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The Market for Conflicted Advice*

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The Market for Conflicted Advice[*]

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Abstract

We present a model of the market for advice in which advisers have conflicts of interest and compete for heterogeneous customers through information provision. The competitive equilibrium features information dispersion and partial disclosure. While conflicted fees lead to distorted information, they are irrelevant for customers' welfare: banning conflicted fees only improves the information quality, not customers' welfare. Instead, financial literacy education for the least informed customers can improve all customers' welfare, because of a spillover effect. Furthermore, although distorted information leads to lower returns, investors find it optimal to trade through advisers, which rationalizes empirical findings.

Keywords: Financial advice, Conflict of interests, Financial Literacy, Information Provision

JEL Codes: G2, D1,D8

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1 Introduction

Providing advice is an important function of intermediaries. The compensation structure of intermediaries, however, often leads to conflicts of interest. For example, broker-dealer firms are compensated by commissions and fund distribution fees, and realtors receive fees only if they close a deal. Financial advice has received considerable attention in the empirical literature and from regulators. It is well documented that conflicted advice leads to lower investment returns, which imposes a substantial loss on households.[1] However, fundamentally, how markets for advice function and their consequences for investors remain unknown. Specifically, can competition correct the adviser's misaligned incentives? If customers differ in their sophistication, do the less sophisticated ones still benefit from receiving advice? Or are they "exploited" by advisers?

To answer these questions, we develop a matching model in which the quality of advice, the customers' investment decisions, and their choice of advisers are determined jointly. To capture the advisers' conflict of interest, we assume that advisers (which we understand to be brokers, retail financial advisers, or realtors) are compensated only if they successfully convince their customers to invest. This setup is designed to capture the fact that brokers are compensated by distribution fees, which are set by fund issuers, and do not charge a separate fee for providing advice.[23] To attract customers, advisers compete by providing information.

[1]Financial advice has attracted considerable interest in the empirical literature, which shows that financial advisers drive customers to chase returns (Linnainmaa et al. (2015)), steer them towards high-fee, actively managed investments (Mullainathan et al. (2012)), and recommend unsuitable products when advisers earn high commissions (Anagol et al. (2017)). As a result, portfolios of advised customers underperform (e.g. Bergstresser et al. (2009), Chalmers and Reuter (2010) and Hoechle et al. (2013)). This has drawn the attention of regulators (CEA (2015)), who have found a substantial welfare loss from conflicted advice in the United States. As a result, the Department of Labor has instituted a new rule holding financial advisers to the fiduciary duty standard. Other countries have enacted a variety of measures, such as banning payments from product providers to advisers (Australia, the Netherlands, the UK) or mandating disclosures about conflicts of interest (Canada, Germany).

[2]See, for example, *Understanding Your brokerage and Investment advisery Relationships* by Morgan Stanley (Dec 2014): "In addition to taking your orders, executing your trades and providing custody services, we also provide investor education, investment research, financial tools and professional, personalized information about financial products and services, including recommendations to our brokerage clients about whether to buy, sell or hold securities. We do not charge a separate fee for these services because these services are part of, or "incidental to," our brokerage services. Similar pay structures are common for other types of financial advisers and may come in the form of sales targets, commissions, or kickbacks. See e.g. CEA (2015), Table 3 for a detailed overview.

[3]On the other hand, there is another type of financial advisers who charge flat fees for their service. We

Taking these misaligned incentives as given,[4] the goal of the paper is to understand to what extent competition can discipline advisers and generate valuable information for customers.

To further shed light on how such conflicts of interest affect different agents in the economy, we take into account the heterogeneity of both customers and advisers. Customers differ in the quality of their ex ante information and advisers differ in the value of their expertise. Precisely, each adviser is perfectly informed about one particular type of asset and assets differ in their information sensitivity. Hence, an adviser with expertise in a more information-sensitive asset effectively has more valuable information.

In our setting, customers understand the misaligned incentives of advisers and optimally select their advisers based on the information provided in equilibrium. After choosing an adviser, the customer receives information, rationally updates his beliefs, and makes his investment decision. Since providing advice takes time, each adviser is subject to a capacity constraint. For simplicity, we assume that each adviser can match with at most one customer and vice versa.[5] Thus, the environment can be understood as a matching model with two-sided heterogeneity, where information provision determines the gains for both sides. The key equilibrium objects are the distribution of information quality, the value of information to customers, the profits of advisers, and the matching patterns.

We derive a notion of information quality, which arises endogenously from the adviser's optimal choice of what information to provide. We show that information is dispersed and distorted because of the conflicted fee structure. Intuitively, since brokers are compensated by fee-upon-investment, they have an incentive to provide biased advice to extract higher profits. This explains why we do not obtain truthful disclosure despite competition.[6]

On the other hand, to attract a particular customer, an adviser must provide sufficiently

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compare our results to these fee-only advisers In Section 4.1.

[4] Inderst and Ottaviani (2009) and Inderst and Ottaviani (2012b) provide micro-foundations for the existence for this type of compensation structure. We derive the optimal contract between adviser and fund issuer in Section 5.1, but for simplicity, we take it as fixed in the main model. *All* results we derive in Section 3 carry through under optimal contracting.

[5] Some firms provide advice to a broad audience, for example via investment newsletters. We instead focus on settings in which an adviser provides information to particular clients. In these settings, information is transmitted in face-to-face interactions and information is not sold to a large audience.

[6] A fundamental difference from the standard model with price competition is that the value of information is not perfectly transferable. Hence, technically, our environment falls into the class of matching problems with imperfectly transferable utilities, which explains why the policy within a pair might not maximize pairwise surplus. Legros and Newman (2007) provides a general sorting condition in an environment with imperfectly transferable utilities.

valuable information, so that the customer does not prefer to match with someone else. This competition disciplines advisers and forces them to provide at least partially informative advice. Specifically, the least informed customers are the easiest to deceive, and all advisers are competing for those customers. Thus, the gain from trading through advisers, relative to trading on their own, is higher for the less informed customers. Nevertheless, they still receive worse information in equilibrium and have lower investment returns. Our result thus shows that investment returns per se do not represent the value of advice.

We use our model to establish important results about regulation. Many countries (e.g. Australia, Italy, the UK and the Netherlands, see CEA (2015), p. 25) have begun restricting adviser fees, using a simple and appealing logic. Since fees are driving the conflict of interest, capping or eliminating them should improve the advice provided to customers. This argument, however, fails to take into account how advisers might change their information policy in equilibrium. We show that when regulators lower the advisers' fees, advisers provide worse information. This is because the utility of a customer in a matching equilibrium is uniquely pinned down such that his matched adviser would not profit from attracting his next-best competitor. Since fees and information quality are substitutes from customers' perspective, it is irrelevant that customers are compensated by better information or lower fees. That is, surprisingly, regulating adviser fees does not improve customers' welfare.

So what can regulators do? Since the utility of a customer is pinned down, what matters is the underlying distribution of financial literary (i.e., the distribution of customers' informedness). As a result, financial literacy education has a positive spillover effect. Specifically, when some customers in the economy become better informed, other customers benefit, although the latter's level of informedness does not change. Intuitively, when a customer becomes better informed, other potential customers become more attractive for advisers. These customers become relatively scarce and receive a higher utility.

Finally, our model has important empirical implications. First, customers who trade through advisers have lower investment returns compared to those who trade on their own. However, this is entirely driven by a selection effect. In equilibrium, the most informed customers do not receive advice, but they realize higher returns simply because their own information is already very precise. Importantly, lower returns do not mean that advice destroys value. Only relatively uninformed customers receive advice. If these customers traded on their own, their returns would be even lower. Second, we can compare advisers with conflicted fees to advisers who only charge up-front payments. The difference in these returns is often interpreted as the cost of conflicted payment, as discussed in CEA (2015). Our theory establishes that this interpretation is in fact misleading: while returns are higher through fee-only advisers, customer's utilities are in fact the same under these two structures.

Last but not least, since whether trading through intermediaries is an endogenous choice of customers, our model offers predictions on which type of customers will actually trade through advisers and how this choice affects their returns (i.e., self-directed vs. broker-client trades). We show that, in equilibrium, investment returns could actually be lower for broker-client trades, thereby rationalizing empirical findings. Precisely, in the parameter regime in which customers only invest if they receive a positive signal that maybe be a false negative, customers may forego some investment opportunities (i.e., be less willing to take risks) without further information. Seeking advice then has two effects: first, it helps customers to better identify the investment opportunities, so that customers are more willing to take risks. However, since the adviser may oversell the product due to the conflict of interest, customers may invest even when the payoff is negative.

Consistent with empirical findings, the model thus predicts that broker clients are more willing to take risks but have a lower return than self-directed investors. This result is related to the existing models based on trust. For example, Gennaioli et al. (2015) considers an environment where money managers can decrease the customers' risk aversion (i.e., anxiety). While we do not model trust, one can interpret how much a customer trusts an adviser's recommendation as being endogenously determined by the quality of information. Since this value is endogenous, our framework sheds light on how returns and welfare would change in response to policy or different fee structures.

Related Literature Methodologically, our work is built on the Bayesian persuasion approach (advanced by Kamenica and Gentzkow (2011))[7] and the literature on matching markets.[8] Compared to models that study competition within Bayesian persuasion frameworks, our model has two important distinctions: first, our paper is the first to study information

[7] Kamenica and Gentzkow (2011) analyzes the environment with a single sender with monopoly power and one receiver.

[8] Building on Becker (1973, Shapley and Shubik (1971), most works in this literature analyze matching patterns with transferable utilities, with a few exceptions: Legros and Newman (2007) provides a general sorting condition in an environment with imperfectly transferable utilities. Chiappori and Reny (2006) studies a risk-sharing problem in a matching model.

provision in a model of decentralized competition with heterogeneity. In particular, the question in Gentzkow and Kamenica (2011) and Au and Kawai (2015) concerns how the degree of competition affects information provision in a setting with multiple senders and one receiver.[9] In our framework, the market is frictionless, but advisers are competing for heterogeneous customers and have a capacity constraint. The question we focus on is how the underlying distribution of financial literacy affects the equilibrium value of information and how it affects different consumers with different levels of sophistication. This is of first-order importance for policy, since generally, less sophisticated customers are perceived as being more at risk.

Second, building on matching models, competition in our framework is captured by agents' matching decisions, and within the match, the information provision is exclusive.[10] This modeling makes our framework very tractable. Despite allowing for a general message space, we provide a closed-form characterization of the optimal policy. Specifically, the value of information to a customer is uniquely pinned down so that it is indeed optimal for him to stay within the match, taking into account the value provided by other advisers.[11]

Regarding the literature on financial advice more specifically, a series of papers by Inderst and Ottaviani (Inderst and Ottaviani (2009) and Inderst and Ottaviani (2012b)) study the optimal compensation for a direct marketing agent (i.e., an adviser) and show that a conflict of interest arises endogenously.[12] They consider a setting in which producers compete through commissions paid to an adviser and advisers are assumed to have concerns for suitability with the customer's needs. We, however, take these misaligned interests as

[9]Gentzkow and Kamenica (2011) study a model in which senders whose preferences differ from those of a single receiver can simultaneously disclose information. They show that adding more senders or making their preferences less aligned improves the quality of information in equilibrium. Au and Kawai (2015) study a game with one receiver and multiple senders with possibly different priors and characterize how equilibrium information changes with each sender's prior.

[10]This also distinguishes our paper from the literature on selling information, where information is generally not exclusive. See Admati and Pfleiderer (1988) for a seminal contribution and García and Sangiorgi (2011) and Malenko and Malenko (2016) for recent work.

[11]Board and Lu (2015) considers a matching model with search frictions in which agents meet randomly, and sellers compete through persuasion with homogeneous buyers. The continuation value in their model has a similar spirit to our equilibrium utilities, although matching is frictionless in our model.

[12]Inderst and Ottaviani (2009) characterizes a monopoly problem and Inderst and Ottaviani (2012a) extend the setting to two competing manufacturers. Inderst and Ottaviani (2012b) further extends the monopoly case to two consumer types: naive consumers who always believe advice and rational consumers who understand adviser incentives.

given, and focus on how advisers compete for heterogeneous consumers. Advisers do not care about the customer's well-being, but they generate valuable information purely because of competition. Stoughton et al. (2011) study the use of kickbacks to advisers when customers differ in wealth levels. They find that kickbacks always decrease customer welfare, whereas we find that customer welfare remains the same with and without kickbacks.[13] One key difference from these previous works is that we allow customers to invest in all assets without contacting an adviser (i.e., intermediary), and more importantly, we solve for the quality of the advice in equilibrium. This distinction allows us to provide empirical predictions on customers' returns between different channels: broker-client vs. self-directed channel. The economics here is also very different, since the value provided by an adviser is purely informational (not about choosing the asset itself).[14]

Finally, our paper relates to the recent literature on persuasion and information disclosure in finance. Recent works include Malenko and Malenko (2016), Malenko and Tsoy (2015), Orlov et al. (2017), and Trigilia (2017).

2 Model

Customers There is a mass of heterogeneous customers who differ in their ex ante information quality. They are indexed by an observable type $b \in B \equiv \left[\underline{b}, \overline{b}\right]$, with $0 \leq \underline{b} < \overline{b} < 1$. Let $Q(b)$ denote the measure of customers with types weakly below b and $Q(b)$ admits a differentiable density $q(b)$. All customers have access to a set of assets $L \equiv [\underline{l}, \overline{l}]$, which differ in their downside risk, indexed by l.[15]The payoff of each asset l is determined by the

[13] Additionally, in their model, advisers directly choose how to allocate each customer's money. In equilibrium, advisers choose the same allocation for each customer, which we can interpret as all customers receiving the same advice. In our paper, advisers can only provide information, and customers ultimately decide whether to invest. We also characterize how different customers receive different advice.

[14] This feature also distinguishes us from the credence goods literature, in which sellers simultaneously offer a good and advice about its suitability. See Dulleck and Kerschbamer (2006) for a recent survey. Calcagno and Monticone (2015) study financial advice between a single customer and adviser.

[15] In Section 5.3, we further consider the case in which assets have different returns and/or volatility and show that our results are robust to this alternative specification.

asset-specific random variable $s \in [0,1]$ and is given by

$$y(s,l) = \begin{cases} r & \text{if} \quad s \geq \lambda \\ -l & \text{if} \quad s < \lambda, \end{cases} \tag{1}$$

where s is distributed with strictly positive and continuous pdf $f(s)$. Thus, each asset gives a positive payoff $r > 0$ only if $s \geq \lambda$. Otherwise, it results in a loss $l > 0$. The cost of investment for all assets is normalized to be zero.[16] Thus, if a customer perfectly observes the state s, he invests if and only if $s \geq \lambda$.

Each customer b can potentially invest in all assets and receives a private signal about each asset's quality, $x(s,b) = \mathbb{1}\{s \geq b\lambda\}$. By construction, if an asset is good ($s \geq \lambda$), customers always receive a positive signal ($x = 1$). However, there are false positives if $b\lambda \leq s < \lambda$: when the asset payoff is negative, customers still receive a positive signal. Thus, a customer with higher b is more informed, since his signal has a lower probability of false positives.

Moreover, since making mistakes is more costly for an asset with higher l and information reduces the likelihood of false positives, an asset with higher l is more information sensitive: information reduces the likelihood of false positives, which is more valuable whenever l is higher.

Advisers There is a unit mass of heterogeneous advisers on the other side of the market. Throughout the paper, we assume that the measure of customers is larger than one so that advisers are on the short side of the market.[17] Each adviser has expertise in one particular type of asset l and is perfectly informed about the realized state s of this asset. Since this is the only dimension in which advisers differ, we use l to denote the type of an adviser. The adviser types are observable. The distribution of adviser types is given by measure $G(l)$, which has differentiable density $g(l)$ and domain L.

Since information reduces the likelihood of false positives and the loss is higher for an asset with a higher l, an asset with a higher l is more information sensitive. In other words,

[16] We can think of the price of all assets being normalized to zero. Alternatively, we can understand the payoff in Equation (1) as the NPV from buying the asset. Keeping the price fixed is reasonable, since individual investors are likely to be price takers.

[17] In Appendix B.1, we also consider the case in which customers are on the short side of the market.

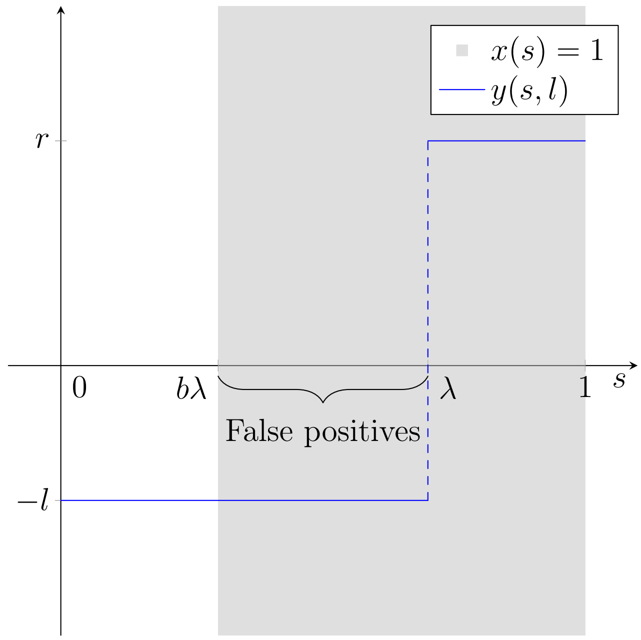

Figure 1: Payoff and Information

an adviser who has expertise in a more information-sensitive asset (i.e. a higher l) effectively has more valuable information.

To capture the conflict of interest between advisers and customers, we assume that an adviser receives a positive payoff $\alpha_c > 0$ from his customer whenever the customer invests the asset. In the context of financial advisers, this payoff represents a commission paid by customers or a kickback that is priced and passed on to customers. Specifically, in the case of mutual funds, customers pay additional distribution fees (e.g., loads and 12b-1 fees) when they buy funds through brokers or advisers, instead of buying a fund directly.[18][19]

Market for Advice We consider a decentralized market for advice in which different advisers compete for different customers through information provision. The two-sided het-

[18] For example, Bergstresser et al. (2009) estimates that broker-channel mutual fund consumers may have paid as much as \$3.6 billion in front-end loads in 2002, \$2.8 billion in back-end loads and another \$8.8 billion in 12b-1 fees, in addition to the investment management fees.

[19] One can easily allow the payoff to have two components: $\alpha_c + \alpha_0$, where $\alpha_c \geq 0$ represents the portion paid by customers, and $\alpha_0 \geq 0$ represents the portion that is not. All our results continue to hold in that case.

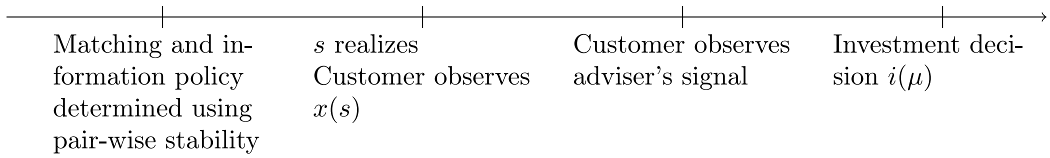

Figure 2: Timeline

erogeneity in our model captures two important dimensions of the market for information: customers (the demand side) differ in their ex ante information and hence their needs for additional information; advisers (the supply side) differ in the value of the information they can provide. Maintaining a relationship between an adviser and a customer requires time or other resources. Thus there is a capacity constraint. For simplicity we assume that each adviser can only serve one customer and that each customer can only obtain advice from one adviser.[20]

Figure 2 summarizes the timeline of our setup. At $t = 0$, the matching takes place. Specifically, the matching decision and information policy within the match are determined simultaneously, subject to pairwise stability. Intuitively, in order to match with a particular customer, the information provided by an adviser must be valuable enough that the customer will not be better off by choosing another adviser.[21] Within a given match, we assume that each adviser can commit to an information policy before the state is realized. Formally, an information policy consists of a signal $\sigma : [0,1] \to \Delta(\mathbb{M})$ that maps the state into a distribution over messages $m \in \mathbb{M}$.[22]

At $t = 1$, after the match is made, the state s realizes, and customers receive information and make their investment decisions. Specifically, each customer first observes his private signal and then the message m sent by the adviser l with whom he matches. He then uses Bayes rule to form a posterior belief about the state and decides whether to invest. If he decides to invest in asset l, he must buy the asset l through the adviser l with whom he

[20]One could instead allow each adviser (customer) to match with a fixed mass of customers (advisers). This would effectively change the underlying distribution and all our results would remain qualitatively unchanged. For example, if an adviser can match with n customers, the model can be solved by setting $\tilde{G}(l) = nG(l)$.

[21]Analogously, matching with a given customer must provide sufficient value to the adviser.

[22]The particular message space $\mathbb{M}$ is irrelevant, as long as it has more than two elements. This is because the customer's action (invest or do not invest) is binary. See Kamenica and Gentzkow (2011), Prop. 1.

matches. Then, the adviser's commission α_c is passed on to him.[23]

For assets about which he does not receive advice, each customer makes his investment decision based on his own private signal. Notice that customers can always make decisions on their own for *all* assets. However, customers are assumed to choose only one asset about which to receive advice. Thus, each customer will choose the adviser that gives him the highest information gain.[24] Our setting is designed so that choosing an adviser is purely about choosing the value of the information provided by the adviser, not about choosing in which asset to invest.[25] Since each customer can invest on his own, regardless of which adviser he chooses, he will choose the adviser who provides him the most valuable information, which is the difference between the expected value under policy σ, and the expected value of self-directed trade. We let $\tilde{U}(b, l, \sigma)$ denote the customer's *gain* from receiving advice throughout the text.

We use the following parameter restrictions throughout the paper.

Assumption. $(A1)\frac{1}{1-F(\underline{b}\lambda)}\left((1-F(\lambda))r-(F(\lambda)-F(b\lambda))\bar{l}\right)>\alpha_c$. *(A2)* $\frac{F(\lambda)-F(\bar{b}\lambda)}{1-F(\lambda)}\underline{l}>\alpha_c$

Assumption (A1) guarantees that the asset is sufficiently valuable that all customers who do not receive any information from an adviser will invest whenever their own signal is high (i.e. $x = 1$).[26] Assumption (A2) guarantees that all advisers can provide positive value to all customers, so that all matches are potentially profitable. Under these two assumptions, if we had a monopolistic adviser, she would choose an information policy that extracts all value from customers and leaves them with their autarky value. This highlights how our results are driven by competition.

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[23]We provide a more explicit construction of the customer's problem in Appendix B.5. As mentioned above, it is irrelevant for our results whether the customer ends up paying the commission (via e.g. fees or loads) or whether the commission is paid by a third party. It only matters that the adviser has an incentive to misrepresent the state s, to persuade the customer to buy.

[24]For tractability, we assume a continuum of assets and the customer optimizes over choosing which asset to receive advice about, even though the impact on his utility is infinitesimal. This captures, in a reduced form, the fact that advice is scarce and customers cannot receive advice about all possible decisions. In Appendix B.5, we consider a variant of the model in which each adviser l knows the state of all assets in an interval centered on l. As that interval shrinks, the equilibrium converges to the one in our main model.

[25]To fix ideas, one can think of an adviser with higher (lower) l as the one specialized in stocks (bonds). This means that if a customer chooses an adviser specialized in stocks, he could still invest bonds based on his own information.

[26]We characterize the case in which Assumption 1 does not hold in Appendix B.4.

Given our assumptions, the expected payoff of a customer under directed trade, denoted by $u^0(b,l)$, yields

$$u^0(b,l) \equiv (1-F(b\lambda))\left(\frac{1-F(\lambda)}{1-F(b\lambda)}r - \left(1-\frac{1-F(\lambda)}{1-F(b\lambda)}\right)l\right) \tag{2}$$
$$= (1-F(\lambda))r - (F(\lambda)-F(b\lambda))l.$$

In Equation (2), with probability $1-F(b\lambda)$, the customer's private signal is high, and Assumption (A1) guarantees that he chooses to invest, his expected payoff of investment is the term in brackets. The first expression is the probability that the asset's payoff is r, conditional on the customer having received a high private signal, times the value r. The second term is the probability that the asset's payoff is $-l$, also conditional on a high private signal times the loss. Equation 2 shows that less informed customers have a lower payoff under self-directed trade than more informed ones, because the former are more likely to make "mistakes."[27] Moreover, such gains are higher for assets that are more information sensitive, as making mistakes for those assets is more costly.

Equilibrium Our equilibrium concept is pairwise stability: if any two agents agree to match, they cannot be better off by matching with others. To formally define the equilibrium, we need to introduce some notation. $H(b,l)$ is the measure of advisers with type below l that match with customers with type below b on the product $\{B\cup\emptyset\}\times\{L\cup\emptyset\}$. $\tilde{V}(b,l,\sigma)$ is the utility of adviser l matching with customer b for a given information policy σ. We define the customer's gain $\tilde{U}(b,l,\sigma)$ analogously.

Definition 1. An equilibrium consists of matching decisions $H(b,l)$, information policies within each pair $\sigma^*(b,l)$, and payoff functions for advisers $V(l)$ and customers $U(b)$ that satisfy the following conditions.

(i) Advisers' optimality: For any $(b,l)\in \operatorname{supp} H$, $V(l)=\tilde{V}(b,l,\sigma^*(b,l))$, and for any (b',σ'), $V(l)\geq \tilde{V}(b',l,\sigma')$ subject to $\tilde{U}(b',l,\sigma')\geq U(b')$.

(ii) Customer's optimality: For any $(b,l)\in \operatorname{supp} H$, $U(b)=\tilde{U}(b,l,\sigma^*(b,l))$, and for any (l',σ'), $U(b)\geq \tilde{U}(b,l',\sigma')$ subject to $\tilde{V}(b,l',\sigma')\geq V(l')$.

[27]See e.g. Lusardi and Mitchell (2014), Section 5, for a discussion of the empirical evidence. Essentially, less informed customers are more likely to commit various mistakes, which in turn lower returns.

(iii) Market Clearing: $\int_L H(b,\tilde{l})d\tilde{l} = Q(b)$ and $\int_B H(\tilde{b},l)d\tilde{b} = G(l)$.

Part (i) means that adviser l prefers her equilibrium match b and equilibrium information policy $\sigma^*(b,l)$ over any other. This is true whenever $V(l)$ exceeds any value the adviser could obtain by matching with a different customer b' under some information policy σ', subject to customer b' also preferring to deviate.[28] The analog holds true for the customer, in part (ii). The last item is a standard market clearing condition. Our setup can be nested in the standard matching model with non-transferable utilities. Existence of equilibria in this class of economies has been established in Kaneko (1982).

2.1 Discussion

We now discuss how our modeling assumptions map to observed patterns in markets for advice, how they relate to existing literature, and how to extend our setting.

Contingent Fees The fee structure we have used is widespread in markets for advice. Financial advisers receive commissions and kickbacks (see e.g. Johannes and Hechinger (2004), Bergstresser et al. (2009), Christoffersen et al. (2013), and Anagol et al. (2017)) whenever they convince a customer to invest. Importantly, this fee is not negotiated between customer and adviser, but is instead set by a fund issuer. This is the case with 12-1b fees, which are split between fund issuer and broker, or with sales commissions.[29] Throughout most of the paper, we treat the fee as fixed from the viewpoints of the adviser and the customer. We can also directly incorporate kickbacks into our model, by defining the adviser's fee as $\alpha = \alpha_0 + \alpha_c$. Here, α_0 is the kickback, which is not directly paid by the customer. All our results carry through in this case.

In Section 5.1, we rationalize the fixed fee structure as the optimal contract between fund issuers and advisers. Under optimal contracting, all our results on matching and information provision remain unchanged.

[28] This is exactly the definition of pairwise stability.

[29] More broadly, realtors are paid a fee only if they facilitate the sale or purchase of a house (e.g. Levitt and Syverson (2008)), lawyers often charge contingent fees conditional on winning a lawsuit, and doctors are paid more if they convince a patient to undergo additional procedures (e.g. Johnson and Rehavi (2016)).

Competition and Commitment We model the market for advice as frictionless matching. While some firms provide information to a broad audience, e.g. via investment newsletters, we focus on settings in which the adviser provides information only to particular customers. Providing advice takes time and the adviser can therefore only contact a limited number of customers.[30] We normalize this number to one for simplicity. If we allowed each adviser to match with a fixed mass of customers, our results would remain qualitatively unchanged. Similarly, customers can only match with a limited number of advisers, which constrains the "amount" of advice they can receive. This is in line with empirical evidence. A survey of equity investors in the US, ICI (2005), shows that two-thirds of customers who receive advice rely on only a single adviser.

Advisers compete by providing information. This type of competition is standard in the literature on credence goods[31] and in other works on advice and certification.[32] We assume that advisers can commit to their information policy before the state is realized, which is standard in the Bayesian persuasion literature. In reality, one can interpret this as advisers using certain predetermined sales systems and/or following certain rules that govern interactions with customers. For example, a review by the Canadian government, Financial Consumer Agency of Canada (2018), finds that retail banks employ automated sales systems.[33] "The bank employee 's computer screen may highlight [...] leads for that customer. These leads tend to be generated by algorithms, prompting the employee to offer a range of products. [...] Staff are generally required to offer products they see in the computer-generated leads." In addition, "employees are provided with conversation cues and scripts, which are intended to ensure that the most important terms, fees, and conditions are disclosed to consumers." Thus, banks commit on disclosing certain aspects of their products. The design of these sales systems and rule sets essentially determines the quality of advice, and they are in place before a given interaction takes place.

[30] In a survey, Chater et al. (2010) find that 80% of purchasers of investment products in Europe bought the product in a face-to-face setting with an adviser.

[31] See Darby and Karni (1973) and Pitchik and Schotter (1987) for early examples and Dulleck and Kerschbamer (2006) for an exhaustive list.

[32] Specifically, advisers or certifiers compete (or attract customers) via the quality of their information in Lerner and Tirole (2006), Inderst and Ottaviani (2009), Inderst and Ottaviani (2012a), Bolton et al. (2012), and many others.

[33] See also http://www.cbc.ca/news/canada/british-columbia/td-tellers-desperate-to-meet-increasing-sales-goals-1.4006743 (accessed April 2018) for a description of such a system.

Finally, we assume that advisers are on the short side of the market. We relax this assumption in Appendix B.1, where we also consider costly entry by advisers. Our main results on matching and advice are unchanged in both extensions.

Assets and Expertise We assume that each adviser knows the payoff of exactly one asset type l. This is meant to capture the fact that different advisers have expertise about different asset classes (or insurance products or types of real estate) with different payoff characteristics. Since advisers do not have expertise in all possible assets, customers must make some decisions on their own. Indeed, many customers who receive financial advice also maintain self-managed investments (see e.g. Hoechle et al. (2015)), which is consistent with our assumption. We consider the case in which advisers know about an interval of assets in Appendix B.5.[34] There, we show that our main results carry through: there is negative assortative matching and less informed customers receive "worse" information, as long as the interval is small.[35]

The structure of asset payoffs captures the fact that different assets have different risks. Since customers are prone to false positives, only the loss l determines the value of advice. Instead of assuming that assets differ in the loss l, we could have them differ in "volatility" v, so that each asset's payoff would be

$$y(s,l) = \begin{cases} v & \text{if} \quad s \geq \lambda \\ -v & \text{if} \quad s < \lambda. \end{cases}$$

In Section 5.3, we further consider the case in which assets have different returns and/or volatility and show that our results are robust to this alternative specification.

Alternatively, we can assume that the adviser's information is a reduced-form representation of the fit between customer and asset (e.g. how the asset matches the customer's risk preference or investment horizon). In this formulation, the state s is asset and customer specific. All our results carry through under this interpretation. Similarly, our results continue to hold if the adviser knows strictly more than the customer. That is, the adviser knows l, and after matching with a customer also learns that customer's private signal.

[34]Precisely, adviser l knows the state of each asset $\hat{l} \in [l - l_0, l + l_0]$ for some fixed $l_0 > 0$.

[35]To be precise, the equilibrium of the model in Appendix B.5 converges to the equilibrium of our main model as $l_0 \to 0$.

Fines and Reputation Effects Advisers may be dissuaded from providing bad advice by penalties imposed by regulators, lawsuits, or a loss of reputation and future business. We can extend our model to take ex post penalties into account in a reduced form. If, after receiving advice, the customer realizes a loss, the adviser must pay an (expected) penalty ρ. As long as $\rho < \alpha$, all qualitative results remain unchanged. In the context of financial advice, Egan et al. (2016) have documented that "advisor misconduct" is pervasive,[36] which suggests that implicit reputational or labor market penalties are not sufficiently severe to dissuade it.

3 Characterization

We now solve the model in two steps. First, we characterize the Pareto optimal policy within any potential match and for any utility level (Section 3.1). We show that it is a threshold policy. Then, we solve for the matching decisions and equilibrium utilities, using pairwise stability (Sections 3.2 and 3.3).

Our equilibrium definition implies that within each equilibrium match, the information policy must be Pareto optimal. Otherwise, a mutually profitable deviation exists without changing the matches.[37] When evaluating deviations, it is also without loss of generality to focus on Pareto optimal information policies.[38] With this in mind, we define $v(b, l, \bar{u})$ as the highest possible utility of adviser l, conditional on providing a customer b with additional information value of $\bar{u}$, which is given by

$$\begin{aligned} v(b, l, \bar{u}) = \max_{\sigma} \tilde{V}(b, l, \sigma) \\ \tilde{U}(b, l, \sigma) \geq \bar{u}. \end{aligned} \tag{3}$$

This function describes the Pareto frontier between customer and adviser within any poten-

[36]In their paper, misconduct is not providing bad advice but defrauding the customer, for example via unauthorized trades.

[37]This follows from Part (i) of Definition 1.

[38]The equilibrium satisfies pairwise stability if and only if there is no pair (b', l') who are not matched in equilibrium and some information policy σ that gives both a weakly higher utility from the match than their equilibrium utilities. If an information policy exists that makes them better off, then a Pareto optimal policy exists. If no policy exists, then there is *a fortiori* no Pareto optimal one.

tial match. We can define $u(b, l, \bar{v})$ analogously, which is the maximum gain of customer b when matched with adviser l, conditional on the adviser receiving at least utility $\bar{v}$.[39]

3.1 Information Policies

We now solve the adviser's problem in Equation (3), keeping the match (b, l) and the customer's utility $\bar{u}$ fixed. This makes Problem (3) a Bayesian persuasion problem. Specifically, we show that within any potential match, the adviser's optimal policy can be characterized by a cutoff $\hat{s} \in [b\lambda, \lambda]$, without loss of generality. The adviser recommends investing if and only if $s \geq \hat{s}$ (Proposition 1). We can interpret $\hat{s}$ as the information quality, since a higher $\hat{s}$ means that customers are less likely to invest in an asset with a negative payoff.

Consider a potential match (b, l). Our information structure implies that whenever a customer receives a bad signal $(x = 0)$, he knows for certain that the asset's payoff is negative.[40] Then, investing is never optimal for any message received from an adviser. We can therefore limit attention to when the customer receives a good signal $(x = 1)$ when describing the optimal information policy. Upon receiving the good signal, the customer learns that $s \geq b\lambda$, and her belief about the state is given by the pdf $\frac{f(s)}{1-F(b\lambda)}$ with domain $[b\lambda, 1]$. We write μ_1 for the corresponding measure.

Reformulation Given any information policy σ and realized message m a customer forms a posterior belief $\mu \in \Delta([0,1])$.[41] Since the message is random, σ induces a distribution P_σ on the space of posteriors beliefs. Under σ, the posterior beliefs must form a martingale, since the customer uses Bayesian updating. Kamenica and Gentzkow (2011), Prop. 1, prove that the converse is also true. Every distribution P on the space of posterior beliefs can be induced by a signal, provided that P satisfies the Bayes-plausibility condition[42]

$$E_P \mu = \mu_1. \tag{4}$$

[39]That is, $u(b, l, \bar{v}) = \max_\sigma \tilde{U}(b, l, \sigma)$ s.t. $\tilde{V}(b, l, \sigma) \geq \bar{v}$. The two definitions are consistent: $v(b, l, \bar{u})$ and $u(b, l, \bar{v})$ are, without loss of generality, supported by the same information policy, as long as $\bar{v} = v(b, l, \bar{u})$ and $\bar{u} = u(b, l, \bar{v})$.

[40]That is, $x = 0$ whenever $s < b\lambda$ which implies that $y(s, l) = -l$ with certainty.

[41]Here, $\Delta([0,1])$ is the space of probability distributions on $[0,1]$.

[42]Equation (4) is a crucial constraint on the sender. Without it, she could choose the receiver's beliefs arbitrarily.

We can therefore represent the adviser's problem 3 as maximizing over Bayes-plausible distributions of posteriors after $x = 1$ is realized and identify any information policy with P.

For a given posterior, $i(\mu) \in \{0, 1\}$ denotes the customer's investment decision after receiving the good signal and the adviser's message.[43] The customer invests whenever his expected payoff from doing so is positive, i.e.

$$i(\mu) = \mathbb{1}\{E_\mu [y(s,l)] - \alpha_c \geq 0\}. \tag{5}$$

The adviser's problem can then be written as[44]

$$\begin{aligned} v(b,l,\bar{u}) \quad &= \quad \max_{P \in \Delta(\Delta([b\lambda,1]))} \alpha_c (1 - F(b\lambda)) E_P [i(\mu)] \\ s.t. \quad & (1 - F(b\lambda)) E_P [i(\mu)(E_\mu [y(s,l)] - \alpha_c)] - u^0(b,l) \geq \bar{u} \\ & E_P \mu = \mu_1. \end{aligned} \tag{6}$$

The payoff to the adviser is the fee α_c times the likelihood of persuading the customer to invest, which is $(1 - F(b\lambda)) \cdot E_P [i(\mu)]$. Here $1 - F(b\lambda)$ is the likelihood that the customer receives the good signal and $E_P [i(\mu)]$ is the likelihood that the customer invests conditional on the good signal $x = 1$ under information policy P.

The customer's gain under information policy P is his expected payoff from investing in asset l minus his outside value of investing in that asset on his own. That is,

$$\tilde{U}(b,l,P) = (1 - F(b\lambda)) E_P [i(\mu)(E_\mu [y(s,l)] - \alpha_c)] - u^0(b,l). \tag{7}$$

Threshold Policy The adviser's optimal policy takes the form of a threshold.[45]

Proposition 1. *Without loss of generality, the optimal policy can be characterized by a threshold $\hat{s} \in [b\lambda, \lambda]$ such that the adviser recommends investing if and only if $s \geq \hat{s}$.*

We prove this result in Appendix A.1. Here are the key steps. The customer's decision is binary: he either invests or does not. The adviser uses a direct recommendation policy

[43] That is, conditional on $x = 1$ and posterior μ.
[44] Here, $\Delta(\Delta([b\lambda, 1]))$ is the space of distributions over posterior beliefs.
[45] The threshold depends on $(b, l, \bar{u})$. That is, $\hat{s} = \hat{s}(b, l, \bar{u})$. We suppress the dependence to save notation.

with two messages, "invest" and "do not invest." In equilibrium, the customer always follows her advice.[46] When the asset pays r (i.e. $s \geq \lambda$), the adviser always recommends investing. Otherwise, the policy would be Pareto dominated.[47] So only the likelihood of investing when the asset pays $-l$ matters for the values of customer and adviser. We prove that any such likelihood can be induced by a threshold policy, which recommends investing whenever $s \geq \hat{s}$.

Proposition 1 does not establish that the threshold policy is uniquely optimal. Instead, any optimal policy can equivalently be expressed as a threshold policy. The threshold represents the quality of information. The higher the threshold $\hat{s}$, the better the information is, since the customer is less likely to have a false positive and invest if the asset's payoff is negative. Because of this, we consider only threshold policies in the following.

The gain of a customer who seeks advice about asset l with information policy $\hat{s}$, relative to self-directed trade, is

$$\tilde{U}(b, l, \hat{s}) \;\; = \;\; (F(\hat{s}) - F(b\lambda))\, l - \alpha_c(1 - F(\hat{s})). \tag{8}$$

The first term is the reduction in false positives relative to investing without advice, and the second is the expected fees paid to the adviser.[48]

The adviser's optimization problem can now be conveniently rewritten as

$$\begin{aligned} v(b, l, \bar{u}) \equiv \max_{\hat{s} \in [b\lambda, \lambda]} \alpha_c(1 - F(\hat{s})) \\ s.t.(F(\hat{s}) - F(b\lambda))l - \alpha_c(1 - F(\hat{s})) \geq \bar{u}. \end{aligned} \tag{9}$$

An adviser is better off with a lower $\hat{s}$, because it implies a higher probability of investment. A customer however is worse off with a lower $\hat{s}$, as it implies a higher probability of making mistakes. The solution is then simply the threshold that provides customer b with utility

[46] This result relies on a similar argument as the revelation principle in mechanism design.

[47] When $s \geq \lambda$, both customer and adviser strictly prefer to invest. If a policy recommends no investment, a simple improvement is possible: always recommend investing when the asset is good, i.e., $s \geq \lambda$, but increase the likelihood of recommending investing when the asset is bad, i.e., $s < \lambda$.

[48] Specifically, under the cutoff rule $\hat{s} \in [b\lambda, \lambda]$, a customer b makes a bad investment and receives a negative payoff $-l$ with probability $F(\lambda) - F(\hat{s})$. If a customer were to have traded based on his own signal, he would have made a mistake with probability $F(\lambda) - F(b\lambda)$. Hence, the added value of cutoff rule $\hat{s}$ to a customer b is given by $(F(\hat{s}) - F(b\lambda))l$, which represents the value of reducing the customer's mistakes.

$\bar{u}$. One can obtain a simple closed form of $v(b, l, \bar{u})$ by substituting in the threshold under which the constraint binds.

3.2 Matching Patterns

We now analyze the matching pattern. First, as we can observe from Problem 3, conditional on providing $\bar{u}$ to a customer, an adviser would prefer to match with a less informed one.[49] Intuitively, for any given information policy, the gain for a less informed customer is higher because he has worse information when he trades in the asset himself. The adviser can hence use a lower threshold and obtain higher expected commissions while still delivering $\bar{u}$ to the customer.[50]

Given that all advisers prefer to match with less informed customers, competition implies that less informed customers receive a higher utility $U(b)$ in equilibrium. Otherwise, every adviser would choose a lower b to match with, which violates market clearing. Thus, the equilibrium gain $U(b)$ must decrease in b.

In equilibrium, an adviser takes customers' utilities $U(b)$ as given and chooses a customer optimally, knowing that to attract a less informed customer, she must provide him a higher gain. The matching outcome is determined by which adviser is willing to provide a higher gain to attract the less informed customers.[51] Specifically, the term $(F(\hat{s}) - F(b\lambda))l$ in Equation (9) implies that the value of information increases with l for any given $\hat{s}$, because mistakes are more costly for an asset with a higher l. Hence, conditional on providing the same level of promised utility to a customer, an adviser with a higher l can provide a lower threshold $\hat{s}$. In this sense, it is less costly for an adviser with a higher l to promise a higher utility to attract a less informed customer. As a result, when two advisers compete to match with a less informed customer, the one with the higher l can outbid his competitor, which leads to the following result.

[49] That is, $v(b, l, \bar{u})$ is decreasing in b.

[50] Precisely, the term $(F(\hat{s}) - F(b\lambda))l$ in the constraint represents the likelihood of mistakes reduced by the information policy $\hat{s}$ relative to his own information on the asset l. Hence, conditional on promising gain $\bar{u}$ to a customer, an adviser is able to give worse information (i.e., a lower threshold $\hat{s}$) if the customer is less informed, which thus implies a higher payoff for an adviser (i.e., $v_b(b, l, \bar{u}) < 0$).

[51] This is analogous to the intuition in the literature on matching with imperfectly transferable utilities, e.g., Legros and Newman (2007).

Lemma 1 (Negative Assortative Matching)**.** *A less informed customer matches with an adviser with more valuable expertise.*

In the proof, we apply Corollary 1 in Legros and Newman (2007) to establish the sorting outcome. Furthermore, we show that $\frac{dv(b,l,U(b))}{dbdl} < 0$. That is, if an adviser with a lower l weakly prefers customer b over customer b' where $b' > b$, then an adviser with a higher l' must strictly prefer the less informed customer b. Given that the adviser's optimality condition can be rewritten as $V(l) = \max_b v(b, l, U(b))$, the sorting outcome can then be alternatively understood as the comparative statics from the optimization problem: the type of customer b that an adviser l chooses must be decreasing in l.

3.3 Equilibrium Advice and Payoffs

Because of Proposition 1, the information policy can be summarized by a simple cutoff rule, which gives us a tractable expression for customers' utilities. Based on this, we now characterize the assignment function $\ell(b)$, the equilibrium payoffs $U(b)$ and $V(l)$, and the threshold used by each adviser $s^*(l)$. Specifically, $\ell : B \to L \cup \{\emptyset\}$ denotes the type of adviser l from whom customer b receives information. Lemma 1 suggests that the assignment function $\ell(b)$ must be weakly decreasing.[52]

Since less informed customers must gain more by participating in the market (i.e., $U(b)$ decreases in b) and the advisers are on the short side of the market, there exists a marginal customer, denoted by b^*, who is indifferent between receiving advice or investing on his own. Only customers with types below b^* receive advice in equilibrium. For agents that are actively matched, the assignment function must then solve the following market clearing condition: $\int_{\underline{b}}^{b} dQ(\tilde{b}) = \int_{\ell(b)}^{\bar{l}} dG(\tilde{l})$. That is, given any b, the measure of customers below b

[52]In Definition 1, the equilibrium consisted of values $U(b)$ and $V(l)$, matching decisions $H(b, l)$, and information policies $\sigma^*(b, l)$. Given the threshold strategy, one can show that $v(b, l, \bar{u})$ and $u(b, l, \bar{v})$ are strictly monotone and continuously differentiable in their arguments. Furthermore, with a continuum of agents, the matching decisions can then be reduced to an assignment function. Thus, $(l, b) \in suppH(b, l)$ if and only if $l = \ell(b)$. We therefore characterize $\ell(b)$ instead of $H(b, l)$. Similarly, we know from Proposition 1 that given $(b, l, \bar{u})$, the optimal information policy is a threshold $\hat{s}(b, l, \bar{u})$. The threshold used in any equilibrium match is therefore $s^*(b, l) = \hat{s}(b, l, U(b))$. The threshold used by adviser l is $s^*(l) = s^*\left(\ell^{-1}(l), l\right)$. To determine the information provided in equilibrium, it is sufficient to characterize $s^*(l)$.

equals the measure of advisers above $\ell(b)$. This yields the differential equation

$$\frac{d\ell(b)}{db} = -\frac{dQ(b)}{dG(\ell(b))} \leq 0, \tag{10}$$

where the boundary condition for the assignment function is $\ell(\underline{b}) = \bar{l}$, because the least informed customer must match with the highest l adviser.

Let $s^*(l)$ denote the threshold policy that an adviser l uses in equilibrium. Observe from Equation (9) that, conditional on the threshold, an adviser does not care about customer type.[53] This means that an adviser l is willing to match with any customer using the threshold policy $s^*(l)$, as such information policy guarantees an adviser receiving her equilibrium utility (i.e., $V(l) = \alpha(1 - F(s^*(l)))$. Taking advisers' utilities (and thus their threshold policies) as given, the customer's optimization problem can then expressed as[54]

$$U(b) = \max_{l} \left(F(s^*(l)) - F(b\lambda)\right) l - \alpha_c(1 - F(s^*(l))). \tag{11}$$

Equation (11) highlights the customer's ability to poach a different adviser. Since an adviser with more valuable expertise is the more attractive type, the adviser's equilibrium profit $V(l)$ must increase in l, and thus, the threshold $s^*(l)$ must decrease in l. That is, an adviser with higher l provides worse information. Thus, Equation (11) shows that customers are effectively trading off between receiving information about a more information sensitive asset vs. receiving information that leads to fewer mistakes. Furthermore, applying the standard envelope theorem to Equation (11) yields

$$U'(b) = -F'(b\lambda)\lambda\ell(b) < 0. \tag{12}$$

That is, the marginal gain of customer b can be understood as her contribution to the information value when matching with her optimal adviser $\ell(b)$. We can solve for customers equilibrium utility $U(b)$, given the assignment function $\ell(b)$ from Equation (10). Moreover, since advisers are on the short side of the market, the marginal customer must gain nothing

[53]This is because in equilibrium, advisers use a threshold $\hat{s} > b\lambda$ whenever $U(b) > 0$. So changing b slightly but keeping $\hat{s}$ the same does not change the adviser's utility.

[54]To attract an adviser l, the threshold policy must be weakly higher than $s^*(l)$. When evaluating deviation from the customer perspective, it is without loss of generality to focus on $s^*(l)$, as it gives the highest possible deviating payoff for customers.

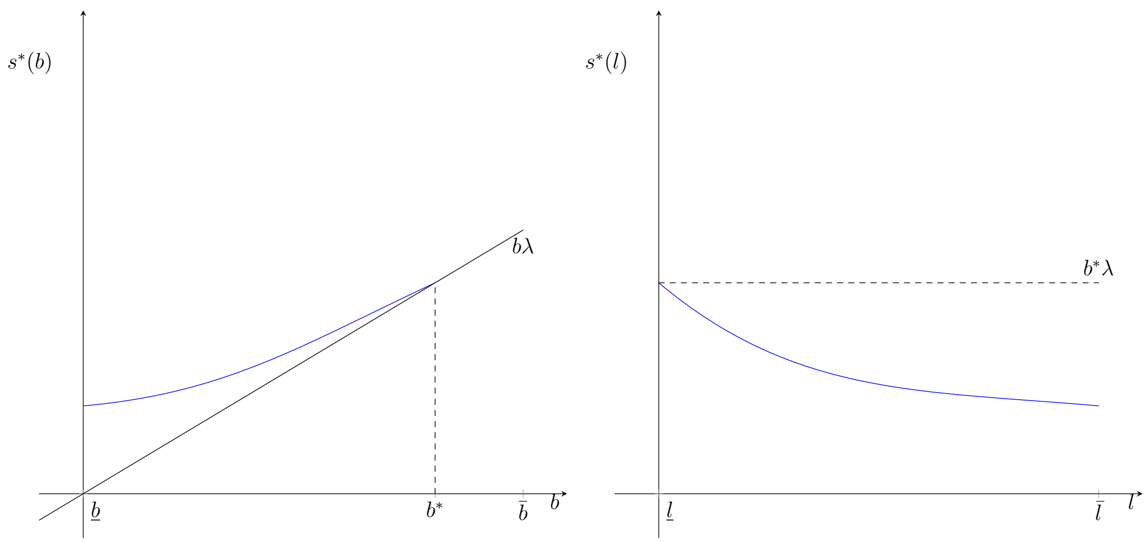

Figure 3: Equilibrium Information Policy with Scarce Advisers ($\alpha_0 > 0$,$\alpha_c = 0$)

relative to investing on his own, which pins down the boundary condition $U(b^*) = 0$. Given the customers' utilities $U(b)$ and assignment function $\ell(b)$, the equilibrium information policy within the match and the adviser's profits are then pinned down accordingly. Specifically, the information policy $s^*(l)$ must solve

$$U(\ell^{-1}(l)) = \left(F(s) - F(\ell^{-1}(l)\lambda)\right) l - \alpha_c(1 - F(s)), \tag{13}$$

where $\ell^{-1}(l)$ denotes the inverse of ℓ.

Proposition 2. *The unique equilibrium is characterized by an assignment function $\ell(b)$, payoff functions $U(b)$ and $V(l)$, and an information policy $s^*(l)$ such that we have the following:*

(1) The marginal customer is given by $Q(b^*) = 1$. For any $b \leq b^*$, $\ell(b)$, and $U(b)$, solve the differential equations (10) and (12) with initial conditions $\ell(\underline{b}) = \bar{l}$ and $U(b^*) = 0$. Customers with $b > b^*$ do not receive advice.

(2) The information policy $s^*(l)$ solves Equation (13), and $V(l) = \alpha_c (1 - F(s^*(l)))$.

(3) $s^*(l)$ is decreasing in l and $U(b)$ is decreasing in b.

In equilibrium, a less informed customer must match with an adviser with a more information-sensitive asset and receive less precise information; nevertheless, he must gain more by participating in the market. An adviser with a more information-sensitive asset, on the other hand, must earn a higher profit. The equilibrium information policy is illustrated in Figure 3.

Information Distortion The policy that maximizes the pairwise surplus is full disclosure. Thus, the information policy is distorted in equilibrium, even though there is competition between advisers. This is because advisers are compensated by a fixed commission,[55] they compete through information provision and the only way that advisers can extract higher profits is to give more biased advice. This is in contrast to standard price competition, where advisers compete through fees. Specifically, since advisers and customers value information differently, this feature leads to imperfectly transferable utilities among agents, and thus, the policy within the pair does not maximize the pairwise surplus.

Dispersed information is driven by the fact that heterogeneous agents compete for the right to match with a more attractive type in the market. As a result, advisers with more valuable expertise must receive a higher payoff and give less precise information. On the other hand, an adviser with a higher l must provide enough value to his equilibrium customer so that the customer will not go to the next-best adviser (i.e., adviser with type $l - \epsilon$). Such competition thus pins down the information quality for each matching pair. If we had homogeneous advisers and they were on the long side of market, then the unique equilibrium would feature full disclosure. The given distribution of l in our model thus captures the idea that expertise is scarce.[56]

Robustness The customer's information structure is modeled as false positives. This setting allows us to summarize the "information quality" with a single threshold. Moreover, the Pareto frontier exhibits a tractable and differentiable form. In Appendix B.3, we consider a more general environment where we do not explicitly specify the customer's information structure. We establish that our main results carry through when (1) a more informed investor receives a higher payoff without advice than a less informed investor and (2) the

[55]As we have explained in Section 2.1, brokers receive conflicted fees that are set by mutual funds (e.g., loads and 12b-1 fees).

[56]Without competition (i.e., in a setting with a single monopolist adviser), Assumption 1 guarantees that the unique equilibrium features no disclosure.

difference in the payoff is higher for more information-sensitive assets. As long as these two conditions hold, less informed customers are matched with advisers with more valuable expertise and receive "worse" advice in equilibrium. In this sense, our results extend to more general frameworks.

In our baseline model with false positives, these two conditions are captured by a less informed investor being more likely to make mistakes (i.e., over-invest) without an adviser and making mistakes is more costly for assets with higher l. If all customers do not invest asset l without advice (i.e., without Assumption A1) and instead receive some default outside option that does not depend on l, then there is no sorting. We characterize this case in Appendix B.4.

The economics here is that less informed customers are more likely to make mistakes without advisers, but the exact source of mistakes is less relevant. In Section 5.2, we consider an environment with false negatives, where customers mistakenly forgo investments without receiving advice. All our results remain intact, because both conditions (1) and (2) are satisfied.

4 Implications for Regulation

4.1 Irrelevance of Fee Structures

To help discipline financial advisers, many countries (e.g. the UK, Australia, and the Netherlands) have restricted the conflicted fees advisers can receive.[57] The intuition is simple: if conflicted fees are distorting the advisers' incentives, then reducing them should improve the quality of advice. This view, however, does not properly take into account how advisers adjust their information policies in equilibrium. We now formally evaluate such a policy via comparative statics on the conflicted fee α_c. Surprisingly, regulating the fee does not improve customer welfare.

In our model, competition disciplines advisers to provide valuable advice: advisers will provide enough value to their customers that the customers do not prefer to match with someone else. Specifically, the utility of customers is described by Equation (12), which

[57]For example, the Netherlands has banned payments from a product issuer to an adviser. See CEA (2015), p. 25 for more examples.

depends only on the assignment function $\ell(b)$, with the boundary condition $U(b^*) = 0$. Given that Lemma 1 holds for any fee structure α_c, the assignment function (i.e., the matching outcome) remains the same. This immediately implies that customers' utility $U(b)$ is the same under any conflicted payment. Restricting the adviser's fees thus has no effect on the customers' utilities. Notice that the above argument holds as long as the assignment function remains the same, so this result is independent of the sorting outcome being positive or negative.

Proposition 3. *For any conflicted fee $\alpha_c \geq 0$ that satisfies Assumption (A1) customers receive the same utility. Specifically, this utility is identical to what they would obtain without conflicted fees.*

Another way to see this is to consider customers' utilities in Equation (11): customers are better off with either a lower fee or better information (i.e., a higher threshold $\hat{s}$). In other words, from the customers' perspective, fees and information are effectively substitutes. Since what matters for competition is customers' utilities, when the fees α_c decrease, an adviser will then respond by providing *worse* information in equilibrium. Hence, any policy that changes the conflicted fee will only affect the information quality, not customers' utilities.

Corollary 1. *Let $s^*(l;\alpha_c)$ denote the equilibrium information under commission α_c. The lower the fee, the worse the information: for any $\alpha_c' < \alpha_c$, $s^*(l;\alpha_c') < s^*(l;\alpha_c)$.*

The second part of Proposition 3 compares our result to the environment where there are no conflicted fees and advisers charge a competitive up-front payment instead. As we explained in the Introduction, this corresponds to a fiduciary duty standard. In both scenarios, customers' utilities are the same. Thus, even regulation that completely eliminates conflicted fees will not improve welfare. With up-front fees only, there is no need for advisers to distort information, and thus, customers will receive full disclosure. However, advisers will extract the customers' values via the up-front fee, so customers are no better off.

Here are the mechanics behind this result. As long as the fees can be adjusted optimally, utility is perfectly transferable and the information policy must maximize pairwise surplus (i.e., full disclosure). Moreover, regardless of whether the fee is contingent on customers' investment decisions, the effective payment will be uniquely pinned down. Thus, without loss of generality, we consider the environment where the flat fees and the information policy are determined jointly. Specifically, suppose that an adviser l charges fees, denoted by $\gamma^*(l)$, in addition to the information policy. Given the fee $\gamma^*(l)$ and the information policy $s^*(l)$, customers make their matching decisions to maximize their expected utility.

Given that the optimal information policy must be full disclosure: $s^*(l) = \lambda \ \forall l$, the pairwise surplus between an adviser l and a customer b then yields $\Omega^{FB}(b,l) = (F(\lambda) - F(b\lambda))\, l$, which represents the added value of full disclosure to a customer b. Since $\Omega^{FB}_{bl}(b,l) < 0$, the equilibrium must feature negative sorting as before. Thus, the assignment function remains the same for both settings. The utility of a customer under the competitive benchmark is thus given by $U^{FB}(b) \equiv \max_l \Omega^{FB}(b,l) - \gamma^*(l)$. By the envelope theorem, we have

$$\frac{dU^{FB}(b)}{db} = -F'(b\lambda)\lambda\ell(b),$$

which is in fact identical to our competitive benchmark in Equation (12), given that the assignment function is the same. This thus establishes that customer's utilities must be the same with or without conflicted fees. The economics is the same as before. Since what matters for competition is customers' utilities, while advisers now provide full disclosure, they will also charge higher fees. Therefore, perhaps surprisingly, completely banning conflicted payment will not improve customers' utilities.

Finally, note that information distortion decreases the pairwise surplus, $(F(s^*(l)) - F(b\lambda))\, l$. This implies that, while information quality does not affect customers' utilities, it must increase the profits of advisers. In other words, advisers bear the cost of information distortions under fixed fees, and their profit is highest under the competitive benchmark (i.e., full disclosure).

Corollary 2. *The information distortion decreases the profit of advisers, but not that of consumers. A higher fee α_c leads to higher profits for all advisers.*

4.2 Spillovers of Financial Literacy Education

We now use our model to evaluate the effects of financial literacy education. Unlike regulating fees, changing the informedness of even a small segment of customers can force advisers to provide better information to all customers. This spillover effect unambiguously improves welfare.

We understand financial literacy education as taking relatively uninformed customers and making them more informed. This changes the distribution of customer types $Q(b)$.[58] Formally, consider two distributions $Q_1(b)$ and $Q_0(b)$: both of them have the same range B, but $Q_1(b)$ first-order stochastically dominates $Q_2(b)$, i.e., $Q_1(b) \leq Q_0(b)$ $\forall b \in B$ and $Q_1(b^*) = Q_0(b^*)$. Intuitively, $Q_1(b)$ has more customers who are relatively informed. We understand Q_1 as the result of a financial literacy intervention starting with Q_0.

When there are more customers who are more informed, the market clearing condition requires that any given customer b must match with an adviser with a (weakly) higher l. That is, $\ell_0(b) \leq \ell_1(b)$, where $\ell_j(b)$ represents the assignment under distribution $Q_j(b)$. Hence, we can show via a comparison theorem on customers' utilities$U(b)$ that the solution to the differential Equation (12), under $Q_1(b)$ must be weakly higher.

Proposition 4. *Consider two distributions with the same range, where $Q_1(b) \leq Q_0(b)$ and $Q_1(b^*) = Q_0(b^*)$. Customers' utilities are weakly higher under $Q_1(b)$ regardless of the fee structures: $U_1(b) \geq U_0(b)$.*

Proposition 4 thus predicts that any given customer $\hat{b}$ is better off when *others* become more informed. Intuitively, advisers compete for less informed customers, since they are the ones who value information most. Thus, for $\hat{b}$, when all other customers $b < \hat{b}$ become more informed, this means that his competitors become less attractive to advisers. Customer $\hat{b}$ must then receive higher utility. This result highlights how competitive forces shape customers' utilities in the market with two-sided heterogeneity. Specifically, in equilibrium, all agents are compensated so that it is indeed optimal for them to match to their counterparty instead of others. Hence, the distribution of other customers and advisers, which represents the potential competitors, is the key determinant of agents' profits.

In an environment with a fixed contingent fee, the only way that advisers can offer a higher payoff to customers is via information policy. Hence, the intuition above implies that advisers must offer better information and earn a lower rent, as formalized below.

Corollary 3. *In an environment with a fixed conflicted fee, consider two distributions with the same range, where $Q_1(b) \leq Q_0(b)$ and $Q_1(b^*) = Q_0(b^*)$. The information quality $s^*(l)$ is weakly higher and the adviser's profit $V(l)$ is weakly lower under $Q_1(b)$.*

[58] For simplicity we assume that this leaves the marginal type b^* who is indifferent between receiving advice or not, unchanged.

One can also consider a similar exercise by changing the shape of the distribution of adviser types $G(l)$. For example, suppose that $G_1(l)$ first-order stochastically dominates $G_0(l)$. That is, there are fewer advisers with valuable information under $G_2(l)$. The same intuition as before applies: an adviser with expertise in a higher l becomes more scarce and as a result, she must earn a higher profit, meaning worse information for customers.

5 Extensions

5.1 Optimal Contracts for Advisers

We have assumed throughout the paper that advisers receive a contingent fee α_c whenever a customer invests after receiving advice. We now rationalize this fee structure as the unique optimal contract in a simple moral hazard model with two tasks. The contracting setup here closely follows Inderst and Ottaviani (2009). We stress that formalizing the contract is not our main contribution. Instead, it is characterizing the equilibrium information provision and the sorting between customers and advisers, which is not done in Inderst and Ottaviani (2009).

As before, we have assets that can be bought freely by all customers. Suppose that some assets are sold directly through advisers[59] and that each such asset has a single issuer and adviser. The issuer acts as the principal and the adviser as an agent. Both are risk neutral. The adviser has limited liability, i.e. she cannot receive negative payments, and she has zero wealth. She also has a reservation utility of zero. Before matching with a customer, the adviser must choose whether to exert a binary effort $e \in \{0, 1\}$ at private cost c. We interpret this effort as expenditures to attract customers and develop expertise. A shirking adviser never matches with a customer and therefore never sells the asset. An adviser who works can match with a customer and she also learns the type of the asset l.[60] Matching then takes place in the same fashion as in our main model. Once the match is made, the adviser's

[59]This is common with mutual funds. Fund issuers will have funds that are exclusively marketed through advisers and others (often with similar risk and return characteristics) that are sold directly to customers. See e.g. Sullivan (2017).

[60]Intuitively, by gaining expertise, the adviser acquires better information about the payoff profile of the asset. Alternatively, we may think of additional "soft" information about the asset that the adviser learns after the contracting stage. For example, market conditions may change and so may the riskiness of the asset. Instead, the adviser's compensation can only be conditioned on whether the customer invests.

message realizes, and then the customer decides whether to invest. Following Inderst and Ottaviani (2009), we assume that only whether the customer invests is contractible.[61] Thus, the contract specifies payments α_c, when the customer invests, and α_n, when he does not, to the adviser. For simplicity, we assume that these payments are priced into the product, so they are effectively borne by the customers.

Finally, there is an exogenous gain from trade, which is received by the issuer whenever the customer invests. When writing the contract, the issuer anticipates the outcome in the matching market. We focus on sufficiently small c, so that the issuer always prefers inducing effort.

Proposition 5. *There is a unique optimal contract. The adviser is paid α_c whenever the customer invests and nothing otherwise. The equilibrium in the market between advisers and customers is the same as in Proposition 2.*

The optimal contract pays the adviser only if the customer invests, because the adviser is risk neutral. Paying her when the customer does not invest does not help provide incentives. Given that the optimal contract is the same as the commission in our main model, we can show that the equilibrium outcome is also the same.

5.2 False Negatives Environment

The environment above assumes that customers may have false positives. We now show that our main result also holds for the case with false negatives. Specifically, less informed customers must match with advisers with more valuable expertise (Proposition 1) and they receive worse information (i.e. a lower $s^* (b)$).

Formally, the signal structure for customers remains the same: $x = \mathbf{1}\{s \geq b\lambda\}$. However, instead of having $b \leq 1$ as in the false positive case, we now assume that $b \in [\underline{b}, \bar{b}]$, where $\underline{b} > 1$ and $\bar{b} \leq \frac{1}{\lambda}$. That is, when $x = 1$, the customer knows that the asset pays r for certain. However, when customers observe a negative signal $x = 0$, the asset can give a positive return (i.e., a false negative). With this assumption, a higher b means the customer is more likely to have a false negative and thus is "less informed". An asset is characterized by the return and the downside risk, denoted by (r, l). Since the information value is now about the

[61] For example, whether an adviser meets a customer or exerts effort to gain expertise may simply be unobservable to the issuer.

upside, we analyze the case in which all assets have the same downside risk l but differ in their upside return r with distribution $G(r)$. In other words, the adviser who knows an asset that pays a higher return r is the type who has more valuable expertise.

Analogous to our Assumption (A1), we assume that customers never invest upon receiving a negative signal ($x = 0$). That is, $r(F(b\lambda) - F(\lambda)) - lF(\lambda) < 0 \forall b$. Under this assumption, the customer's outside option is $u^0(b, r) = (1 - F(b\lambda))\, r$. That is, he only invests if he receives the signal $x = 1$ and then receives r for sure. Intuitively, this captures the idea that, without any further information, customers will forgo some investment opportunities.

As before, we can show that the information policy of the adviser can again be described with a threshold $\hat{s} \leq \lambda$. Equation (8), which represents the gain of a customer who seeks advice about asset r with information policy $\hat{s}$, relative to self-directed trade, thus yields

$$U(b, r, \hat{s}) - u^0(b, r) = r(F(b\lambda) - F(\lambda)) - l(F(\lambda) - F(\hat{s})) - (1 - F(\hat{s}))\alpha_c. \tag{14}$$

Advice now has two roles: on the one hand, the adviser helps a customer identify positive investment opportunities. That is, he provides value by ensuring that the agent invests whenever the asset pays r. Specifically, without advice, customers would have forgone investment opportunities, which is captured by the first term $r(F(b\lambda) - F(\lambda))$. On the other hand, to extract value from the customer, the adviser also sometimes oversells, which is represented by the second term, $l(F(\lambda) - F(\hat{s}))$. The last term, as before simply captures the additional cost of the commission compared to the direct-trading channel.

By the same logic as before, advisers with more valuable expertise (i.e., a higher r in this case) must earn a higher payoff and thus must give worse information in equilibrium. That is, $s^*(r)$ must decrease in r. From the customer perspective, choosing an adviser is then effectively trading off between the upside return r and information quality $s^*(r)$, where

$$U^*(b) = \max_r \left\{ U(b, r, s^*(r)) - u^0(b, r) \right\}. \tag{15}$$

Observe from Equation (14) that there is complementarity between the investors' information needs (i.e., a higher b in the case of false negative) and the value of expertise r. That is, for any $r' > r$, if customer b prefers an adviser with more valuable expertise r', then a less informed customer $b' > b$ must also prefer adviser r'.[62] This thus shows that Proposition 1 remains intact. As before, less informed customers (a higher b in this case) must match with advisers with more valuable expertise. The equilibrium can be characterized similarly as before.

5.3 Assets with Multidimensional Heterogeneity

For both information environments, we have thus far assumed that assets only differ in one dimension, that is, downside risk (l) for the false positive case and upside return (r) for the false negative case. We now show that our result can easily be extended for more general asset payoffs.

For both cases, the dimension of heterogeneity is designed to capture the value of information. In the case of false positives, one can see that, according to Equation (8), only the downside risk l is relevant for customers' value, while the upside return r is not. This is because choosing an adviser is not about choosing which asset to invest in[63] but purely about maximizing the information gain, which is only a matter of reducing the downside risk in this case. Thus, as long as the underlying returns satisfy Assumption A1, the upside return is payoff irrelevant, and thus all the characterization remains the same.

In the case of false negatives, as shown in Equation (14), one can see that both the returns r and l matter for customers' utilities. However, what matters for the complementarity is again also the upside return r. In other words, the sorting result will not be affected even when we allow for heterogeneous downside risk. Because of this, customers' utilities $U^*(b)$ remain the same.

However, since a higher l suggests that making mistakes is more costly, the threshold strategy will then be affected by the value of l. Specifically, let (r, l_r) denote the payoff of the asset r.[64] Given $U^*(b)$, the information policy for each customer then solves:

$$U^*(b) = r\left(F\left(b\lambda\right) - F\left(\lambda\right)\right) - l_r\left(F\left(\lambda\right) - F\left(s^*(b)\right)\right) - (1 - F(s^*(b)))\alpha_c.$$

[62]Formally, for any $b' > b$ and $r' > r$, if $U\left(b, r, s^*(r')\right) - u^0(b, r') \geq \left[U\left(b, r, s^*(r)\right) - u^0(b, r)\right]$, then $U\left(b', r, s^*(r')\right) - u^0(b', r') > U\left(b', r, s^*(r)\right) - u^0(b', r)$.

[63]Recall that customers can always invest in the asset themselves.

[64]Again, we allow for any arbitrary pair (r, l_r) such that customers never invest upon receiving a negative signal: $r\left(F\left(b\lambda\right) - F\left(\lambda\right)\right) - l_r F\left(\lambda\right) < 0\ \forall b$.

That is, conditional on giving $U^*(b)$ to customers b, $s^*(b)$ is higher (lower) when the asset r has a higher (lower) downside risk l_r. Intuitively, if the asset has a higher (lower) downside risk l_r, overselling is more (less) costly, and thus the adviser r must give better (worse) information to compensate the customer.

5.4 Customers with Heterogeneous Wealth

Thus far, we have focused on the case in which customers differ in their informedness. Our framework nevertheless can be applied to different notions of heterogeneity. For example, another important dimension of heterogeneity in the financial market is the wealth of customers. The empirical literature has documented that wealthier clients are more likely to receive advice,[65] and it seems reasonable to believe that wealthier clients also have access to more knowledgeable advisers.[66] We now derive this result when customers have heterogeneous wealth.

Assume that, for any given asset l, customers differ in the amount of capital that they can invest, which we denote by w, but they have the same level of information. The Pareto frontier within each match is then given by

$$\begin{aligned} v(w, l, \bar{u}) &= \max_{\hat{s}\in[b\lambda,\lambda]} \alpha_c (1 - F(\hat{s}))\, w \\ s.t. &\quad w\left\{(F(\hat{s}) - F(b\lambda))\, l - \alpha_c(1 - F(\hat{s}))\right\} \geq \bar{u} \end{aligned} \tag{16}$$

Clearly, all advisers would like to attract customers with higher wealth (i.e., $v_w > 0$). Intuitively, a customer with higher wealth has a higher demand for information. Thus, fixing any information quality, wealthy customers benefit most as they have more skin in the game. This thus suggests that they must gain more in equilibrium: $U^*(w)$ must increase in w.

Moreover, observe that there is complementary between information demand (w) and the value of expertise (l). Hence, the adviser with the most valuable expertise attracts the wealthiest customer. Given the sorting, the allocation $\ell(w)$ and customers' utilities $U^*(w)$ can be pinned down as before. The information received by customer w in equilibrium is

[65] See e.g. Hackethal et al. (2012) and Hoechle et al. (2017).

[66] Many banks have wealth management services that clients can only access if they have a sufficiently large portfolio.

then such that a customer receives value $U^*(w)$ when he matches with $\ell(w)$.[67]

Proposition 6. *When customers differ in wealth, a wealthier customer gains more by participating in the market for advice, and he is matched with an adviser with more valuable expertise.*

5.5 Naïve Customers

The key contribution of our model is to provide a competitive benchmark when agents are rational. Our framework, however, can easily capture the case in which customers are naïve. That is, customers are not Bayesian and take an adviser's recommendation at face value. In this case, advisers will not have any incentive to disclose information. That is, an adviser always recommends to buy as long as the customer's signal is positive. As a result, the payoff of adviser l who matches with customer b is then given by $\tilde{v}(b,l) = \alpha_c(1 - F(b\lambda))$. Since naïve customers believe that advisers always tell the truth, by matching with an adviser l, a naïve customer thus believes that he gains $\tilde{u}(b,\lambda) = \{F(\lambda) - F(b\lambda)\}l - \alpha_c(1 - F(\lambda))$.

Given the payoffs $\tilde{v}(b,l)$ and $\tilde{u}(b,\lambda)$, the matching outcome in this case can then be solved as a special case when utilities are non-transferable. Specifically, since all advisers prefer less informed customers and all customers prefer advisers with more valuable expertise, the unique equilibrium features the same sorting outcome as before, where less informed customers are matched with advisers with more valuable expertise. Since advisers have no incentive to provide information in this case, an increase in fees simply increases advisers' payoff and decreases customers' utilities, which is in sharp contrast with the case when customers are rational.

6 Implications for Empirical Work

Empirically, it has been well documented that conflicted advice leads to lower investment returns (e.g., Bergstresser et al. (2009), Chalmers and Reuter (2010) and Hoechle et al. (2013)),[68] which raises the question of why customers seek advice in the first place. The same phenomenon occurs in our model, yet some customers rationally choose to receive advice.

[67] See the detailed derivation in Appendix A.5.

[68] See also the detailed summary in the White House report (CEA (2015)).

This is driven by two factors. First, in the context of our model, observed investment returns do not fully reflect the customer's value from participating in the market for advice, because they do not take the customer's outside option into account. Intuitively, since less informed customers make worse investment decisions on their own, they prefer to participate in the market even when they receive worse advice and realize lower returns. Second, there is a selection effect. Customers who receive no advice have higher investment returns than those who do, but this is because only the most informed customers choose to forgo advice in equilibrium.[69]

Return Comparison to Direct Investment To clearly see the selection effect, compare the returns between broker-client trades vs. self-directed trades (i.e., trading without a broker). In our model, the expected return from investing in asset l after receiving advice is

$$R\left(l\right) = \frac{(1 - F(\lambda))r - \left(F(\lambda) - F(s^*\left(l\right))\right)l}{(1 - F(s^*\left(l\right)))} - \alpha_c. \tag{17}$$

Compare this to the return of a customer who does not receive any advice (i.e., $b \geq b^*$in Proposition 2) and invests in the same asset. For this customer, the return is

$$\hat{R}\left(l\right) = \frac{1 - F\left(\lambda\right)r - \left(F\left(\lambda\right) - F\left(b\lambda\right)\right)l}{1 - F\left(b\lambda\right)}.$$

This return is higher.[70] Thus, in our model broker-client trades have a lower return, which is purely driven by a selection effect, since customers who invest through advisers are the less informed. This, however, does not mean that advice destroys value. Indeed,, less informed customers rationally accept advice that yields lower returns, because it improves on what they could achieve by investing on their own.

In the case of false negatives in Section 5.2, this result is even more stark. There, if a customer invests without receiving any advice, his return is always r. When he receives advice

[69] Chalmers and Reuter (2012) exploit institutional changes in access to financial advice in the Oregon State University retirement plan to control for this selection. Hoechle et al. (2017) are able to observe contacts between advisers and clients in a bank and use this information to classify trades as advised and unadvised for each client. Guiso et al. (2018) build a structural model of financial advice in the Italian mortgage market.

[70] From Proposition 2, we can see that $s^*\left(l\right) \leq b^*\lambda$, which implies that $R\left(l\right) \leq \hat{R}\left(l\right)$.

however, the adviser sometimes recommends that he invest in an asset with a negative return, so conditional on receiving advice, the return is always lower. However, advice still improves value for customers because without advice, they would reject too many investments that have positive returns. This again shows that it is optimal for customers to seek conflicted advice, even though it leads to a lower return.

Return Comparison to Fee-Only Advisers Another common way to measure the cost of conflicted advice is to compare the return across fee-only and commission-based advisers. The difference in these returns is often interpreted as the cost of conflicted payment, as discussed in CEA (2015). In our model, customers of fee-only advisers can have higher investment returns, but their utility from receiving advice is the same.

Specifically, with fee-only advisers, customers always receive full disclosure. Thus, the return after the fee is simply r minus the competitive flat fee $\gamma(l)$, $R_{NC}(l) = r - \gamma(l)$. One can see that the return is in fact higher than that under conflicted payment, $R_{NC}(l) > R(l)$; however, as we have shown in Proposition 3, customer utility will be the same across the two settings. Customers simply pay in different ways.[71]

7 Conclusion

The paper studies a matching model for information in which advisers are subject to conflicts of interest. In contrast to existing models that assume naïve customers, our model analyzes the quality of information with competition between advisers and rational customers with different levels of sophistication. We are thus able to establish new insights and predictions regarding customers' welfare and asset returns. Specifically, we show that it is the underlying distribution of financial literacy that determines the consumers' welfare. When advisers are scarce, the fee structure of advisers is irrelevant for the welfare of consumers. We further show that the existing empirical measures of conflicted advice can be misleading if they fail to take into account the selection effect of the characteristics of funds and customers.

[71] To see this, $(1 - F(\lambda))\, R_{NC}(l) > (1 - F(\lambda))\, r - \gamma(l) = (1 - F(s^*(l)))\, R(l)$, where the equality follows from our irrelevant result. Given that, $1 - F(\lambda) \leq 1 - F(s^*(l))$, we thus have $R_{NC}(l) > R(l)$.

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A Proofs

A.1 Proof of Proposition 1

To prove the Proposition, we must establish a few preliminary results. The following lemma shows that the optimal policy induces at most two posterior beliefs.

Lemma 2. *For any information policy P such that $\tilde{U}(b,l,P) > u^0(b,l)$, there exists an equivalent policy $\hat{P}$, which only places weight*

$$p := P(\mu : i(\mu) = 1)$$

on the posterior $\mu_I = E_P[\mu | i(\mu) = 1]$ and $(1-p)$ on the posterior $\mu_N = E_P[\mu | i(\mu) = 0]$.[72]

Proof. The result follows from the law of iterated expectations. Consider an information policy P. The values of the adviser and customer are

$$\begin{aligned} \tilde{V}(b,l,P) &= \alpha(1 - F(b\lambda)) E_P[i(\mu)] \\ \tilde{U}(b,l,P) &= (1 - F(b\lambda)) E_P\left[\max\left\{r\int_\lambda^1 d\mu(s) - l\int_{b\lambda}^\lambda d\mu(s) - \alpha_c, 0\right\}\right]. \end{aligned}$$

Let $\mathcal{M}_0 = \{\mu : i(\mu) = 0\}$ denote the subset of the space of posterior beliefs where the customer does not invest and let $\mathcal{M}_1 = \{\mu : i(\mu) = 1\}$ denote the subset where he does. Since $U(p) > u^0(b,a)$, both $\mathcal{M}_0$ and $\mathcal{M}_1$ are non-empty.[73] We have

$$\tilde{V}(b,l,P) = \alpha(1 - F(b\lambda)) p(\mathcal{M}_1)$$

and

$$\tilde{U}(b,l,P) = (1 - F(b\lambda)) p(\mathcal{M}_1) E_p\left[r\int_\lambda^1 d\mu(s) - l\int_{b\lambda}^\lambda d\mu(s) - \alpha_c | \mu \in \mathcal{M}_1\right].$$

[72] In equilibrium, all customers who participate in the market will receive a value strictly higher than $u^0(b,l)$. Throughout this section, we therefore focus attention on the case in which $\tilde{U}(b,l,P) > u^0(b,l)$. If $\tilde{U}(b,l,P) = u^0(b,l)$, the customer receives no information. P then simply assigns probability one to the customer's prior.

[73] If $\mathcal{M}_1$ were empty, then $U(p) = 0$, which is below the customer's no-information value $u^0(b,a)$. If $\mathcal{M}_0$ were empty, then the customer would always invest, but then necessarily $U(p) = u^0(b,a)$.

Now define $\mu_N = E_p[\mu|\mu \in \mathcal{M}_0]$, $\mu_I = E_p[\mu|\mu \in \mathcal{M}_1]$, and $p = P(\mathcal{M}_1)$. Consider an information policy that assigns mass p to μ_I and mass $1-p$ to μ_N. We have $i(\mu_h) = 1$ and $i(\mu_l) = 0$, and the ex ante values under this alternative policy must be the same as under P by the law of iterated expectations. $\square$

Based on Lemma 2, it is without loss of generality to place weight on two posteriors μ_I and μ_N, which are the posteriors conditional on investing and not investing. The Bayes plausibility condition then becomes

$$p\mu_I(B) + (1-p)\mu_N(B) = \mu_1(B)$$

for any Lebesgue measurable set $B \subset [b\lambda, 1]$ and any $p \in [0,1]$. This implies that μ_I and μ_N are absolutely continuous with respect to μ_1. That is, if $\mu_1(B) = 0$, the equation can only hold if $\mu_I(B) = \mu_N(B) = 0$. Since μ_1 admits a density $\frac{f(s)}{1-F(b\lambda)}$, it is absolutely continuous with respect to the Lebesgue measure. Then, μ_I and μ_N are also absolutely continuous with respect to the Lebesgue measure. Therefore, they both must admit densities. With slight abuse of notation, we denote by $\mu_I(s)$ and $\mu_N(s)$. Now, p is simply the likelihood that the customer invests, conditional on receiving the signal $x = 1$.

The Bayes plausibility condition now becomes

$$p\mu_I(s) + (1-p)\mu_N(s) = \frac{f(s)}{1-F(b\lambda)} \; \forall s \geq b\lambda \tag{18}$$

and the customer's value in Equation (7) becomes

$$(1-F(b\lambda))\,p\left(r\int_{\lambda}^{1}\mu_I(s)\,ds - l\int_{b\lambda}^{\lambda}\mu_I(s)\,ds - \alpha_c\right). \tag{19}$$

We next show that any optimal policy is without loss of generality a threshold policy.[74] We do this by showing that threshold policies span the Pareto frontier within the match between customer and adviser.[75]

First, any efficient policy must have $\mu_N(s) = 0$ for any $s \geq \lambda$. Otherwise, one can

[74] We define a threshold policy as revealing truthfully to the customer whether s is above some threshold $\hat{s}$.

[75] We call any policy that achieves the Pareto frontier *efficient*.

increase $\mu_I(s)$ somewhere on the set $[\lambda, 1]$ to make the customer strictly better off. This implies that

$$\mu_I(s) = \frac{f(s)}{1 - F(b\lambda)} \frac{1}{p} \tag{20}$$

for all $s \geq \lambda$. Moreover, any efficient p exceeds $\frac{1-F(\lambda)}{1-F(b\lambda)}$, because the customer should always invest when the asset pays r. Thus, a threshold policy can be efficient only if it uses a threshold $\hat{s} \in [b\lambda, \lambda]$. If it does, it achieves the same posterior as in Equation (20) for $s \geq \lambda$.

Second, under an efficient policy, the customer's value is solely determined by p. To see this, combine Equation (19) with Equation (18). This yields

$$r \int_{\lambda}^{1} f(s)\, ds - l(1 - F(b\lambda))\, p \int_{b\lambda}^{\lambda} \mu_I(s)\, ds - (1 - F(b\lambda))\, p\alpha_c.$$

Since $\mu_I(s)$ is a probability density, it must integrate to one, i.e.

$$\int_{b\lambda}^{\lambda} \mu_I(s)\, ds + \int_{\lambda}^{1} \mu_I(s) = 1.$$

Using Equation (20), we obtain

$$\int_{b\lambda}^{\lambda} \mu_I(s)\, ds = 1 - \frac{1 - F(\lambda)}{1 - F(b\lambda)} p.$$

Therefore, the customer's value is

$$(r + l)(1 - F(\lambda)) - (l + \alpha_c)(1 - F(b\lambda))\, p,$$

which depends on the information policy only via p. Similarly, the adviser's value in Equation (6) becomes

$$\alpha_c (1 - F(b\lambda))\, p.$$

Thus, the Pareto frontier is completely characterized by $p \in \left[\frac{1-F(\lambda)}{1-F(b\lambda)}, 1\right]$. It only remains to show that any such p can be achieved by a threshold $\hat{s} \in [b\lambda, \lambda]$. Since the customer invests whenever he learns that $s \geq \hat{s}$, we have

$$p(\hat{s}) \equiv \frac{1 - F(\hat{s})}{1 - F(b\lambda)},$$

which is strictly decreasing and continuous in $\hat{s}$ and satisfies $p(\lambda) = \frac{1-F(\lambda)}{1-F(b\lambda)}$ and $p(b\lambda) = 1$. This establishes the result.

A.2 Proof of Lemma 1

Since the payoff of a adviser (customer) increases (decreases) with the threshold, the solution to the problem (9) is the threshold that gives customer b her utility level $\bar{u}$ (i.e., the constraint always binds). Substituting the threshold into adviser's problem, we then obtain the following expression for $v(b, l, \bar{u})$:

$$v(b, l, \bar{u}) = \alpha_c \left(\left(\frac{(1 - F(b\lambda))l - \bar{u}}{l + \alpha_c} \right) \right). \tag{21}$$

This thus implies that $v_{bl} < 0$ and $v_{ul} > 0$. According to Corollary 1 in Legros and Newman (2007), this implies that less informed customers are matched with advisers with higher l.[76] That is, we have negative assortative matching.

Equivalently, this means that $\frac{dv(b,l,U(b))}{dbdl} = v_{bl}(b, l, U(b)) + v_{ul}(b, l, U(b))U'(b) < 0$, as $U'(b) < 0$. Given that an adviser's choice of customers in equilibrium can be expressed as $\max_b v(b, l, U(b))$,one can then understand the sorting outcome using the standard argument from monotone comparative statics, which suggests that an adviser with a higher l must choose a lower b.

A.3 Proof of Proposition 2

Since customers and advisers' problem are symmetric, the optimality condition of advisers guarantees the optimality condition of customers.[77] We thus establish the optimality for

[76] Note that to apply Corollary 1 in Legros and Newman (2007), set $a = 1 - b$ so that we have $\phi(a, l, \bar{u}) \equiv v(1 - b, l, \bar{u})$ is type increasing. Thus by condition (i) in Corollary 1, a higher l must match with a higher a.

[77] To see this, suppose that there exists a threshold policy and new matches (l', σ') that make customers strictly better off $\tilde{U}(b, l', \sigma') > U(b)$ and $\tilde{V}(b, l', \sigma) \geq V(l)$. Since the utility of customers (advisers) strictly increases (decreases) with the threshold $\hat{s}$. Then, there exists a policy $\hat{s} - \epsilon$ such that $\tilde{U}(b, l', \sigma') \geq U(b)$ and

advisers below. Recall that $v(b,l,\bar{u})$ represents the highest possible payoff to an adviser when matching with customer b and provides him with utility $\bar{u}$. Taking $U(b)$ as given, the adviser's optimality condition can be rewritten as

$$\begin{aligned} V(l) &= \max_{\tilde{b}} v(\tilde{b},l,U(\tilde{b})) \\ &= \max_{\tilde{b}} (\alpha_0+\alpha_c)\left(1-\left(\frac{F(\tilde{b}\lambda)l}{l+\alpha_c}+\frac{U(\tilde{b})+\alpha_c}{l+\alpha_c}\right)\right). \end{aligned}$$

Thus, $U'(b)$ must satisfy the FOC of advisers: $U'(b)=F'(b\lambda)\lambda\ell(b)$. The SOC is also satisfied by construction:

$$\begin{aligned} \frac{d^2 v(b,l,U(b))}{d^2 b}|_{b=\ell^{-1}(l)} &= -\frac{(\alpha_0+\alpha_c)}{(l+\alpha_c)}\left((F''(\ell^{-1}(l)\lambda)\lambda^2 l+U''(\ell^{-1}(l))\right) \\ &= \frac{(\alpha_0+\alpha_c)}{(l+\alpha_c)}F'(\ell^{-1}(l)\lambda)\ell'(b)<0 \end{aligned}$$

where as $U'(b)=-F'(b\lambda)\lambda\ell(b)$ and $U''(b)=-F''(b\lambda)\lambda^2\ell(b)-F'(b\lambda)\lambda\ell'(b)$. This thus proves optimality for advisers. That is, there are no other matches or information policy that would make the adviser better off: there does not exist (b',σ') such that $\tilde{V}(b',l,\sigma')>V(l)$ and $\tilde{U}(b',l,\sigma')\geq U(b')$

We can apply the Picard–Lindelöf theorem to show that the differential equations characterizing the assignment function and customer utility (Equations (10) and (12) respectively) have unique solutions given the initial conditions for $U(b^*)$ and $\ell(b^*)$. As we argued in Section 3.2, in any equilibrium, $\ell(b)$ and $U(b)$ must satisfy these differential equations. Therefore, the equilibrium is unique.

A.4 Optimal Contract: Proof of Proposition 5

Before proving the result, let us be more precise about the equilibrium concept. The issuer designs a contract for the adviser $\phi=\{\alpha_c,\alpha_n,e\}$, which consists of a recommended effort level and payments conditional on convincing the customer to invest and on failing to to so. In doing this, the issuer takes into account the incentive compatibility condition of the adviser,

$\tilde{V}(b,l',\sigma)\geq V(l)$ are also satisfied.

and the equilibrium in the matching market, which is described by an assignment function $\ell(b)$ and customer and adviser utilities $U(b)$ and $V(l)$. The equilibrium thus consists of contract ϕ, the payoff for customers $U(b)$ and advisers $V(l)$, and an assignment function $\ell(b)$ such that (1) given the contract ϕ, $\{U, V, \ell\}$ satisfies pairwise stability in the matching game; (2) given $\{U, V, \ell\}$, the contract ϕ maximizes issuer's profit.

Given customer utility $U(b)$, define $\hat{s}(l, b, \alpha_c)$ as the policy such that $\tilde{U}(b, l, \hat{s}) - u^0(b, l) = U(b)$. We first guess (and verify later) that $\alpha_n^* = 0$, and show that the optimal contract α_c^* is given by the following:

$$\alpha_c = \frac{c}{\int \left(1 - F\left(\hat{s}(l, \ell^{-1}(l), \alpha_c)\right)\right) dG(l)} \tag{22}$$

Recall that the solution to Problem 3 yields

$$v(b, l, U(b)) = \max_b \alpha_c \left(1 - \left(\frac{F(b\lambda)l + U(b) + \alpha_c}{l + \alpha_c}\right)\right).$$

From the FOC of b, $F'(b\lambda)\lambda l = U'(b)$, which is independent of the fee α_c. Hence, taking equilibrium $U(b)$ as given, when an adviser l has a different fee $\tilde{\alpha}_c$, he will still choose the same customer $\ell^{-1}(l)$. Given that customers must receive the same utilities, an increase in the fee means that the quality of information must increase. Thus, $\hat{s}(l, \ell^{-1}(l), \alpha_c)$ increases with α_c.

The the problem of each issuer is then given by

$$\begin{aligned} & \max\nolimits_{\alpha_c, \alpha_n} \delta \int \left(1 - F\left(\hat{s}\left(l, \ell^{-1}(l), \alpha_c\right)\right)\right) dG(l) \\ s.t. \quad & \alpha_c \int \left(1 - F\left(\hat{s}\left(l, \ell^{-1}(l), \alpha_c\right)\right)\right) dG(l) \geq c. \end{aligned}$$

Here, δ is the gain from trade that the issuer receives whenever a customer buys her asset. Since we assumed that c is small, we know that the optimal contact induces effort. The inequality is the IC constraint of the adviser. Exerting effort costs c. The LHS is the adviser's ex ante payoff from matching with a customer. α_n is the adviser's payoff conditional on observing no investment, and α_c is the payoff conditional on the customer investing. Given that $\alpha_n = 0$, advisers' profits increase with α_c, Equation (22) thus means that α_c^* is the smallest fee for which the IC is binding. One can easily see that any $\alpha_c > \alpha_c^*$ lowers the issuer's profit since customers invest with a lower probability. Thus, such a deviation is not profitable. Lastly, one can easily show that increasing α_n is also not profitable because customers must be compensated with better advice, which implies lower profits for the issuer.

A.5 Customers with Heterogeneous Wealth

A.5.1 Proof of Proposition 6

Analogous to the optimization problem (16), one can define the maximum gain for a customer w when matching with an adviser l with utility $\bar{v}$, denoted by $u(w, l, \bar{v})$. Let $\hat{s}(w, l, \bar{v})$ be the policy that gives the adviser l utility $\bar{v}$ when matching with customer w. That is, $\hat{s}(w, l, \bar{v})$ solves: $\alpha_c (1 - F(s)) w = \bar{v}$. The expression of $u(w, l, \bar{v}) = \tilde{U}(w, l, \hat{s}(w, l, \bar{v}))$ thus yields

$$
\begin{aligned}
u(w, l, \bar{v}) &= \tilde{U}(w, l, \hat{s}(w, l, \bar{v})) \\
&= \{F(\hat{s}(w, l, \bar{v})) - F(b\lambda)\} wl - \alpha_c (1 - F(\hat{s}(w, l, \bar{v}))) w \\
&= l(w - \frac{\bar{v}}{\alpha_c}) - F(b\lambda) lw - \bar{v}
\end{aligned}
$$

Since $u_{wv} = 0$ and $u_{wl} = 1 - F(b\lambda) > 0$,according to Corollary 1 in Legros and Newman (2007), customers with higher w will then match with an adviser with more valuable expertise.

A.5.2 Characterization

Given the sorting, the market clearing condition thus becomes $\int_w^{\bar{w}} dQ(\tilde{b}) = \int_{\ell(w)}^{\bar{l}} dG(\tilde{l})$,which pins down the assignment function $\ell(w)$ with the boundary condition $\ell(\bar{w}) = \bar{l}$. Customer's utilities are given by $U^*(w) = \max_l u(w, l, V^*(l))$, and thus

$$
\frac{dU^*(w)}{dw} = \ell(w)(1 - F(b\lambda)). \tag{23}
$$

As before, when advisers are scarce, the marginal customers w^* must earn zero. Thus, $U^*(w)$ is given by Equation (23) with the boundary condition $U^*(w^*) = 0$, which then pins down the policy function within the match. Note that the adviser's payoff from matching with customer w and using threshold $\hat{s}$ is $\alpha_c w (1 - F(\hat{s}))$. Hence, in contrast to our main model, an information policy that guarantees an adviser receiving her equilibrium utility now depends on customer type, which is the policy that solves $V(l) = \alpha_c w\left(1 - F\left(\hat{s}\right)\right).$

B Robustness and Additional Extensions

B.1 When Customers are on the Short Side

If customers are on the short side of the market, all customers actively participate in the market, i.e. $b^* = \bar{b}$ and the marginal adviser $l^* < \bar{l}$ is given by $Q(\bar{b}) = 1 - G(l^*)$. Moreover, customer b^* must receive full disclosure from the marginal adviser l^*, thereby obtaining the highest payoff possible. Otherwise, other inactive advisers could outbid the marginal advisers by providing better information to attract customer b^*. That is, the boundary condition for the marginal customer now becomes $U(b^*) = \bar{U}(\bar{b}) \equiv F(\lambda) - F(\bar{b}\lambda)l^* - \alpha_c(1 - F(\lambda))$. Everything else in Propostion 2 on the other hand, remains the same. Thus, as before,less informed customers are matched with advisers with more valuable expertise and receive worse information. On the other hand, any change in α_c changes the boundary condition for the marginal customer, which means that it shifts $U(b)$ up to a constant. Specifically, an increase in α_c necessarily decreases all customers' utilities by $\alpha_c(1 - F(\lambda))$. Nevertheless, the main intuition that information and fees are substitutes remains the same as Corollary 1 remains intact. Moreover, the additional gain for customer b relative to his next-best competitor $b + \epsilon$, as characterized by Equation (12), also remains the same and is independent of the fee structure.

One can further consider an extension with free entry of advisers to determine whether customers or advisers are on the short side of the market. Specifically, suppose that advisers can enter at a cost c. We can understand this as a "learning cost" that the adviser must incur to gain expertise. After the adviser enters, he draws his type l from the distribution $G(l)$.[78] Free entry then implies that $\mathbb{E}_l V(l) = c$,that is, the ex ante gain from entering is zero. In this setup, all our results carry through. The only difference is that the mass of advisers is endogenously determined. That is, when c is large (small), then advisers will be on the short (long) side.

[78] Intuitively, the adviser does not have perfect control over which "trading opportunities" she discovers.

B.2 Alternative Setting with Two-Stage Game

This section provides an alternative interpretation of our setup with the following extensive form: first, each adviser posts a menu of information policies contingent on customers' types b to maximize her expected payoff. That is, each adviser l posts a menu $\sigma_m(b, l)$. Second, customers choose their adviser given the posted information policy. Then, as before, the state realizes and customers make their investment decisions based on the information they receive.

One can then use the equilibrium concept from the literature on large games (e.g.,Mas-Colell (1984)), where the payoff of each agent is determined by his own decision and by the distribution of others' actions in the market. Given the distribution of other agents' actions in the market, one can then define the equilibrium utility as the highest utility the agent can obtain. The adviser then chooses the information policy and the customer type she wants to attract, taking the equilibrium utility as given.

We now show that the equilibrium outcome $\{U(b), V(l), H(b,l), \sigma^*(b,l)\}$ in our matching game in Section 3 is also an equilibrium in this two-stage game, where the menu posted by an adviser l for a customer b, denoted by $\sigma_m(b,l)$,solves:

$$\max_{\sigma}\{\tilde{U}(b,l,\sigma)|\tilde{V}(b,l,\sigma) \geq V(l)\}. \tag{24}$$

As we argued in Section 3, $\sigma^*(b,l)$ in our matching equilibrium must be Pareto optimal on the equilibrium path. Therefore, we have $\sigma^*(b,l) = \sigma_m(b,l)$. This implies that the constructed menu gives all agents the same equilibrium utilities within the equilibrium matches.

To see that posting the menu $\sigma_m(b,l)$ is indeed an equilibrium in this setting, one needs to check two conditions. First, given the posted information $\sigma_m(b,l)$, a customer b will not benefit from choosing a different an adviser l' that is not on the support of $H(b,l)$. That is, $U(b) = \tilde{U}(b,l,\sigma_m(b,l)) \geq \tilde{U}(b,l',\sigma_m(b,l'))\ \forall l$. The main difference from our main setup is that, when a customer chooses a different adviser l',the information policy that he obtains in this two-stage game is $\sigma_m(b,l')$; our original equilibrium allows for any deviating policy as long as it attracts adviser l'. Our equilibrium condition (ii) guarantees that there is no policy that can make a customer better off while making the adviser also (weakly) better off. The constructed menu $\sigma_m(b,l')$ in Equation (24) guarantees that an adviser l' receives her equilibrium utility when matching with any customer b. Hence,choosing $\sigma_m(b,l')$ cannot be profitable for customer b in this two-stage setting. Otherwise, matching with adviser l' with information policy $\sigma_m(b, l')$ constitutes a profitable deviation in our matching game, which would violate pairwise stability.

Second, advisers will not choose a different combination of customer and information policy. The argument above guarantees that, given the menu $\sigma_m(b, l)$, customer b only chooses the adviser l that is the support of $H(b, l)$. Hence, if an adviser wants to successfully attract a different customer b', the policy that she provides must ensure that the customer receives at least $U(b')$. Hence, her maximum payoff under this deviation is given by $\max_\sigma\{\tilde{V}(b', l, \sigma)|\tilde{U}(b', l, \sigma) \geq U(b')\} \leq V(l)$, where the last inequality follows from our equilibrium Condition (i).

Two observations are in order: first, we know that it is WLOG to focus on the threshold strategy. The solution $\sigma_m(b, l)$ to Equation (24) is then the threshold that gives the adviser her equilibrium utility. That is, it must be the case that $\tilde{V}(b, l, \hat{s}) = V(l)$. Second, from Equation (9), conditional on the threshold $\hat{s}$, advisers do not care about the customers' type, which implies that the threshold strategy that solves $\tilde{V}(b, l, \hat{s}) = \alpha(1 - F(\hat{s})) = V(l)$ is the same for all customers. That is, $\sigma_m(b, l) = s^*(l)\ \forall(b, l)$. In other words, all advisers post a single policy $s^*(l)$ for all customers b in this two-stage game.

Moreover, the above argument suggests that the equilibrium outcome in our matching model in Section 2 remains an equilibrium in a setting where customers' types are not observable. This is because, under the constructed policy $s^*(l)$, the optimization problem of customers is described by (8) with solution $\ell(b)$. Thus, customers will optimally sort themselves into their equilibrium matches. When posting a different policy, an adviser now has a rational belief about which type to attract. Nevertheless, as before, he can only attract a customer type conditional on giving him his equilibrium utility. However, we have argued that an adviser will not benefit from attracting a different customer under the constructed equilibrium; thus, for the same reason, there is no profitable deviation for advisers.

B.3 Robustness of Sorting Outcomes

The sorting result implies that customers who demand more information are matched with more advisers who have more valuable information. Below, we establish how our results remain robust in a more general setting for the customer's information structure and under different settings for the adviser's ability.

Customer's Information Structure Let b index the quality of a customer's information, in the sense that more informed customers have a higher payoff under self-directed trade for all assets: for $b' > b$, $u^0(b', l) > u^0(b, l)$. In general, depending on a customer's signal structure, the optimal information policy might no longer be a threshold policy. We restrict our attention to environments where the optimal information policy σ is such that customers always invest when the asset payoff is positive. That is, under the optimal information policy, a customer receives return r with probability $1 - F(\lambda)$ ex ante. Let $q_L(b, l, \sigma)$ denote the ex ante probability that a customer b invests asset l when the payoff is negative under policy σ. The gain for the customer under this information policy (relative to investing by himself) can then be expressed as

$$\tilde{U}(b, l, \sigma) = (1 - F(\lambda))r - q_L(b, l, \sigma)l - \alpha_c \{1 - F(\lambda) + q_L(b, l, \sigma)\} - u^0(b, l).$$

Since the customer's utility decreases in the probability of over-investment q_L, we say that the information is worse when it induces a higher q_L. The payoff of the adviser, on the other hand, increases in q_L,which yields

$$\tilde{V}(b, l, \sigma) = \alpha_c \{1 - F(\lambda) + q_L(b, l, \sigma)\}.$$

The Pareto frontier within each possible match can then be expressed as

$$u(b, l, \bar{v}) = (1 - F(\lambda))r - \left(\frac{\bar{v}}{\alpha_c} - (1 - F(\lambda))\right) l - \bar{v} - u^0(b, l).$$

We assume that an adviser with a higher l is more valuable to customers, in the sense that, fixing any information quality (and thus adviser utility $\bar{v}$),a customer is better off knowing about the asset with a higher l. That is, for $l' > l$, $u(b, l', \bar{v}) > u(b, l, \bar{v})$.

Proposition 7. *For any information policy σ under which customers always invest when the asset payoff is positive, less informed customers are matched with advisers with more valuable expertise and receive worse information, as long as, for any $b' > b$ and $l' > l$,*

$$u^0(b', l') - u^0(b, l') > u^0(b', l) - u^0(b, l). \tag{25}$$

Proof. To establish the sorting, we employ the condition identified in Proposition 1 in Legros and Newman (2007). That is, suppose that a more informed customer b' is indifferent between l and l'; then a less informed customer b must strictly prefer to match with an adviser with a higher l. Suppose that a customer b' receives the same utility $\bar{u}$ when matching with adviser l and with adviser l'. Given that $u(b', l', v(b', l', \bar{u})) = u(b', l, v(b', l, \bar{u}))$, we have

$$\begin{aligned}
&(1 - F(\lambda))r - \left(\frac{v(b', l', \bar{u})}{\alpha_c} - (1 - F(\lambda))\right) l' - v(b', l', \bar{u}) - u^0(b', l') \\
&= (1 - F(\lambda))r - \left(\frac{v(b', l, \bar{u})}{\alpha_c} - (1 - F(\lambda))\right) l - v(b', l, \bar{u}) - u^0(b', l)
\end{aligned}$$

Condition (25) implies the following inequality,

$$\begin{aligned}
&(1 - F(\lambda))r - \left(\frac{v(b', l', \bar{u})}{\alpha_c} - (1 - F(\lambda))\right) l' - v(b', l', \bar{u}) - u^0(b, l') \\
&> (1 - F(\lambda))r - \left(\frac{v(b', l, \bar{u})}{\alpha_c} - (1 - F(\lambda))\right) l - v(b', l, \bar{u}) - u^0(b, l)
\end{aligned}$$

Hence, $u(b, l', v(b', l', \bar{u})) > u(b, l, v(b', l, \bar{u}))$. Thus, by the condition in Proposition 1 in Legros and Newman (2007), the equilibrium must feature negative sorting. Intuitively, trading the more information sensitive assets directly is relatively costly for the less informed customers; hence, they gain more by matching with adviser with higher l. Furthermore, since advisers with more valuable expertise must receive a higher equilibrium payoff, it thus means that his matching customer (i.e., a less informed customer) must receive worse information in the sense that he over-invests with higher probability. □

Thus, our results are robust to perturbing our model. For example, suppose that customer b learns the state s with probability b. With probability $1 - b$, she learns nothing. Or, suppose that that the customer's signal is represented by a partition consisting of finitely many intervals and the customer learns which interval the state belongs to.[79] In both cases, we can show that condition (25) holds. Then, we still get negative assortative matching.

[79] Specifically, for an even number $N \geq 2$, there exist intervals $\{[0, \underline{s}), [\underline{s}, s_1), [s_1, s_2), ..., [s_N, \bar{s}), [\bar{s}, 1]\}$, so that the "middle" interval is centered on $s = \lambda$. Each interval $[s_k, s_{k+1})$ has the same length $1/b$. The intervals on the "edges," $[0, \underline{s})$ and $[\bar{s}, 1]$ adjust so that the partition covers $[0, 1]$. While this is not the most general definition, it is a natural way to define a partition which leaves a customer with higher b more informed without running into too many edge cases.

Advisers' Expertise We now consider an alternative modeling of "better" advisers. Consider an environment where all advisers know about the same asset l but differ in how informed they are. Specifically, let δ index adviser's types, where adviser with type δ learns about the state s with probability δ but learns nothing with probability $1-\delta$ and cannot give advice. Customers' information structure and preferences remains the same as before, where the customer receives a positive signal subject to false positives. Likewise, a threshold policy remains optimal for advisers. Hence, the gain of customer b when matching with adviser δ under threshold $\hat{s}$ can then be expressed as

$$\tilde{U}(b,\delta,\hat{s}) = \delta\{(1-F(\lambda))\,r - (F(\lambda)-F(\hat{s}))\,l - \alpha_c(1-F(\hat{s}) - u^0(b,l)\}.$$

While the adviser's payoff can be expressed as

$$\tilde{V}(b,\delta,\hat{s}) = \alpha\delta\,(1-F(\hat{s}))\,.$$

The Pareto frontier within each possible match can then be expressed as

$$u(b,\delta,\bar{v}) = \delta\left\{(1-F(\lambda))r - \left(\frac{\bar{v}}{\alpha\delta} - (1-F(\lambda))\right)l\right\} - \bar{v} - \delta u^0(b,l).$$

The sorting outcome can be established by using the same argument for the Proposition above, where the relevant term that determines the sorting is $\delta u^0(b,l)$. That is, less informed customers match with more informed advisers.

Algebraically, it is as if the outside option in Proposition 7 becomes $\delta u^0(b,l)$. For any $\delta' > \delta$, we have $\delta'(u^0(b',l) - u^0(b,l)) > \delta(u^0(b',l) - u^0(b,l))$.

B.4 Case Without Sorting

The condition identified in Appendix B.3 highlights that the customer's outside option is important for our result. We now characterize the environment where condition (25) is violated. Specifically, we assume that without requesting advice, a customer will not invest in any asset l, but instead obtains a default outside option, denoted by $u^0(b)$. Note that one can also interpret this environment as if advisers act as gatekeepers so that customers can only invest in asset l through them. In this case, the value of trading through an adviser l

with policy $s^*(l)$ yields

$$\tilde{U}(b, l, s^*(l)) = (1 - F(\lambda))r - (F(\lambda) - F(s^*(l)))l - \alpha_c(1 - F(s^*(l))) - u^0(b).$$

Given that $\tilde{U}_{bl}(b, l, s^*(l)) = 0$, there is no sorting. Hence, every customer must be indifferent between requesting advice for all asset. Moreover, in this case, choosing advisers is effectively choosing the asset. Since a higher l asset has lower ex-ante value, advisers must compensate customers with providing better information, which lowers their profits. That is, an adviser with a higher l now becomes less attractive. This is in contrast to our setting where choosing an adviser is purely about choosing the highest information value.

For any active b and l, the policy $s^*(l)$ must be such that all active customers are indifferent. Thus, $s^*(l)$ must satisfy the following ODE:

$$\frac{ds^*(l)}{dl} = \frac{F(\lambda) - F(s^*(l))}{(l + \alpha_c)F'(s^*(l))} > 0. \tag{26}$$

That is, $s^*(l)$ is increasing, so advisers with higher l provide better information. In the case in which $u_b^0(b) > 0$ (i.e. a more informed customer has a higher outside option), then, as before, a less informed customer always has a higher incentive to participate. When advisers are on the short side of the market, the information provided by the worst asset $s^*(\bar{l})$ must be such that the marginal customer is indifferent. That is, the initial condition $s^*(\bar{l})$ solves $\tilde{U}(b^*, \bar{l}, s^*(\bar{l})) = u^0(b^*)$. On the other hand, if $u^0(b) = u^0$ for all customers, then all customers are effectively the same from perspective of advisers. As a result, the initial condition $s^*(\bar{l})$ is such that all customers are indifferent between trading through advisers or receiving their outside option u^0.

B.5 Customer's Problem

In the model setup, we have assumed that advisers are infinitesimal in that they know about only one asset and that customers choose the adviser who yields them the highest gain relative to the outside option of investing based on their own information. We have made this assumption to keep the model tractable and to have a clean characterization of the information provided by each adviser as a single threshold. In this section, we provide an explicit microfoundation for the customer's decision problem when advisers are not infinitesimal in the sense that they have information over a strictly positive interval of assets. We show that the customer indeed maximizes the gain when choosing an adviser and that the tradeoff between adviser and customer utilities that determines matching in our model approximates the one we found in Section 3.1. That is, the economics of our model remain unchanged.

Assets are indexed by $l \in \left[\underline{l}, \bar{l}\right]$ and customers by $b \in \left[\underline{b}, \bar{b}\right]$ as before. Each adviser learns the state of assets in the interval $[l - l_0, l + l_0] \subset \left[\underline{l}, \bar{l}\right]$ for some l and fixed $l_0 > 0$. Intuitively, each adviser has information about assets that are similar to l. We denote a generic asset in $[l - l_0, l + l_0]$ by $\hat{l}$ and we can again index advisers by l without loss of generality. The set of adviser types is now $\left[\underline{l} + l_0, \bar{l} - l_0\right]$. We assume that l_0 is sufficiently small that this interval has strictly positive size. After matching with adviser l and receiving information, the customer then decides whether to buy each asset. To keep notation minimal we set $\alpha_c = 0$ throughout this section.

His investment decision for asset $\hat{l}$ is the same as in equation (5). We can use the same argument as in Section 3.1 to show that without loss of generality, the adviser recommends that the customer invest in asset $\hat{l}$ whenever $s_{\hat{l}}$ is above a threshold $\hat{s}_{\hat{l}} \in [b\lambda, \lambda]$. The only difference from the main model is thus that the adviser provides the customer with multiple thresholds $(\hat{s}_{\hat{l}})_{\hat{l} \in [l - l_0, l + l_0]}$. Given this policy, the utility of customer b from receiving the adviser's information is

$$\tilde{U}(b, l) = \int_{l - l_0}^{l + l_0} \left[r(1 - F(\lambda)) - \hat{l}\left(F(\lambda) - F(\hat{s}_{\hat{l}})\right) \right] d\hat{l}.$$

His utility from investing in asset $\hat{l}$ without advice is given by

$$u^0\left(b, \hat{l}\right) = r(1 - F(\lambda)) - \hat{l}\left(F(\lambda) - F(b\lambda)\right).$$

The customer's problem of choosing adviser l thus becomes

$$\max_{l \in \left[\underline{l} - l_0, \bar{l} + l_0\right]} \tilde{U}(b, l) + \int_{\hat{l} \notin [l - l_0, l + l_0]} u^0\left(b, \hat{l}\right) d\hat{l}.$$

That is, in choosing adviser l, the customer understands that the adviser only has information about assets $\hat{l} \in \left[\underline{l} + l_0, \bar{l} - l_0\right]$. The customer's utility from receiving this advice is the first term in the maximization problem. For all other assets, he must invest based on his own information, which is the second term. We can rewrite this equation as

$$\max_{l\in\left[\underline{l}-l_0,\bar{l}+l_0\right]} \tilde{U}\left(b,l\right) - \int_{l-l_0}^{l+l_0} u^0\left(b,\hat{l}\right) d\hat{l} + \int_{\underline{l}}^{\bar{l}} u^0\left(b,\hat{l}\right) d\hat{l},$$

which shows that the customer's utility from choosing adviser l is the gain from receiving information about assets in $[l-l_0, l+l_0]$, which is represented by the first two terms, plus the utility he would receive by simply investing in all assets by himself. This problem is equivalent to solving

$$\begin{aligned} & \max\nolimits_{l\in\left[\underline{l}-l_0,\bar{l}+l_0\right]} \tilde{U}\left(b,l\right) - \textstyle\int_{l-l_0}^{l+l_0} u^0\left(b,\hat{l}\right) d\hat{l} \\ = \;& \max\nolimits_{l\in\left[\underline{l}-l_0,\bar{l}+l_0\right]} \textstyle\int_{l-l_0}^{l+l_0} \left[\hat{l}\left(F\left(\hat{s}_{\hat{l}}\right) - F\left(b\lambda\right)\right)\right] d\hat{l}, \end{aligned}$$

which is the analog of Equation (8) in Section 3.1.

In choosing advisers with given information policies, the customer thus chooses the one with the highest gain. This is analogous to how we have modeled the customer's behavior in the main sections of the paper. Extending the model in this way does not qualitatively alter the equilibrium, except that we now have a vector of thresholds for each adviser. To see this, we can write the problem of adviser l conditional on having to promise a certain gain $l_0\bar{u}$ to attract customer b, which is the analog of the adviser's problem in Equation (9),

$$\begin{aligned} v\left(b,l,\bar{u}\right) &= \max_{\left(\hat{s}_{\hat{l}}\right)_{\hat{l}\in[l-l_0,l+l_0]}} \alpha \int_{l-l_0}^{l+l_0} \left(1-F\left(\hat{s}_l\right)\right) d\hat{l} \\ & \int_{l-l_0}^{l+l_0} \left[\hat{l}\left(F\left(\hat{s}_{\hat{l}}\right) - F\left(b\lambda\right)\right)\right] d\hat{l} \geq l_0\bar{u}. \end{aligned}$$

Substituting the constraint into the objective yields the expression

$$v\left(b,l,\bar{u}\right) = 2l_0\alpha\left(1-\left(\frac{u}{l}+F\left(bl\right)\right)+o\left(l_0\right)\right).$$

That is, for sufficiently small l_0, the adviser's value function within each match has approximately the same shape as that in our original model. We can show that as l_0 becomes small, any equilibrium must converge to the equilibrium we derived in Proposition 2.