apply to equilibria with default.

In a default-free equilibrium, agent $(j-1) < (N-1)$ can rationally anticipate that agent j will collect the anticipated equilibrium face value d_{j+1} from agent $(j+1)$, which is helpful for predicting agent j's wealth.[4] Given this, agent $(j-1)$ anticipates that agent j will not default on an offer d_j as long as:

$$d_j \leq v_j + d_{j+1}. \tag{2}$$

For agent $(j-1)$, proposing a new face value d_j is equivalent to choosing a *marginal debtor type* $v_j^* = d_j - d_{j+1}$ that would be just indifferent between defaulting and not defaulting at date $t = 2$. All date-2 debtor types greater or equal to v_j^* will be *included* by this offer, in the sense that they will not default on the new face value. All debtor types below v_j^* will be *excluded* in the sense that they will default at date $t = 2$. Correspondingly, we can write agent $(j-1)$'s optimization problem as choosing a marginal debtor type v_j^* to maximize his expected equity value given the signal s_{j-1} he received at date $t = 1$:

$$\begin{aligned} \Pi_{j-1}(v_j^*) = \; & \Pr[v_j < v_j^*] \cdot \mathbb{E}\left[\max\left(v_{j-1} + (1-\rho)(v_j + d_{j+1}) - d_{j-1}\;,\; 0\right) \mid s_{j-1}, v_j < v_j^*\right] \\ & + \Pr[v_j \geq v_j^*] \cdot \mathbb{E}\left[\max\left(v_{j-1} + v_j^* + d_{j+1} - d_{j-1}\;,\; 0\right) \mid s_{j-1}\right]. \end{aligned} \tag{3}$$

The equity value (3) shows that agent $(j-1)$'s renegotiation offer will generally depend on the private information he has about the value of his own endowment asset v_{j-1}, as represented by the signal s_{j-1}. The max operators in equation (3) reflect the limited liability agent $(j-1)$ has himself: whenever his total payoff would be negative after paying his debt, agent $(j-1)$ defaults on his debt and gets a payoff of zero instead. The extent to which he anticipates using limited liability depends on his information about the future value of his own endowment asset, but also on the renegotiation

[4] Agent $(N-1)$ differs from agents 1 to $(N-2)$ in that his debtor, agent N, does not have a debt claim to another agent's assets (or equivalently, if there was an agent $(N+1)$ but with $\overline{d}_{N+1} = 0$).

offer d_{j-1} he expects to receive from agent $(j-2)$.

Agent 1. The first agent in the credit chain is special in that he does not owe anything to another agent. We can again write this agent's expected equity value for a given signal s_1 as a function of the marginal debtor type:

$$\Pi_1(v_2^*) = \mathbb{E}[v_1 \mid s_1] + \Pr[v_2 < v_2^*] \cdot (1-\rho)(\mathbb{E}[v_2 \mid v_2 < v_2^*] + d_3) \\ + \Pr[v_2 \geq v_2^*] \cdot (v_2^* + d_3) \ . \tag{4}$$

3.2 Default-Free Renegotiation

To ensure a default-free equilibrium outcome, each agent must choose to renegotiate the debt level of his borrower to the lowest possible *total* asset value that this debtor might have at date $t = 2$. Conditional on his information at $t = 1$, an agent j's endowment asset delivers at least a value equal to $\underline{v}_j$ at date $t = 2$. Moreover, in a *default-free* equilibrium, agent j will also collect the renegotiated face value d_{j+1} from agent $(j+1)$ with probability 1. As a result, the *total* value of agent j's assets at date $t = 2$ is bounded from below at $(\underline{v}_j + d_{j+1})$. Note that $\underline{d}_j$ is not per se the lowest possible value of an agent's total assets. Rather it is the lowest possible value conditional on his information at $t = 1$ *and* on being in an equilibrium in which agents $(j+1)$ through N do not default, that is, the newly proposed face values are indeed collected with probability 1. In contrast, if a default did occur among agents $(j+1)...N$, then agent j would possibly have less financial wealth than $(\underline{v}_j + d_{j+1})$ after accounting for default costs.

The new face values proposed by the lenders in a default-free equilibrium correspondingly satisfy the recursive relation:

$$\underline{d}_j \equiv \underline{v}_j + d_{j+1}, \tag{5}$$

provided that the initial face value $\overline{d}_j$ exceeds this value, that is, $\overline{d}_j \geq \underline{d}_j$. Otherwise, the face value remains at its initial level. Moreover, if $\overline{d}_j \geq \underline{d}_j$ for all j, we obtain the following explicit formulae:

$$\underline{d}_j = \sum_{i=j}^{N} \underline{v}_i. \tag{6}$$

Whereas equation (6) indicates that the default-free renegotiated face values represent the accumulated lower bounds of the endowment asset values, higher renegotiated face values would apply if we introduced additional default costs that are internalized by the debtors, such as for example, reputation cost of defaulting (see Section 5 for details). Whereas these costs increase the renegotiated face values, the default costs internalized by the creditor, as captured by the parameter ρ in our model, do not. The reason for this difference is that in a default-free equilibrium, an agent j's borrower, agent $(j+1)$ is collecting the full face value from his borrower, agent $(j+2)$, so default costs are not incurred in equilibrium. Yet, the marginal borrower type (and the associated renegotiated face value) is increased when default costs are internalized by the debtor, as he is then willing to pay that extra cost to avoid default.

3.3 The Case with Binomially Distributed Asset Values

To illustrate our main insights in an intuitive way, we first consider the case in which each agent's endowment asset value is binomially distributed. Specifically, each agent j owns an asset that might either be worth $\underline{v}_j$ or $\overline{v}_j$ at date $t = 2$, where $\underline{v}_j < \overline{v}_j$. Furthermore, to make every negotiation decision non-trivial, we assume that $\overline{d}_j > \sum_{i=j}^{N} \underline{v}_i$ for each agent j, that is, all initial face values are greater than their default-free counterparts (relaxing this assumption would simply add credit relationships that do not need to be renegotiated). Finally, we assume that $\overline{d}_j < \overline{v}_j + \underline{d}_{j+1}$ so that no agent j defaults with probability 1 if his debt is not renegotiated down.

Agent (j – 1). In a default-free equilibrium, agent $(j-1)$ expects that the debt of any agent $k \neq j$ will be renegotiated to $\underline{d}_k = \sum_{i=k}^{N} \underline{v}_k$. Thus, agent $(j-1)$ faces the following renegotiation choices. First, he can keep agent j's debt at its initial level $\overline{d}_j$. Agent j then makes the promised debt payment when his type is $v_j = \overline{v}_j$ at date $t = 2$, but defaults when it is $v_j = \underline{v}_j$. That is, the marginal included type is $\overline{v}_j$; the low type $\underline{v}_j$ is excluded. Given this strategy, agent $(j-1)$'s equity value at $t=1$ is:

$$\Pi_{j-1}(\overline{v}_j) = F_j(\underline{v}_j)\cdot\mathbb{E}\left[\max\left(v_{j-1}+(1-\rho)(\underline{v}_j+\underline{d}_{j+1})-\underline{d}_{j-1}\ ,\ 0\right)\mid s_{j-1}\right] + (1-F_j(\underline{v}_j))\cdot(\mathbb{E}[v_{j-1}\mid s_{j-1}]+\overline{d}_j-\underline{d}_{j-1}). \tag{7}$$

Alternatively, agent $(j-1)$ can renegotiate agent j's liabilities to a default-free level $\underline{d}_j$, in which case the marginal included type is $\underline{v}_j$. Agent $(j-1)$ then collects:

$$\begin{aligned}\Pi_{j-1}(\underline{v}_j) &= \mathbb{E}\left[\max\left(v_{j-1}+\underline{d}_j-\underline{d}_{j-1}\ ,\ 0\right)\mid s_{j-1}\right]\\ &= \mathbb{E}[v_{j-1}\mid s_{j-1}]-\underline{v}_{j-1}.\end{aligned} \tag{8}$$

Correspondingly, efficient renegotiation to the default-free level is privately optimal for agent $(j-1)$ whenever $\Pi_{j-1}(\underline{v}_j)\geq\Pi_{j-1}(\overline{v}_j)$ for all possible signal realizations s_{j-1}. This condition for the optimality of efficient renegotiation simplifies to:

$$\underbrace{\frac{1-F_j(\underline{v}_j)}{F_j(\underline{v}_j)}\left(\overline{d}_j-\underline{d}_j\right)}_{\text{Gain from higher debt face value}} \leq \mathbb{E}[\min(\underbrace{\rho\cdot\underline{d}_j}_{\text{Default cost}}\ ,\ \underbrace{v_{j-1}-\underline{v}_{j-1}}_{\text{Information rent}})\mid s_{j-1}]. \tag{9}$$

One can think of the left-hand side of condition (9) as the benefit of following a tough renegotiation strategy and the right-hand side as its cost. When the benefit is lower than the cost (i.e., condition (9) holds), agent $(j-1)$ is lenient, ensuring that agent j does not default. The left-hand side of condition (9) shows that the benefit of following a tough renegotiation strategy is higher when the initial face value $\overline{d}_j$ is large relative to the level that would be required to avoid default, $\underline{d}_j$. Moreover, the benefit of following this strategy is affected by the relative odds of facing a high borrower type vs. a low type, as represented by the ratio $\frac{1-F_j(\underline{v}_j)}{F_j(\underline{v}_j)}$.

On the other hand, the cost of following a tough renegotiation strategy, as represented by the right-hand side of condition (9), is affected by two channels. Default costs are incurred through a tough renegotiation strategy. If agent j is the low type, he has assets worth $\underline{d}_j$. Under this renegotiation strategy, this low type defaults, leading to default costs $\rho\cdot\underline{d}_j$. If agent $(j-1)$ did not have liabilities, he would fully internalize those losses, but since agent $(j-1)$ has outstanding liabilities, he internalizes losses only to the extent that there is equity value to absorb them. In a default-free equilibrium, agent $(j-1)$'s equity value at $t=2$ is given by $(v_{j-1}-\underline{v}_{j-1})$. As such, in each state of the world at $t=2$, agent $(j-1)$ at the margin internalizes at most the minimum of the default costs and his equity value (absent default), as indicated by the min operator in equation (9). The possibility of a positive equity value for agent $(j-1)$ stems from an information rent. In a default-free equilibrium, agent $(j-1)$'s lender, agent $(j-2)$, chooses a new face value that allows agent $(j-1)$ to avoid default even if he is the lowest type $\underline{v}_{j-1}$. Agent $(j-2)$ optimally makes that choice given the limited information he has about agent $(j-1)$'s asset values and given that agent $(j-1)$ himself makes a renegotiation offer to agent j at a time when he has incomplete information about his own future asset value v_{j-1}. As a result, when agent $(j-1)$'s actual type exceeds this lowest type, he realizes strictly positive equity values at date $t=2$. A higher equity value and the associated skin-in-the-game, in turn, discourage agent $(j-1)$ from choosing a tough renegotiation strategy for his own borrower, agent j.

Discussion: The role of private information and the timing of renegotiations. The magnitude of this skin-in-the-game effect depends on the private signal s_{j-1} that agent $(j-1)$ obtains at date $t=1$. The worse the signal, the less likely it is that agent $(j-1)$ will have an asset worth more than the lower bound $\underline{v}_{j-1}$ at date $t=2$. Thus, agent $(j-1)$ is less willing to renegotiate down his borrower's liabilities when he receives a bad interim signal about the value of his own endowment asset. In fact, if agent $(j-1)$ is perfectly informed about v_{j-1} and observes a bad asset value realization $v_{j-1}=\underline{v}_{j-1}$, the right-hand-side of condition (9) takes the value zero. Given our initial assumption that $\overline{d}_j \geq \underline{d}_j = \sum_{i=j}^{N} \underline{v}_i$, condition (9) then cannot be satisfied. The economic intuition for this is simple. If at the renegotiation stage agent $(j-1)$ already observes that his asset value is $\underline{v}_{j-1}$, he knows that a default-free renegotiation strategy will generate zero equity value for him. Thus, he is better off gambling for a positive profit by keeping agent j's debt level at the initial level $\overline{d}_j$.

An immediate implication of this channel is that the timing of the arrival of private information, relative to the renegotiation process, is an important determinant default risk in a debt chain. If the timing of the renegotiations ($t = 1$) is such that agents still learn a substantial amount of information about their asset values after the renegotiation, the information rent component entering condition (9) is larger, facilitating efficient renegotiation. In practice, agents are likely to learn more after the renegotiation process if the renegotiation takes place early, relative to the actual payment date. As a result, *early* renegotiation after a large shock tends to be desirable, as it tends to lead to more efficient outcomes. We will investigate this channel in more detail in Section 4.

Agent 1. As stated above, agent 1's decision to renegotiate with agent 2 is different due to the fact that agent 1 does not owe liabilities to another agent. In a default-free equilibrium, if agent 1 keeps agent 2's face value at $\overline{d}_2$, agent 1 can expect to collect:

$$\Pi_1(\overline{v}_2) = \mathbb{E}[v_1 \mid s_1] + F_2(\underline{v}_2)(1-\rho)(\underline{v}_2 + \underline{d}_3) + (1 - F_2(\underline{v}_2))\overline{d}_2. \tag{10}$$

If, on the other hand, agent 1 renegotiates agent 2's debt to its default-free level $\underline{d}_2$, agent 1 can expect to collect:

$$\Pi_1(\underline{v}_2) = \mathbb{E}[v_1 \mid s_1] + \underline{d}_2. \tag{11}$$

The condition for the optimality of agent 1's efficient renegotiation simplifies to:

$$\frac{1 - F_2(\underline{v}_2)}{F_2(\underline{v}_2)} \left(\overline{d}_2 - \underline{d}_2\right) \leq \rho \cdot \underline{d}_2. \tag{12}$$

Agent 1 fully internalizes the default costs that are triggered when the marginal agent-2 type $\underline{v}_2$ defaults, because agent 1 himself does not have any lenders that would share in absorbing these losses.

Discussion: The role of lender indebtedness. An important force in our model is the fact that lenders are generally indebted themselves. We now illustrate how this channel reduces downstream agents' incentives to renegotiate with their own borrowers.

In our baseline setting, agent 1 is by definition the first chain member and thus, solely a lender. Without any liabilities, agent 1 is willing to renegotiate with agent 2 if and only if condition (12) is satisfied. Suppose instead that agent 1 owed $\overline{d_1}$ to a fictional agent 0. Using the derivations above, we know that in a default-free equilibrium agent 1 would then be willing to renegotiate with agent 2 if and only if:

$$\frac{1-F_2(\underline{v}_2)}{F_2(\underline{v}_2)}(\overline{d}_2-\underline{d}_2) \le \mathbb{E}[\min(\rho\cdot\underline{d}_2\ ,\ v_1-\underline{v}_1)\mid s_1]. \tag{13}$$

The condition associated with agent 1 having a liability himself is more restrictive than in our original environment for a given signal s_1 whenever:

$$\mathbb{E}[\min(\rho\cdot\underline{d}_2\ ,\ v_1-\underline{v}_1)\mid s_1] \le \rho\cdot\underline{d}_2. \tag{14}$$

This inequality is guaranteed to be weakly satisfied and is strictly satisfied if default costs are large and the signal agent 1 receives is bad (which means $v_1-\underline{v}_1<\rho\cdot\underline{d}_2$ with positive probability for at least some realizations of s_1). As highlighted above, a lender deviating from a default-free renegotiation strategy fully internalizes the default costs when he is not indebted, but he only internalizes a fraction of the default costs when he owes debt to another agent.

This result shows that agent j has lower incentives to renegotiate down the debt of agent $(j+1)$ if agent j also has a liability to agent $(j-1)$. Importantly, agent $(j-1)$'s bargaining power generally leads to higher liabilities for agent j, rendering this channel more severe. Agent $(j-1)$ is expected to use his bargaining power to extract any safe gains that agent j might secure when renegotiating with agent $(j+1)$. Thus, agent j has stronger incentives to take risks by opting for a tougher renegotiation stance with agent $(j+1)$.

Numerical example. We now further illustrate the intuition behind our first results through a numerical example. We assume that $N=3$, where agent 3 owes $\overline{d}_3=\$125K$ to agent 2 who owes $\overline{d}_2=\$325K$ to agent 1. Each agent j has an endowment asset that is equally likely to take the values $\underline{v}_j=\$100K$ or $\overline{v}_j=\$250K$. We set $\rho=0.6$, that is, only 40% of the borrower's asset value can be recovered in case of default.

In this scenario, agent 2 can choose between keeping agent 3's debt at its existing level $\overline{d}_3 =$ \$125K or renegotiating it to its default-free level $\underline{d}_3 = \underline{v}_3 = \$100K$. Now, suppose that before renegotiating with agent 3, agent 2 receives a signal s_2 that either updates the probability of his own endowment asset value being high to 0.75 (i.e., the good signal) or to 0.25 (i.e., the bad signal). If he expects agent 1 to renegotiate his debt to $\underline{d}_2 = \underline{v}_2 + \underline{v}_3 = \$200K$, agent 2 is unwilling to renegotiate agent 3's debt to $\underline{d}_3 = \$100K$ after the bad signal realization. Agent 2 prefers to keep asking agent 3 for \$125K, despite the 50% probability of default, than renegotiating agent 3's debt to its default-free level of \$100K. Formally, condition (9) is violated when agent 2 receives the bad signal:

$$\begin{aligned}&\frac{1-F_3(\underline{v}_3)}{F_3(\underline{v}_3)}\left(\overline{d}_3-\underline{d}_3\right)=\left(\frac{1-0.5}{0.5}\right)(\$125K-\$100K)=\$25K\\ &>\mathbb{E}\left[\min\left(\rho\cdot\underline{d}_3\ ,\ v_2-\underline{v}_2\right)\mid s_2\right]=0.25\cdot 0.6\cdot\$100K+0.75\cdot\$0=\$15K.\end{aligned}\tag{15}$$

Moreover, even if he expects agent 2 to renegotiate down agent 3's debt (which will not happen in equilibrium), agent 1 is unwilling to renegotiate agent 2's debt from its existing level $\overline{d}_2 = \$325K$ to its default-free level of $\underline{d}_2 = \underline{v}_2 + \underline{v}_3 = \$200K$. Formally, condition (12) is violated:

$$\begin{aligned}&\frac{1-F_2(\underline{v}_2)}{F_2(\underline{v}_2)}\left(\overline{d}_2-\underline{d}_2\right)=\left(\frac{1-0.5}{0.5}\right)(\$325K-\$200K)=\$125K\\ &>\rho\cdot\underline{d}_2=0.6\cdot\$200K=\$120K.\end{aligned}\tag{16}$$

Thus, we have a parameterization where, absent any outside intervention, no lender is willing to renegotiate his borrower's debt to an efficient, default-free level, even if he conjectures that the other lender would do so. We will revisit this example later when we analyze the impact of government interventions.

3.4 The Case with Continuously Distributed Asset Values

While the binomial case above provided a simple and intuitive illustration of some of our main insights, our results are by no means specific to this distributional assumption. In this section, we investigate the case with continuously distributed asset values. Specifically, the distribution of the asset values v_j has a density function $f_j(v_j)$, which takes strictly positive and finite values everywhere on the support $v_j \in [\underline{v}_j, \overline{v}_j]$. Thus, unlike in the binomial setting where a lender's renegotiation decision effectively involves choosing either $\overline{d}_j$ or $d_j = \underline{v}_j + d_{j+1}$ as the face value, the continuous setting enriches the renegotiation stage in that the lender optimally chooses from a larger set of marginal borrower types $v_j^* \in [\underline{v}_j, \overline{v}_j]$ and corresponding face values $d_j = v_j^* + d_{j+1}$.

For tractability, we impose the standard regularity condition that the hazard rate $\frac{f_j(v_j)}{1-F_j(v_j)}$ is increasing on the support $[\underline{v}_j, \overline{v}_j]$. This condition ensures that (local) first-order conditions are sufficient for global optimality. Moreover, agent j obtains a signal $s_j \in [\underline{s}_j, \overline{s}_j]$ at date $t = 1$ that implies that the conditional density of the value of his endowment asset at date $t = 2$ is given by $f_j(v_j|s_j)$. We assume that this conditional density takes finite values everywhere on the support $[\underline{v}_j, \overline{v}_j]$, independent of the signal realization $s_j \in [\underline{s}_j, \overline{s}_j]$, which avoids having to consider point masses in the conditional distribution.

Agent (j–1). Agent $(j-1)$'s marginal benefit of increasing the marginal debtor type v_j^* is given by (see the Appendix for a derivation):

$$\begin{aligned}\Pi'_{j-1}(v_j^*) = {} & f_j(v_j^*) \cdot \mathbb{E}\left[\max\left(v_{j-1} + (1-\rho)(v_j^* + d_{j+1}) - d_{j-1}\ ,\ 0\right)\ \mid s_{j-1}\right] \\ & - f_j(v_j^*) \cdot \mathbb{E}\left[\max\left(v_{j-1} + v_j^* + d_{j+1} - d_{j-1}\ ,\ 0\right)\ \mid s_{j-1}\right] \\ & + (1 - F_j(v_j^*)) \Pr[v_{j-1} \geq d_{j-1} - d_{j+1} - v_j^* \mid s_{j-1}]. \end{aligned} \tag{17}$$

Given the regularity condition we imposed on the hazard rate, the sufficient condition for an equilibrium in which agent $(j-1)$ charges a face value that ensures that agent j does not default is:

$$\Pi'_{j-1}(\underline{v}_j) \leq 0, \tag{18}$$

that is, $v_j^* = \underline{v}_j$ is the optimal choice for agent $(j-1)$. This condition can be rewritten as follows:

$$\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq \frac{\mathbb{E}\left[\max\left(v_{j-1}+\underline{v}_j+d_{j+1}-d_{j-1}\ ,\ 0\right)\ |\ s_{j-1}\right]}{\Pr[v_{j-1} \geq d_{j-1}-d_{j+1}-\underline{v}_j \mid s_{j-1}]} - \frac{\mathbb{E}\left[\max\left(v_{j-1}+(1-\rho)(\underline{v}_j+d_{j+1})-d_{j-1}\ ,\ 0\right)\ |\ s_{j-1}\right]}{\Pr[v_{j-1} \geq d_{j-1}-d_{j+1}-\underline{v}_j \mid s_{j-1}]}. \tag{19}$$

A default-free equilibrium requires that this condition holds for all possible signal realizations that agent $(j-1)$ might observe, $s_{j-1} \in [\underline{s}_{j-1}, \bar{s}_{j-1}]$.

Plugging in the relation for default-free debt levels derived in Section 3, we obtain the conditions (see Appendix for the derivation):

$$\underbrace{\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)}}_{\text{Gain from marginally higher debt face value}} \leq \mathbb{E}[\min(\underbrace{\rho \cdot \underline{d}_j}_{\text{Default cost}}\ ,\ \underbrace{v_{j-1}-\underline{v}_{j-1}}_{\text{Information rent}}) \mid s_{j-1}]. \tag{20}$$

This condition is closely related to the one we obtained in the case of binomially distributed asset values (see condition (9)). In particular, the right-hand sides of the two conditions (9) and (20) are identical. Moreover, the left-hand side of condition (20) is the continuous analogue of the corresponding terms in condition (9). While a lender considered a discrete deviation from $\underline{d}_j$ to $\overline{d}_j$ in the binomial setting, he now considers a marginal increase from $\underline{d}_j$. As a result, the initial face value $\overline{d}_j$ does not explicitly enter the condition for a default-free equilibrium in the case with continuously distributed asset values.

Agent 1. As explained previously, agent 1's decision to renegotiate with agent 2 is special since agent 1 does not owe debt to another agent. With continuously distributed asset values, the condition for the optimality of agent 1's efficient renegotiation simplifies to:

$$\frac{1-F_2(\underline{v}_2)}{f_2(\underline{v}_2)} \leq \rho \cdot \underline{d}_2. \tag{21}$$

4 Policies Supporting Efficient Renegotiation

In this section, we analyze and compare different types of government interventions and their effect on renegotiation outcomes in a debt chain. Specifically, we consider policies that incentivize early renegotiation, provide subsidies to struggling businesses, and mandate debt reductions. Our analysis highlights the endogenous responses of all debt-chain members to policies that only target a subset of agents. Understanding how such policies affect renegotiation behavior throughout a chain can facilitate the design of government interventions, even though it might also be affected by forces and constraints that are outside the scope of our analysis (e.g., budgetary, social, or political costs of injecting funds or forgiving debt for a certain number of firms).

4.1 Early Renegotiation

In Section 3.3, we discussed the insight that a lender's information at the time of the renegotiation affects his incentives to efficiently renegotiate his borrower's liabilities. If at the renegotiation stage an agent expects his asset value to be low, he knows that a default-free renegotiation strategy will generate little equity value for him. Thus, he is better off gambling for a positive profit by keeping his borrower's debt at a higher level. An immediate implication is that the timing of the renegotiation process is an important determinant of inefficiencies. In practice, agents are likely to know less about how an economic shock will affect their financial conditions if the renegotiation takes place right after the shock hits. As a result, a government policy that promotes early renegotiation after a large economic shock could be socially desirable, as it would lead to more efficient renegotiation outcomes.

Numerical example. We now revisit the numerical example from Section 3 and analyze how reducing the quality of agent 2's private signal, perhaps by initiating the renegotiation process earlier, would affect his incentives to renegotiate with his borrower. For now, assume that agent 2 expects his existing debt to agent 1 to be renegotiated to its efficient, default-free level $\underline{d}_2 = \underline{v}_2 + \underline{v}_3 = \$200K$. Consider a stark situation where the government can force agent 2 to decide whether to renegotiate agent 3's liabilities from $\overline{d}_3 = \$125K$ to $\underline{d}_3 = \$100K$ before agent 2 has collected any private information about the value of his endowment asset v_2. Thus, just like other agents, agent 2 believes that his endowment asset is equally likely to be worth $\bar{v}_2 = \$250K$ and $\underline{v}_2 = \$100K$. (Clearly, the benefits of early renegotiation apply more generally to cases for which the lender's private information is not completely eliminated by accelerating the timing of the renegotiation.) Using these unconditional probabilities, agent 2's efficient-renegotiation condition is satisfied:

$$\begin{aligned} & \frac{1-F_3(\underline{v}_3)}{F_3(\underline{v}_3)}(\bar{d}_3 - \underline{v}_3) = \left(\frac{1-0.5}{0.5}\right)(\$125K - \$100K) = \$25K \\ \leq \; & \mathbb{E}\left[\min\left(\rho \underline{v}_3 \; , \; v_2 - \underline{v}_2\right)\right] = 0.5 \cdot 0.6 \cdot \$100K + 0.5 \cdot \$0 = \$30K. \end{aligned} \tag{22}$$

Unlike in the original example from Section 3, agent 2 is now willing to renegotiate agent 3's debt to its default-free level $\underline{d}_3 = \$100K$.

Note, however, that early renegotiation does not change agent 1's willingness to renegotiate down agent 2's debt as agent 1's private information does not enter his efficient renegotiation condition. Thus, other interventions like those we analyze below are needed to incentivize agent 1 to provide concessions. Yet, the example illustrates how a government can reduce the inefficiencies associated with defaults by designing policies aimed at accelerating when renegotiation among debt-chain members occurs.

4.2 Subsidies

We now show how providing a subsidy to the business of a struggling borrower does not solely improve the recipient's ability to make his payments, but also further incentivizes upstream lenders to renegotiate the debt that is owed to them to default-free levels. Since agent j's decision is affected by the accumulated value of the assets owned by downstream borrowers, a subsidy to a borrower relaxes the efficient-renegotiation conditions for all upstream lenders. Thus, ceteris paribus, a given amount of subsidy to downstream borrowers can have a large effect through the recursivity of renegotiations.

Formally, suppose the government gives a subsidy g_k to agent k. Since this is equivalent to replacing v_k by $v_k + g_k$ in the analysis of Section 3, we can rewrite the efficient renegotiation condition for agent $j-1<k$ in the binomial case as:

$$\frac{1-F_j(\underline{v}_j)}{F_j(\underline{v}_j)}\left[\overline{d}_j-\left(g_k+\sum_{i=j}^{N}\underline{v}_i\right)\right]\leq\mathbb{E}\left[\min\left(\rho\cdot\left(g_k+\sum_{i=j}^{N}\underline{v}_i\right)\ ,\ v_{j-1}-\underline{v}_{j-1}\right)\ \middle|\ s_{j-1}\right], \tag{23}$$

and in the continuous case as:

$$\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)}\ \leq\ \mathbb{E}\left[\min\left(\rho\cdot\left(g_k+\sum_{i=j}^{N}\underline{v}_i\right)\ ,\ v_{j-1}-\underline{v}_{j-1}\right)\ \middle|\ s_{j-1}\right]. \tag{24}$$

Clearly, a subsidy $g_k>0$ makes these conditions for efficient renegotiation less restrictive. Yet, it is worth noting that this type of subsidy does not alter the conditions for efficient renegotiation offers applying to agents $j\geq k$. While the notion that providing subsidies to a struggling business can prevent default by upstream lenders would also apply in models without strategic renegotiation, our model highlights that subsidies can also relax the efficient-renegotiation conditions of upstream lenders, without affecting downstream borrowers. The optimal renegotiation channel featured in our model shows that government subsidies can reduce default even when the amount injected is not large enough to make up for a borrower's shortfall $d_j-(v_j+d_{j+1})$; the lender might optimally respond to the associated change in the distribution of this shortfall by efficiently renegotiating agent j's liabilities to a default-free level. Moreover, our analysis shows that providing a subsidy to a downstream borrower can have a larger effect, compared to providing it to an upstream borrower, due to the recursivity of optimal renegotiation decisions in a debt chain.

Numerical example. Consider a government intervention where the government gives a subsidy $g_3=\$20K$ to agent 3 to help him meet his financial obligations. Note that when agent 3's asset value is low (i.e., when $v_3=\$100K$) the subsidy $g_3=\$20K$ is insufficient to allow him to make his debt payment $\overline{d_3}=\$125K$. However, in that case, agent 2's efficient-renegotiation condition after observing the bad signal s_2 is satisfied:

$$\begin{aligned}&\frac{1-F_3(\underline{v}_3)}{F_3(\underline{v}_3)}(\overline{d}_3-(\underline{v}_3+g_3))=\left(\frac{1-0.5}{0.5}\right)(\$125K-(\$100K+\$20K))=\$5K\\ \leq\ &\mathbb{E}[\min(\rho(\underline{v}_3+g_3)\ ,\ v_2-\underline{v}_2)\mid s_2]=0.25\cdot0.6\cdot\$120K+0.75\cdot\$0=\$18K.\end{aligned} \tag{25}$$

Agent 2 is then willing to renegotiate agent 3's debt to a new default-free level of $\underline{d}_3 = \underline{v}_3 + g_3 = \$120K$. Moreover, agent 1 is also willing to renegotiate agent 2's debt since his efficient-renegotiation condition is satisfied:

$$\frac{1-F_2(\underline{v}_2)}{F_2(\underline{v}_2)}\left(\overline{d}_2 - (\underline{v}_2 + \underline{v}_3 + g_3)\right) = \left(\frac{1-0.5}{0.5}\right)(\$325K - \$220K) = \$105K$$
$$\leq \rho \cdot (\underline{v}_2 + \underline{v}_3 + g_3) = 0.6 \cdot \$220K = \$132K. \tag{26}$$

Overall, while a subsidy of $\$20K$ is not enough to enable agent 3 to fulfill his existing liabilities of $\$125K$ with probability 1, it is enough to incentivize agent 2 to renegotiate agent 3's debt from $\overline{d}_3 = \$125K$ to its new default-free level $\underline{d}_3 = \underline{v}_3 + g_3 = \$120K$. And while the subsidy is far from covering agent 2's shortfall of $\overline{d}_2 - \underline{v}_2 - \underline{v}_3 = \$125K$ when both asset values happen to be $v_j = \$100K$, it is enough to convince agent 1 to renegotiate agent 2's debt from $\overline{d}_2 = \$325K$ to its new default-free level $\underline{d}_2 = \underline{v}_2 + \underline{v}_3 + g_3 = \$220K$. In both renegotiation decisions, the default-free debt level (i.e., the minimal value of the borrower's assets) is increased by the subsidy, which contributes to making efficient debt renegotiation more attractive to the lender. This example thus illustrates that subsidies to downstream agents can incentivize upstream lenders to renegotiate down their borrowers' liabilities to avoid default.

4.3 Mandated debt reductions

We now turn our attention to how government interventions targeting the private bargaining process can be effective in preventing default waves. Eliminating a lender's bargaining power by mandating a debt reduction or backing his borrower's liabilities to prevent a default outcome can also incentivize downstream lenders to renegotiate the liabilities owed to them to default-free levels. As observed above, the fact that agents $j > 1$ owe debt to a lender with bargaining power weakens their incentives to efficiently renegotiate down the debt of their downstream borrowers. As a result, if the government were to forgive or reduce part of the debt an agent owes, it would relax this agent's efficient-renegotiation conditions with downstream borrowers.

Formally, suppose that agent $(j-1)$ currently has a level of debt outstanding d_{j-1} such that a default-free outcome is possible when renegotiating down the debt of agent j:

$$d_{j-1} \leq \underline{v}_{j-1} + \underline{d}_j. \tag{27}$$

When expecting no downstream defaults (i.e., $d_i = \underline{d}_i$ for all $i > j$), agent $(j-1)$ optimally renegotiates down agent j's debt in the binomial case if:

$$\frac{1 - F_j(\underline{v}_j)}{F_j(\underline{v}_j)} \left(\overline{d}_j - \sum_{i=j}^{N} \underline{v}_i \right) \leq \mathbb{E}\left[\min\left(\rho \cdot \sum_{i=j}^{N} \underline{v}_i \ , \ v_{j-1} + \sum_{i=j}^{N} \underline{v}_i - d_{j-1} \right) \middle| \ s_{j-1} \right] \tag{28}$$

and in the continuous case if:

$$\frac{1 - F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq \ \mathbb{E}\left[\min\left(\rho \cdot \sum_{i=j}^{N} \underline{v}_i \ , \ v_{j-1} + \sum_{i=j}^{N} \underline{v}_i - d_{j-1} \right) \middle| \ s_{j-1} \right]. \tag{29}$$

In both cases, the efficient renegotiation condition is made less restrictive by a reduction of agent $(j-1)$'s liabilities d_{j-1}. Thus, forgiving agent $(j-1)$'s debt to agent $(j-2)$ might incentivize agent $(j-1)$ to renegotiate down agent j's debt and avoid default, which then incentivizes agent j to do the same with agent $j+1$'s debt and so on.

The impact of this intervention on upstream lenders greatly depends on what happens to agent $(j-2)$. If the government simply reduces the amount that is transferred from agent $(j-1)$ to agent $(j-2)$, this intervention reduces agent $(j-3)$'s incentives to efficiently renegotiate how much agent $(j-2)$ owes him. By reducing how much agent $(j-2)$ collects from agent $(j-1)$, the government effectively lowers how much agent $(j-2)$ and all upstream agents can pay without defaulting – it thus makes efficient renegotiation behavior less attractive for upstream lenders. A poorly designed intervention can therefore lead to higher default risk in the debt chain. As a result, debt reduction policies that do not involve subsidies for the lenders are more effective when the debt is owed to lenders that are still expected to have their upstream liabilities renegotiated down after the intervention, or lenders that have low levels of liabilities (like agent 1, who has none). If on the other hand, the government lowers agent $(j-1)$'s debt owed to agent $(j-2)$ but also gives agent $(j-2)$ the difference between the renegotiated debt amount without intervention and the new debt amount, then the efficient-renegotiation conditions of upstream lenders is unchanged by the intervention. This intervention relaxes downstream lenders' efficient-renegotiation conditions without affecting upstream lenders'.

Numerical example. Consider a government policy that reduces agent 2's debt to agent 1 from $\overline{d}_2 = \$325K$ to $d_2 = \$175K$. In this case, the lower level of debt entices agent 2 to renegotiate agent 3's debt from $\overline{d_3} = \$125K$ to $\underline{d}_3 = \$100K$, even after observing the bad signal s_2, since:

$$\frac{1-F_3(\underline{v}_3)}{F_3(\underline{v}_3)}(\overline{d}_3-\underline{v}_3) = \left(\frac{1-0.5}{0.5}\right)(\$125K-\$100K) = \$25K$$

$$\leq \mathbb{E}\left[\min\left(\rho\underline{v}_3\ ,\ v_2+\underline{v}_3-d_2\right) \mid s_2\right] = 0.25\cdot 0.6\cdot \$100K+0.75\cdot \$25K = \$33.75K. \tag{30}$$

Thus, by effectively forgiving part of agent 2's debt to agent 1, the government can incentivize agent 2 to renegotiate down agent 3's debt and avoid default. In a debt chain, upstream debt reductions can incentivize downstream lenders to renegotiate their borrowers' liabilities to default-free levels.

Discussion: Debt reductions vs. subsidies. As shown above, a government can help eliminate default waves in a debt chain by providing subsidies to a subset of borrowers or by mandating that their liabilities are reduced. These two policies might look similar at first as they both reduce the gap between a targeted borrower's assets and liabilities. However, our analysis shows that these policies affect renegotiation outcomes differently when the targeted credit relationship is part of a debt chain. First, subsidies relax the efficient-renegotiation conditions of all upstream lenders, whereas mandated debt reductions can relax the efficient-renegotiation conditions of downstream lenders. Which credit relationships in the chain are most likely to exhibit inefficient renegotiation outcomes should inform the choice and design of government interventions. Second, comparing conditions (28-29) with (23-24) shows that mandated debt reductions also differ from subsidies in how they affect renegotiation within a given credit relationship in a chain. A subsidy increases the costs a lender faces when his borrower defaults. In contrast, a mandated debt reduction increases the information rent a lender might lose by defaulting on his own liabilities. Which of these two costs of inefficient renegotiation needs to be amplified to ensure efficient renegotiation should also inform the choice and design of government interventions.

5 Robustness

In this section, we discuss the robustness of our key insights to considering alternative forms of default costs, introducing dependence between asset values, and analyzing regions of the parameter space where default-free equilibria do not obtain.

5.1 Borrower-specific default costs

In our baseline model, we assumed that the only inefficiency associated with default emanates from liquidation costs that reduce the value of the assets a lender can recover from a borrower. The parameter ρ was used to capture these proportional deadweight costs associated with default. Going beyond these costs, it is plausible that borrowers also internalize a subset of the inefficiencies triggered by default. For example, a defaulting borrower might experience a loss of reputation, which can affect his future labor market outcomes and limit his access to capital markets for future projects. In this section, we highlight that introducing these types of default costs does not alter our model's key insights.

Formally, suppose each borrower internalizes an additional fixed cost equal to $\phi > 0$ upon default. In this case, borrower j agrees to pay his debt if $d_j \leq v_j + d_{j+1} + \phi$, that is, the introduction of borrower-specific costs makes defaulting less attractive for the borrower. As a result, his lender can choose a higher debt level without triggering default. Moreover, borrowers' default costs increase the default-free debt level for each credit relationship, which is now given by:

$$\underline{d}_j \equiv \sum_{i=j}^{N} \underline{v}_i + (N+1-j)\phi. \tag{31}$$

The conditions ensuring that the renegotiation offers by agents $(j-1) = 1, \dots, (N-1)$ yield a default-free equilibrium outcome for the whole debt chain then take the following form in the case of continuous distributions:

$$\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq \mathbb{E}\left[\min\left(\rho\cdot\left(\sum_{i=j}^{N}\underline{v}_i+(N-j)\phi\right)+\phi\ ,\ v_{j-1}-\underline{v}_{j-1}\right)\ \middle|\ s_{j-1}\right]. \tag{32}$$

As to be expected, this condition reduces to our previous condition (20) when $\phi = 0$.

These results highlight that borrower-specific default costs increase the default-free debt levels and loosen lenders' efficient renegotiation conditions, yet they do so without qualitatively impacting our key insights.

5.2 Dependence between asset values

In our baseline model, endowment asset values were independently distributed across agents. Thus, agent $(j-1)$ did not use his signal realization s_{j-1} to update the distribution of agent j's asset value, $F_j(v_j)$. In contrast, if agent $(j-1)$'s signal was also informative about agent j's asset value, due to a dependence between asset values, the distribution $F_j(v_j)$ would be replaced by the updated distribution $F_j(v_j|s_{j-1})$. Moreover, if the lower bound of the support of v_j was still $\underline{v}_j$ under this updated distribution, then the default-free debt level $\underline{d}_j$ would stay the same as in the baseline model, and agent $(j-1)$'s efficient renegotiation condition under the binomial distribution (i.e., the analogue of condition (9)) would be:

$$\frac{1-F_j(\underline{v}_j|s_{j-1})}{F_j(\underline{v}_j|s_{j-1})}\left(\overline{d}_j-\sum_{i=j}^{N}\underline{v}_i\right) \leq \mathbb{E}\left[\min\left(\rho\cdot\sum_{i=j}^{N}\underline{v}_i\ ,\ v_{j-1}-\underline{v}_{j-1}\right)\ \middle|\ s_{j-1}, v_j=\underline{v}_j\right]. \tag{33}$$

This result highlights that positively correlated asset values would partially mitigate the effect that bad signals have on a lender's renegotiation tradeoff. Whereas a bad signal s_{j-1} reduces the information rents agent $(j-1)$ expects to earn on the default-free path (as pointed out in our baseline analysis), it also increases the probability that agent j defaults if $d_j > \underline{d}_j$. While introducing dependence between asset values enriches the role of signals in our model, it does not alter the key insights we derived from our baseline analysis.

5.3 Default risk

In our baseline analysis, we solved for the conditions under which lenders' renegotiation decisions lead to default-free equilibrium outcomes in a debt chain. These conditions allowed us to characterize which economic forces and policies support efficient outcomes and which do not. Moreover, focusing on the conditions for default-free equilibria facilitated the tractability of our analysis, which involves N strategic, privately informed agents. Specifically, we did not have to keep track of a plethora of cases that exist when any combination of agents can default in equilibrium. Yet, it is possible that the economic conditions following a large economic shock do not allow to achieve a socially efficient outcome without any default risk. Thus, in the Appendix we analyze a simple version of our model with $N = 3$ agents and binomially distributed asset values to show that our main insights also apply when the conditions for default-free equilibria are violated.

We first consider the case in which agent 2 expects agent 1 not to provide any concessions, giving rise to a positive probability that agent 2 defaults on his liabilities $\overline{d}_2$. We show that, as in the baseline model, agent 2's liabilities can impact his willingness to renegotiate agent 3's debt to a default-free level. In several parametric regions, a policy that mandates a debt reduction by an upstream lender, like agent 1, incentivizes a downstream lender, like agent 2, to renegotiate with his borrower more efficiently.

We then consider the case in which agent 1 expects agent 2 not to provide debt relief to agent 3, giving rise to a positive probability that agent 3 defaults on his liabilities $\overline{d}_3$. Again, in several parametric regions, subsidizing the business of a downstream borrower, like agent 3, makes it more likely that an upstream lender, like agent 1, will prefer to renegotiate his borrower's debt to a default-free level, instead of maintaining the original face value and the elevated default risk associated with it.

6 Conclusion

When an economy is exposed to a large shock such as the ongoing COVID-19 pandemic or the most recent financial crisis, most businesses' ability to fulfill their existing financial obligations is challenged, especially so if these businesses are members of a debt chain. As a result, governments and private parties are evaluating possible solutions to avoid large-scale default waves. To analyze the effectiveness of private and public interventions in this context, we developed a tractable model of strategic renegotiation in a debt chain. Our model illustrates how private renegotiation decisions are generically interrelated: a lender's willingness to provide concessions to his borrower depends on how he expects his own liabilities to be renegotiated. Whereas a tough renegotiation strategy may be privately optimal for the lender, it may create negative externalities to renegotiation efforts elsewhere in the chain. In fact, an unaccommodating renegotiation strategy by one lender in a chain can trigger tough renegotiations and increased default probabilities throughout the whole chain.

Our policy analysis shows how government subsidies to downstream borrowers do not only improve the recipients' ability to make payments, but they also further incentivize upstream lenders to renegotiate debts to default-free levels. Once we account for the recursivity of the optimal renegotiation decision of each agent, we show that awarding subsidies to downstream borrowers can be highly effective in preventing default waves compared to awarding the same subsidies to upstream borrowers. We also show how forgiving a struggling borrower's debt or backing it to prevent default can further incentivize downstream lenders to efficiently renegotiate the debt of their borrowers. Finally, we highlight that facilitating early debt renegotiations after a large shock tends to increase incentives for providing concessions, thus reducing default risk. In sum, our analysis not only sheds light on the implications of different types of government interventions but also reveals which members of a debt chain should be targeted to maximize the effectiveness of a given policy.

Appendix: Derivations Omitted from Main Text

A Optimality Conditions with Continuous Distributions

In this section, we derive the optimality conditions in a more general version of our model in which a borrower internalizes some of the default costs, substantiating our discussion on this issue in Section 5. Specifically, we consider a setting in which a borrower incurs a loss equal to ϕ when defaulting.

Agent (j-1). In a default-free equilibrium, agent j collects from agent $(j+1)$ the face value d_{j+1}. Agent j, in turn, does not default on an offer d_j when:

$$v_j + d_{j+1} - d_j \geq -\phi. \tag{A1}$$

Agent $(j-1)$ chooses a marginal debtor type $v_j^* = d_j^* - d_{j+1} - \phi$ to maximize his expected payoff (note that the new proposed face value is then: $d_j^* = v_j^* + d_{j+1} + \phi$):

$$\begin{aligned} &\Pi_{j-1}(v_j^*) \\ =& \int_{\underline{v}_{j-1}}^{\overline{v}_{j-1}} \int_{\underline{v}_j}^{v_j^*} \max(v_{j-1} + (1-\rho)(v_j + d_{j+1}) - d_{j-1}\ ,\ -\phi) \cdot f_j(v_j) \cdot f_{j-1}(v_{j-1}|s_{j-1}) dv_j dv_{j-1} \\ &+ (1 - F_j(v_j^*)) \cdot \mathbb{E}[\max(v_{j-1} + v_j^* + d_{j+1} + \phi - d_{j-1}\ ,\ -\phi) \mid s_{j-1}], \end{aligned} \tag{A2}$$

reflecting that agent $(j-1)$ gets a signal s_{j-1} on his income realization v_{j-1} and can predict the renegotiation offer d_{j-1} from agent $(j-2)$. To derive first-order conditions, we compute the following derivates of terms in equation (A2):

$$\begin{aligned} &\frac{\partial \int_{\underline{v}_{j-1}}^{\overline{v}_{j-1}} \int_{\underline{v}_j}^{v_j^*} \max(v_{j-1} + (1-\rho)(v_j + d_{j+1}) - d_{j-1}\ ,\ -\phi) f_j(v_j) dv_j f_{j-1}(v_{j-1}|s_{j-1}) dv_{j-1}}{\partial v_j^*} \\ =& f_j(v_j^*) \mathbb{E}[\max(v_{j-1} + (1-\rho)(v_j^* + d_{j+1}) - d_{j-1}\ ,\ -\phi) \mid s_{j-1}], \end{aligned} \tag{A3}$$

and

$$
\begin{aligned}
&\frac{\partial\,\mathbb{E}[\max(v_{j-1}+v_j^*+d_{j+1}+\phi-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]}{\partial v_j^*}\\
=&\,\mathbb{E}[\mathbf{1}_{\{(v_{j-1}+v_j^*+d_{j+1}+\phi-d_{j-1})\geq-\phi\}}\mid s_{j-1}]\\
=&\,\Pr[v_{j-1}\geq d_{j-1}-d_{j+1}-v_j^*-2\phi\mid s_{j-1}].
\end{aligned}
\tag{A4}
$$

Using these results, we can write the marginal net-benefit of increasing v_j^* as follows:

$$
\begin{aligned}
\Pi'_{j-1}(v_j^*)=&f_j(v_j^*)\cdot\mathbb{E}[\max(v_{j-1}+(1-\rho)(v_j^*+d_{j+1})-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]\\
&-f_j(v_j^*)\cdot\mathbb{E}[\max(v_{j-1}+v_j^*+d_{j+1}+\phi-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]\\
&+(1-F_j(v_j^*))\Pr[v_{j-1}\geq d_{j-1}-d_{j+1}-v_j^*-2\phi\mid s_{j-1}].
\end{aligned}
\tag{A5}
$$

The necessary condition for an equilibrium in which agent j does not default is:

$$
\Pi'_{j-1}(\underline{v}_j)\leq 0,
\tag{A6}
$$

that is, $v_j^*=\underline{v}_j$ is the optimal choice for agent $(j-1)$. This condition can be rewritten as:

$$
\begin{aligned}
\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)}\leq&\frac{\mathbb{E}[\max(v_{j-1}+\underline{v}_j+d_{j+1}+\phi-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]}{\Pr[v_{j-1}\geq d_{j-1}-d_{j+1}-\underline{v}_j-2\phi|s_{j-1}]}\\
&-\frac{\mathbb{E}[\max(v_{j-1}+(1-\rho)(\underline{v}_j+d_{j+1})-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]}{\Pr[v_{j-1}\geq d_{j-1}-d_{j+1}-\underline{v}_j-2\phi|s_{j-1}]}.
\end{aligned}
\tag{A7}
$$

For a default-free equilibrium to exist, this condition has to hold for all possible signals agent $(j-1)$ might receive, $s_{j-1}\in[\underline{s}_{j-1},\bar{s}_{j-1}]$.

Renegotiated debt values in a default-free equilibrium. The typical renegotiated face values are recursively defined as:

$$
d_j=\underline{d}_j\equiv\underline{v}_j+d_{j+1}+\phi,
\tag{A8}
$$

assuming that we have $\overline{d}_j \geq \underline{d}_j$ for all j. Otherwise, if $\overline{d}_j < \underline{d}_j$, the offer will be just matching the previous offer, that is, $d_j = \overline{d}_j$. If $\overline{d}_j \geq \underline{d}_j$ for all j, we obtain the following explicit formulae:

$$d_N = \underline{v}_N + \phi, \tag{A9}$$

$$d_{N-1} = \underline{v}_{N-1} + d_N + \phi = \underline{v}_{N-1} + \underline{v}_N + 2\phi, \tag{A10}$$

$$d_{N-2} = \underline{v}_{N-2} + d_{N-1} + \phi = \underline{v}_{N-2} + \underline{v}_{N-1} + \underline{v}_N + 3\phi, \tag{A11}$$

$$d_j = \sum_{i=j}^{N} \underline{v}_i + (N - j + 1) \cdot \phi. \tag{A12}$$

Note that the borrower-specific default costs ϕ enter these debt values, whereas the proportional default costs captured by ρ do not. The reason for this is that in a default-free equilibrium, an agent j's borrower, agent $(j+1)$ is collecting the full face value from his borrower, agent $(j+2)$ (remember that default costs do not apply in equilibrium). Yet, the marginal borrower type (and the associated debt value) can be increased by the default cost ϕ in excess of the collateral, since a borrower is willing to pay that extra cost to avoid default.

Suppose that the following default-free face values are charged in equilibrium:

$$\underline{d}_{j-1} = \sum_{i=j-1}^{N} \underline{v}_i + (N - j + 2) \cdot \phi, \tag{A13}$$

$$\underline{d}_{j+1} = \sum_{i=j+1}^{N} \underline{v}_i + (N - j) \cdot \phi, \tag{A14}$$

which requires that the initial face value satisfies: $\overline{d}_j \geq \underline{d}_j$. Note that:

$$\begin{aligned} d_{j+1} - d_{j-1} &= \left[\sum_{i=j+1}^{N} \underline{v}_i + (N - j) \cdot \phi \right] - \left[\sum_{i=j-1}^{N} \underline{v}_i + (N - j + 2) \cdot \phi \right] \\ &= -\underline{v}_{j-1} - \underline{v}_j - 2\phi. \end{aligned} \tag{A15}$$

Using this result, we can now simplify the following terms entering our key efficiency condition (A7):

$$
\begin{aligned}
&\mathbb{E}[\max(v_{j-1}+\underline{v}_j+d_{j+1}+\phi-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]\\
=&\mathbb{E}[\max(v_{j-1}+\underline{v}_j+\phi-\underline{v}_{j-1}-\underline{v}_j-2\phi\ ,\ -\phi)\mid s_{j-1}]\\
=&\mathbb{E}[\max(v_{j-1}-\underline{v}_{j-1}-\phi\ ,\ -\phi)\mid s_{j-1}]\\
=&\mathbb{E}[v_{j-1}\mid s_{j-1}]-\underline{v}_{j-1}-\phi,
\end{aligned} \tag{A16}
$$

$$
\begin{aligned}
&\Pr[v_{j-1}\geq d_{j-1}-d_{j+1}-\underline{v}_j-2\phi|s_{j-1}]\\
=&\Pr[v_{j-1}\geq \sum_{i=j-1}^{N}\underline{v}_i+(N-j)\cdot\phi-\left(\sum_{i=j+1}^{N}\underline{v}_i+(N-j)\cdot\phi\right)-\underline{v}_j|s_{j-1}]\\
=&\Pr[v_{j-1}\geq \sum_{i=j-1}^{N}\underline{v}_i-\sum_{i=j}^{N}\underline{v}_i|s_{j-1}]\\
=&\Pr[v_{j-1}\geq \underline{v}_{j-1}]\\
=&1,
\end{aligned} \tag{A17}
$$

$$
\begin{aligned}
&\mathbb{E}[\max(v_{j-1}+(1-\rho)(\underline{v}_j+d_{j+1})-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]\\
=&\mathbb{E}[\max(v_{j-1}+(1-\rho)\underline{v}_j-\rho d_{j+1}+d_{j+1}-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]\\
=&\mathbb{E}[\max(v_{j-1}+(1-\rho)\underline{v}_j-\rho d_{j+1}-\underline{v}_{j-1}-\underline{v}_j-2\phi\ ,\ -\phi)\mid s_{j-1}]\\
=&\mathbb{E}[\max(v_{j-1}-\underline{v}_{j-1}-\rho\underline{v}_j-\rho d_{j+1}-2\phi\ ,\ -\phi)\mid s_{j-1}],
\end{aligned} \tag{A18}
$$

and

$$
\begin{aligned}
&\mathbb{E}[v_{j-1}|s_{j-1}] - \underline{v}_{j-1} - \phi - \mathbb{E}[\max(v_{j-1} - \underline{v}_{j-1} - \rho\underline{v}_j - \rho d_{j+1} - 2\phi\ ,\ -\phi) \mid s_{j-1}] \\
=&\mathbb{E}[v_{j-1} - \underline{v}_{j-1} - \phi - \max(v_{j-1} - \underline{v}_{j-1} - \rho\underline{v}_j - \rho d_{j+1} - 2\phi\ ,\ -\phi) \mid s_{j-1}] \\
=&\mathbb{E}[\min(v_{j-1} - \underline{v}_{j-1} - \phi - (v_{j-1} - \underline{v}_{j-1} - \rho\underline{v}_j - \rho d_{j+1} - 2\phi), v_{j-1} - \underline{v}_{j-1} - \phi + \phi) \mid s_{j-1}] \\
=&\mathbb{E}[\min(v_{j-1} - v_{j-1} + \rho(\underline{v}_j + d_{j+1}) + \phi, v_{j-1} - \underline{v}_{j-1}) \mid s_{j-1}] \\
=&\mathbb{E}[\min(\rho(\underline{v}_j + d_{j+1}) + \phi, v_{j-1} - \underline{v}_{j-1}) \mid s_{j-1}]. \qquad \text{(A19)}
\end{aligned}
$$

Using these simplifications, we can rewrite condition (A7) as follows:

$$
\frac{1 - F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq \mathbb{E}[\min(\rho \cdot (\underline{v}_j + d_{j+1}) + \phi, v_{j-1} - \underline{v}_{j-1}) \mid s_{j-1}]. \qquad \text{(A20)}
$$

Policy: Mandated debt reductions. We start again with our general condition for a default-free equilibrium:

$$
\begin{aligned}
\frac{1 - F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq &\frac{\mathbb{E}[\max(v_{j-1} + \underline{v}_j + d_{j+1} + \phi - d_{j-1}\ ,\ -\phi) \mid s_{j-1}]}{\Pr[v_{j-1} \geq d_{j-1} - d_{j+1} - \underline{v}_j - 2\phi | s_{j-1}]} \\
&- \frac{\mathbb{E}[\max(v_{j-1} + (1-\rho)(\underline{v}_j + d_{j+1}) - d_{j-1}\ ,\ -\phi) \mid s_{j-1}]}{\Pr[v_{j-1} \geq d_{j-1} - d_{j+1} - \underline{v}_j - 2\phi | s_{j-1}]}. \qquad \text{(A21)}
\end{aligned}
$$

Suppose that we start with a debt level d_{j-1} such that

$$
d_{j-1} - d_{j+1} - \underline{v}_j - 2\phi < \underline{v}_{j-1}, \qquad \text{(A22)}
$$

or equivalently:

$$
\underline{v}_{j-1} + \underline{v}_j + d_{j+1} + \phi - d_{j-1} > -\phi. \qquad \text{(A23)}
$$

Then the initial condition can be written as:

$$\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq \mathbb{E}[(v_{j-1}+\underline{v}_j+d_{j+1}+\phi-d_{j-1}) \mid s_{j-1}] \tag{A24}$$

$$-\mathbb{E}[\max(v_{j-1}+(1-\rho)(\underline{v}_j+d_{j+1})-d_{j-1}\ ,\ -\phi) \mid s_{j-1}], \tag{A25}$$

which further simplifies to:

$$\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq \mathbb{E}[\min(\rho\cdot(\underline{v}_j+d_{j+1})+\phi\ ,\ v_{j-1}+\underline{v}_j+d_{j+1}-d_{j-1}+2\phi) \mid s_{j-1}]. \tag{A26}$$

We can take the derivative of the right hand side with respect to d_{j-1} and get:

$$-\Pr[v_{j-1}+\underline{v}_j+d_{j+1}-d_{j-1}+2\phi < \rho\cdot(\underline{v}_j+d_{j+1})+\phi]$$

$$=-\Pr[v_{j-1} < d_{j-1}-(1-\rho)\cdot(\underline{v}_j+d_{j+1})-\phi]. \tag{A27}$$

Note that we had assumed to begin with that:

$$\underline{v}_{j-1} > d_{j-1}-(\underline{v}_j+d_{j+1})-2\phi. \tag{A28}$$

Thus, as long as:

$$d_{j-1} \in (\underline{v}_{j-1}+(1-\rho)(\underline{v}_j+d_{j+1})+\phi\ ,\ \underline{v}_{j-1}+(\underline{v}_j+d_{j+1})+2\phi), \tag{A29}$$

this probability is strictly positive. That is, a *decrease* in d_{j-1} loosens the condition for agent $(j-1)$ to pick a renegotiated debt level that leads to no default.

B Extension with Default Risk

In our baseline analysis, we solved for lenders' efficient renegotiation conditions in equilibria with no default. Focusing on a default-free path kept our bargaining model tractable despite the involvement of N strategic, privately informed agents. In this Appendix, we analyze the binomial case with

$N = 3$ agents to show that our main insights survive when default is probable along the chain.

We first consider the case in which agent 2 expects agent 1 to keep asking for $\overline{d}_2$ instead of renegotiating the debt to its default-free level of $\underline{d}_2$. Moreover, we assume that $\underline{v}_2 + \underline{v}_3 < \overline{d}_2 \leq \overline{v}_2 + \underline{v}_3$, which ensures that default will occur with positive probability in equilibrium. Then, agent 2 must choose whether to renegotiate agent 3's debt to $\underline{d}_3 = \underline{v}_3$ or to keep asking for $\overline{d}_3$. If agent 2 chooses $\underline{d}_3$, he can expect to collect:

$$\begin{aligned} & F_2(\underline{v}_2) \cdot \max(\underline{v}_2 + \underline{v}_3 - \overline{d}_2\ ,\ 0) + (1 - F_2(\underline{v}_2)) \cdot \max(\overline{v}_2 + \underline{v}_3 - \overline{d}_2\ ,\ 0) \\ = \quad & (1 - F_2(\underline{v}_2))(\overline{v}_2 + \underline{v}_3 - \overline{d}_2). \end{aligned} \tag{B1}$$

If instead agent 2 chooses $\overline{d}_3$, he can expect to collect:

$$\begin{aligned} & F_2(\underline{v}_2) \cdot F_3(\underline{v}_3) \cdot \max(\underline{v}_2 + (1-\rho)\underline{v}_3 - \overline{d}_2\ ,\ 0) \\ + \quad & F_2(\underline{v}_2)(1 - F_3(\underline{v}_3)) \cdot \max(\underline{v}_2 + \overline{d}_3 - \overline{d}_2\ ,\ 0) \\ + \quad & (1 - F_2(\underline{v}_2))F_3(\underline{v}_3) \cdot \max(\overline{v}_2 + (1-\rho)\underline{v}_3 - \overline{d}_2\ ,\ 0) \\ + \quad & (1 - F_2(\underline{v}_2))(1 - F_3(\underline{v}_3)) \cdot \max(\overline{v}_2 + \overline{d}_3 - \overline{d}_2\ ,\ 0), \end{aligned} \tag{B2}$$

which given the inequalities above simplifies to:

$$\begin{aligned} & F_2(\underline{v}_2)(1 - F_3(\underline{v}_3)) \cdot \max(\underline{v}_2 + \overline{d}_3 - \overline{d}_2\ ,\ 0) \\ + \quad & (1 - F_2(\underline{v}_2))F_3(\underline{v}_3) \cdot \max(\overline{v}_2 + (1-\rho)\underline{v}_3 - \overline{d}_2\ ,\ 0) \\ + \quad & (1 - F_2(\underline{v}_2))(1 - F_3(\underline{v}_3)) \cdot (\overline{v}_2 + \overline{d}_3 - \overline{d}_2). \end{aligned} \tag{B3}$$

Agent 2 finds it optimal to renegotiate agent 3's liabilities down whenever:

$$\begin{aligned} (1 - F_2(\underline{v}_2))(\overline{v}_2 + \underline{v}_3 - \overline{d}_2) \quad \geq \quad & F_2(\underline{v}_2)(1 - F_3(\underline{v}_3)) \cdot \max(\underline{v}_2 + \overline{d}_3 - \overline{d}_2\ ,\ 0) \\ + \quad & (1 - F_2(\underline{v}_2))F_3(\underline{v}_3) \cdot \max(\overline{v}_2 + (1-\rho)\underline{v}_3 - \overline{d}_2\ ,\ 0) \\ + \quad & (1 - F_2(\underline{v}_2))(1 - F_3(\underline{v}_3))(\overline{v}_2 + \overline{d}_3 - \overline{d}_2). \end{aligned} \tag{B4}$$

If $\overline{d}_2$ is high enough so that it can only be repaid when agent 3 pays his debt and agent 2's asset is worth $\overline{v}_2$, agent 2 is willing to renegotiate agent 3's liabilities to $\underline{d}_3$ whenever:

$$(1-F_2(\underline{v}_2))(\overline{v}_2+\underline{v}_3-\overline{d}_2) \quad \geq \quad (1-F_2(\underline{v}_2))(1-F_3(\underline{v}_3))(\overline{v}_2+\overline{d}_3-\overline{d}_2), \tag{B5}$$

or equivalently:

$$\overline{d}_3 \leq \underline{v}_3 + \frac{F_3(\underline{v}_3)}{1-F_3(\underline{v}_3)}(\overline{v}_2+\underline{v}_3-\overline{d}_2). \tag{B6}$$

Thus, as in our baseline model, agent 2's liabilities impact his willingness to renegotiate agent 3's debt to an efficient level. Moreover, under the current parametric restrictions, a policy that mandates a debt reduction by upstream lenders can incentivize downstream lenders to renegotiate with their own borrowers more efficiently outside of the default-free region.

If instead $\overline{d}_2$ is sufficiently low that it can be repaid as long as either agent 2's asset is worth $\overline{v}_2$ or agent 3 repays $\overline{d}_3$, agent 2 is willing to renegotiate agent 3's liabilities down whenever:

$$\begin{aligned}(1-F_2(\underline{v}_2))(\overline{v}_2+\underline{v}_3-\overline{d}_2) \quad &\geq \quad F_2(\underline{v}_2)(1-F_3(\underline{v}_3))(\underline{v}_2+\overline{d}_3-\overline{d}_2)\\ &+ \quad (1-F_2(\underline{v}_2))F_3(\underline{v}_3)(\overline{v}_2+(1-\rho)\underline{v}_3-\overline{d}_2)\\ &+ \quad (1-F_2(\underline{v}_2))(1-F_3(\underline{v}_3))(\overline{v}_2+\overline{d}_3-\overline{d}_2).\end{aligned} \tag{B7}$$

Notice that in this case, renegotiating agent 3's debt to $\underline{d}_3$ makes agent 2 default with probability $F_2(\underline{v}_2)$ whereas keeping his debt at $\overline{d}_3$ makes him default with probability $F_2(\underline{v}_2)\cdot F_3(\underline{v}_3)$ and makes agent 3 default with probability $F_3(\underline{v}_3)$. Thus, the expected cost of default with $\underline{d}_3$ is:

$$F_2(\underline{v}_2)\cdot\rho(\underline{v}_2+\underline{v}_3), \tag{B8}$$

whereas with $\overline{d}_3$ it is:

$$F_2(\underline{v}_2)\cdot F_3(\underline{v}_3)(\rho\underline{v}_2+[1-(1-\rho)^2]\underline{v}_3)+(1-F_2(\underline{v}_2))F_3(\underline{v}_3)\rho\underline{v}_3. \tag{B9}$$

In contrast with the default-free case featured in the baseline analysis and with the first case considered above, agent 2 refusing to renegotiate agent 3's debt down can be the socially efficient strategy here. Specifically, it is the case when:

$$F_3(\underline{v}_3)\underline{v}_3 \leq \frac{F_2(\underline{v}_2)}{1+(1-\rho)F_2(\underline{v}_2)}[(1-F_3(\underline{v}_3))\underline{v}_2+\underline{v}_3]. \tag{B10}$$

In this case, the condition for efficient renegotiation by agent 2 becomes:

$$\overline{d}_3 \quad \geq \quad F_2(\underline{v}_2)(\overline{d}_2-\underline{v}_2)+\frac{(1-F_2(\underline{v}_2))[1-(1-\rho)F_3(\underline{v}_3)]}{1-F_3(\underline{v}_3)}\underline{v}_3. \tag{B11}$$

While this case qualitatively differs from the main analysis and from the first case considered above, it still features the insights that (i) agent 2's liabilities impact his willingness to renegotiate agent 3's debt to an efficient level, and (ii) a policy that mandates a debt reduction by upstream lenders can incentize downstream lenders to renegotiate with their own borrowers more efficiently.

We now turn our attention to the case in which agent 1 expects agent 2 to keep asking for $\overline{d}_3$ rather than renegotiating agent 3's liabilities to their default-free level of $\underline{d}_3$. Agent 1 is then expecting agent 3 to default on his debt to agent 2 when $v_3 = \underline{v}_3$. When agent 1 decides whether to renegotiate agent 2's debt to a default-free level, he compares his expected payoff from a default-free debt level:

$$\underline{v}_2+(1-\rho)\underline{v}_3 \tag{B12}$$

to the expected payoff from sticking with the higher level of debt $\overline{d}_2$ on which agent 2 might default.[5]

If $\overline{d}_2$ is sufficiently high so that agent 2 would default on his debt to agent 1 unless his asset is

[5] Note that other renegotiated debt levels must also be considered in order to identify the optimal strategy, but to keep the analysis tractable, we focus our attention on the comparison of the existing debt level $\overline{d}_2$ and the renegotiated default-free level $\underline{v}_2+(1-\rho)\underline{v}_3$.

valued at $\bar{v}_2$ and agent 3 pays $\bar{d}_3$, agent 1's expected payoff from not renegotiating is:

$$\begin{aligned} & (1-F_2(\underline{v}_2))(1-F_3(\underline{v}_3))\bar{d}_2 \\ + \quad & F_2(\underline{v}_2)(1-F_3(\underline{v}_3))(1-\rho)(\underline{v}_2+\bar{d}_3) \\ + \quad & (1-F_2(\underline{v}_2))F_3(\underline{v}_3)(1-\rho)(\bar{v}_2+(1-\rho)\underline{v}_3) \\ + \quad & F_2(\underline{v}_2)\cdot F_3(\underline{v}_3)(1-\rho)(\underline{v}_2+(1-\rho)\underline{v}_3). \end{aligned} \tag{B13}$$

If instead $\bar{d}_2$ is sufficiently low so that agent 2 would only default if his asset is valued at $\underline{v}_2$ and agent 3 defaults on his debt $\underline{d}_3$, then agent 1's expected payoff from not renegotiating becomes:

$$\begin{aligned} & (1-F_2(\underline{v}_2)\cdot F_3(\underline{v}_3))\bar{d}_2 \\ + \quad & F_2(\underline{v}_2)\cdot F_3(\underline{v}_3)(1-\rho)(\underline{v}_2+(1-\rho)\underline{v}_3). \end{aligned} \tag{B14}$$

If the government awards a subsidy of g_3 to agent 3, which is not large enough to prevent agent 3's default, this subsidy would increase the expected payoff from renegotiating the debt to a default-free level by $(1-\rho)g_3$. In comparison, it would increase the expected payoff from not renegotiating by $F_3(\underline{v}_3)(1-\rho)^2 g_3$ in the case considered above where $\bar{d}_2$ is high and by $F_2(\underline{v}_2)F_3(\underline{v}_3)(1-\rho)^2 g_3$ in the case considered above where $\bar{d}_2$ is low. In both cases considered, this subsidy benefits the strategy of renegotiating the debt to a default-free level more than the strategy of keeping the debt at its initial level. Thus, as in the baseline model, subsidies to downstream borrowers can strengthen upstream lenders' relative incentives to efficiently renegotiate their borrowers' liabilities from their initial level to a default-free level, even though default risk exists on the equilibrium path.

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