FTG Working Paper Series

Sentiment, Liquidity and Asset Prices*

by

William Fuchs
Brett Green
William Fuchs

Working Paper No. 00034-00

Finance Theory Group

www.financetheory.com

Sentiment, Liquidity and Asset Prices

Vladimir Asriyan, William Fuchs, and Brett Green*

First draft, June 2017. This draft, January 2018.

Abstract

We study a dynamic market for durable assets, in which asset owners are privately informed about the quality of their assets and experience occasional productivity shocks that generate gains from trade. An important feature of our environment is that buyers worry not only about asset quality, but also about the prices at which they can re-sell the assets in the future. We show that this interaction between adverse selection and re-sale concerns generates an inter-temporal coordination problem and gives rise to multiple self-fulfilling equilibria. We construct sentiment equilibria, in which sunspots generate large fluctuations in asset prices, market liquidity, output and welfare. Furthermore, we show that the strategic nature of trade in our setting disciplines the set of possible sentiments as a function of the parameters of the model. The theory has implications for empirical work in asset pricing and macroeconomics.

JEL: D82, E32, E44, G12.

Keywords: Sentiment; Liquidity; Asset Prices; Capital Reallocation; Business Cycles; Sunspots.

---

*Asriyan: CREi, UPF and Barcelona GSE, vasriyan@crei.cat. Fuchs: McCombs School of Business, UT Austin and Universidad Carlos III Madrid, wfuchs@gmail.com. Green: Haas School of Business, UC Berkeley, greenb@berkeley.edu. Asriyan acknowledges financial support from the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563). Fuchs gratefully acknowledges support from the ERC Grant 681575. We thank Manuel Amador, Fernando Broner, Jason Donaldson, Hugo Hopenhayn, Sergei Kovbasyuk, Ricardo Lagos, Alberto Martin, Vincent Maurin, Guillermo Ordoñez, Giorgia Piacentino, Giacomo Ponzetto, Edouard Schaal, Anton Tsoy, Jaume Ventura, Victoria Vanasco, Venky Venkateswaran, Pierre-Olivier Weill and seminar participants at CEMFI, CREi-UPF, London Business School, Sciences Po, Universidad Carlos III de Madrid, UT Austin, OTC Markets and Securities Workshop at LAEF in Santa Barbara, Transpyrenean Macro Workshop CREi-TSE, the Workshop on Information Asymmetries in Macro-Finance at the Barcelona GSE Summer Forum, 8th Summer Workshop in Macro-Finance in Paris, 6th Rome Workshop on Macroeconomics (a.k.a. Pizzanomics), 13th CSEF-IGIER Symposium on Economics and Institutions in Capri, meetings of the Society for Economic Dynamics (SED) in Edinburgh, Finance Theory Group (FTG) in Minnesota and Macro Finance Society (MFS) in Boston for useful suggestions and feedback.

1 Introduction

In a frictionless market, all gains from trade are realized and durable assets or securities always end up being held by parties that value them the most. As a result, asset prices reflect not only the current but also all expected future gains from trade. Instead, in the presence of frictions, some gains from trade may remain unrealized and, thus, asset prices may be depressed. In such an environment, there is a close connection between *liquidity* – the ease with which assets are re-allocated – and asset prices. In this paper, we show that if the frictions result from *information asymmetries*, there can be multiple self-fulfilling equilibria. Even though all agents are fully rational and asset prices always reflect fundamentals, the mix of assets that is traded can depend on *sentiments* – the agents' expectations about future market conditions. We show that there is a set of sentiment-driven equilibria, in which sunspots generate large fluctuations in asset prices, market liquidity, output and welfare.

We consider a dynamic market for assets (Lucas trees), in which asset owners are privately informed about the quality of their assets (the fruit to be harvested). Gains from trade arise stochastically over time because the current asset owners experience "productivity" or "liquidity" shocks that change their value of holding or employing assets relative to other agents in the economy. Buyers compete for assets, but they may face a lemons problem as in Akerlof (1970), since they do not observe the quality of the owners' assets nor the motive for their sale. The buyers who purchase assets in any given period become asset owners in the next period. The important feature of our environment is that the buyers must worry not only about the quality of the assets for which they currently bid, but also about market prices were they to resell the assets in the future.

When information is symmetric, all asset owners with (productivity) shocks immediately sell their assets and, in the unique equilibrium, asset prices are equal the expected discounted value of asset cash-flows at their most efficient allocation (Proposition 1). Furthermore, this economy features no aggregate fluctuations in asset prices, output or welfare.

Instead, when information is asymmetric, the owners of low quality assets want to mimick the owners of high quality assets, and their presence in the market depresses the buyers' willingness to pay. Absent resale considerations (or in a static setting), the buyers only care about the flow payoff that they expect to receive from holding the asset in the current period. As a result, when the proportion of high quality assets is sufficiently low, the buyers' willingness to pay drops below the reservation value of shocked owners of high quality assets. In this case, only low quality assets trade in equilibrium, asset allocation is inefficient, and asset prices are depressed to reflect this. On the other hand, when the proportion of high quality assets is sufficiently high, the buyers' willingness to pay remains above the reservation value of the shocked owners of high quality assets. In this case, all shocked asset owners trade, asset allocation is efficient and asset prices are high to reflect this. Therefore, depending on parameters, there can be two possible equilibria but, more importantly, the equilibrium is unique.[1]

Our first main result is that the interaction between information frictions and resale concerns generates an inter-temporal coordination problem, which can give rise to multiple equilibria (Theorem 1). The reason is that, when buyers anticipate the need to sell assets in the future, their willingness to pay for them today depends on their beliefs about future market conditions. If buyers believe that the market will be liquid and asset prices will be high tomorrow, they will bid more aggressively for assets today, and thus be able to attract a better pool of assets today, and vice versa. To show how these concerns about future market conditions can generate multiplicity of equilibria, we first construct two types of what we term *constant price equilibria*. A defining property of these equilibria is that both asset prices and asset allocations among different owner types are fixed over time.

This class includes an *efficient trade* equilibrium, in which all shocked asset owners trade their assets immediately and, as a result, the asset prices, output and welfare are permanently high. We show that there exists a lower bound $\bar{\pi}_{ET}$ on the proportion π of high quality assets, such that the *efficient trade* equilibrium exists when π is greater than $\bar{\pi}_{ET}$. Then, we construct an *inefficient trade* equilibrium, in which only low quality asset owners trade and, as a result, the asset prices, output and welfare are permanently low. We show that there exists an upper bound $\bar{\pi}_{IT}$ on the proportion of high quality assets, such that the *inefficient trade* equilibrium exists when π is smaller than $\bar{\pi}_{IT}$. Importantly, we show that $\bar{\pi}_{ET} < \bar{\pi}_{IT}$ and, therefore, the two equilibria coexist for intermediate π.

We then capture the notion of *sentiments* as coordinated beliefs about future market conditions. To do so, we introduce a sunspot process z_t and we look for equilibria in which agents coordinate on efficient or inefficient trade depending on z_t. We demonstrate that the coexistence of multiple constant price equilibria and sufficient persistence of the process z_t is necessary and sufficient for the existence of sentiment equilibria (Proposition 4 and Theorem 2). Moreover, the amount of persistence needed to support sentiment-driven equilibria depends critically on the underlying primitives. That is, unlike static coordination problems, sentiment equilibria cannot be driven by an arbitrary stochastic process, but rather the necessary properties of its evolution are disciplined by the parameters of the model.

Our model shows that sentiments can actually affect the fundamental value of assets by

[1]When buyers are strategic, as in our setting, the static Akerlof (1970) model also has a unique equilibrium, as was first noted by Wilson (1980).

changing the mix of assets that are traded and, therefore, the extent to which gains from trade are realized. Thus, market sentiments cannot be separated from fundamentals, and both are essential in determining asset valuations. In particular, even when there is no intrinsic information about changes in the characteristics of the assets, sentiments can lead to large price swings. Thus, our model can provide a fully rational explanation for the documented excess volatility in asset prices.[2] Our theory also illustrates that sentiments can be an important source of macroeconomic volatility. Although measured total factor productivity (TFP) may fluctuate, it may not be the driver of output volatiltiy, as both can instead be driven by shocks to expectations. Thus, an econometrician, who cannot directly observe sentiments, must be cautious when estimating and interpreting measures of TFP, so as to avoid over-estimating the role of technology shocks.

1.1 Related Literature

Our paper naturally relates to the recent and growing literature that embeds adverse selection in a macro-finance context.[3] Daley and Green (2016) and Fuchs et al. (2016) explicitly model re-trade considerations.[4] Unlike us, these papers focus on the role of time-on-the-market as a signal of quality; furthermore, the equilibria in their settings are essentially unique. Although both papers can generate time varying liquidity, a fact that is particularly stressed in Daley and Green (2016), this variation is not driven by inter-temporal coordination and expectations of future market liquidity. Instead, it is driven by whether the current beliefs about the asset quality are above or below the critical threshold at which pooling is an equilibrium.

Janssen and Karamychev (2002) and Janssen and Roy (2002) have used a competitive framework to highlight another source of time-varying liquidity (even in the absence re-trade). Namely, that when the gains from trade are persistent (something we purposefuly abstract away from in our core analysis), past liquidity has a negative effect on current liquidity. Intuitively, if more of the gains from trade were realized yesterday, there will be more adverse selection in the market today. This can lead to deterministic liquidity cycles, as recently has also been pointed out by Maurin (2016) in a search-theoretic environment.

Some other recent work in the area considers markets with search frictions rather than a

[2]See, for example, LeRoy (2004) and Shiller (2005).

[3]See, for example, Eisfeldt (2004), Martin (2005), Kurlat (2013), Guerrieri and Shimer (2014), Bigio (2015), Chari et al. (2010), Gorton and Ordoñez (2014, 2016), Benhabib et al. (2014), Daley and Green (2016) and Fuchs et al. (2016).

[4]The importance of re-trade considerations in asset markets goes back to Harrison and Kreps (1978). See also Lagos and Zhang (2015, 2016) for recent related work within search-theoretic environment.

competitive environment like ours. The closest within this literature are the papers by Chiu and Koeppl (2011), Maurin (2016) and Mäkinen and Palazzo (2017). In addition to differences in market structure, these papers have a very different focus from ours. The main consideration in Chiu and Koeppl (2011) is the interaction between adverse selection and search frictions, and it is largely motivated by the recent financial crisis: they mainly discuss policy interventions when the fraction of low quality assets in the market is so large that there would be no trade absent an intervention. Although Maurin (2016) notes that there is a possibility for multiple equilibria, as previously noted, his main contribution is the construction of equilibria with cycles. Unlike our sentiment equilibria, these equilibria are deterministic and are not driven by inter-temporal coordination. Finally, Mäkinen and Palazzo (2017) have a more general search and matching technolodgy that allows for congestion externalities. Their focus is on the additional negative effect (and policies to overcome it) from the fact that unshocked traders stay in the market trying to trade away their lemons and creating congestion externalities for shocked sellers.

The papers by Plantin (2009) and Malherbe (2014) are also related to our work, although the strategic considerations in their papers are contemporaneous rather than dynamic. In Malherbe (2014), firms must make a portfolio choice decision between holding cash versus assets with privately known quality. He shows that multiple equilibria are possible due to complementarities in firms' cash-holding decisions. If a firm decides to increase its cash-holdings in the first period, then if that firm trades in the second period, it is less likely that the trade is the result of a liquidity shock. As a result, there are less gains from trade in the second period and there is more adverse selection in the market. This in turn makes it more attractive for other firms to also hoard cash. Thus, there can be two equilibria, one in which firms expect other firms not to hoard cash and the second period market to work well, and another in which firms expect other firms to hoard cash and, as a result, the second period market dries-up. A similar mechanism is present in Plantin (2009). Although there is no cash-hoarding by firms in his setting, the number of investors who decide to buy the bond in the first period affects the potential market size for the bonds and hence their price in the future. As in Malherbe (2014), this contemporaneous complementarity can lead to self-fulfilling market failures. It is important to highlight that equilibrium multiplicity in these papers arises due to static coordination failures. Indeed, as in the global games literature, Plantin (2009) is able to obtain uniqueness of equilibrium by introducing a noisy private signal about the probabilities of default of the bonds.

The inter-temporal aspect of the coordination leading to multiplicity of equilibria relates our work with the broad literature on fiat money and rational bubbles.[5] There is an important difference between our work and most of that literature. In our setting, the value of assets is always pinned down by fundamentals and we do not rely on a violation of the "No-Ponzi games" condition for assets to have positive prices. Somewhat closer to our model is the contemporaneous work of Donaldson and Piacentino (2017), who motivate potential runs on banks as arising from failures of coordination in the re-trading of"money-like" bank obligations. In their setting, trading frictions are exogenous, there is no adverse selection and trade completely breaks down, whereas adverse selection is the source of the endogenous frictions in our model. Furthermore, there is always some trade in our model.

Finally, there has been an increased interest among macroeconomists to understand how sentiments – in the form of correlated shocks to agents' information sets,– can be drivers of aggregate fluctuations. Some recent papers include Lorenzoni (2009), Hassan and Mertens (2011), Angeletos and La'O (2013), and Benhabib et al. (2015). In this literature, the dispersion of information among agents about aggregate economic conditions is an essential ingredient. We contribute to this literature by showing that, in the presence of adverse selection, sentiments which coordinate agents' expectations about future market conditions can generate aggregate fluctuations even when the information about aggregate variables is common to all economic agents at all times.

The rest of the paper is organized as follows. In Section 2, we present our baseline model. In Section 3, we conduct our main analysis. In Section 4, we consider some extensions, and we conclude in Section 5. All proofs are relegated to the Appendix.

2 The Model

Time is infinite and discrete, indexed by $t \in \{0, 1, ...\}$. There is a mass of indivisible assets or Lucas trees, indexed by $i \in [0, 1]$, which are identical in every respect except their quality. These trees are long-lived and each tree can either be of high or low quality, which we denote by $\theta_i \in \{L, H\}$. A tree of quality θ_i can potentially produce x_{θ_i} units of output per period, where $x_H > x_L > 0$. The probability that a given tree is of high quality is $P(\theta_i = H) = \pi \in (0, 1)$, which is also assumed to be the fraction of high quality trees in the economy. For expositional simplicity, we suppose that asset qualities are fixed; we extend our analysis to incorporate asset quality shocks in the Appendix.

There is a a large mass M of ex-ante identical risk-neutral agents, indexed by $j \in [0, M]$,

[5] See, for example, the early papers by Samuelson (1958), Tirole (1985), Weil (1987), Santos and Woodford (1997), and the more recent work by Martin and Ventura (2012) and Dong et al. (2017).

who discount payoffs with a factor $\delta \in (0,1)$. Each of these agents can operate only one unit of the Lucas tree, and we refer to those who currently operate assets as *owners* and to the rest as potential *buyers*. We introduce gains from asset trade by supposing that owners experience occasional productivity shocks that depress their asset valuation relative to the potential buyers. In particular, each period owner j can have two possible productivities, denoted by $\omega_j \in \{\chi, 1\}$, which implies that she can produce $\omega_j x_{\theta_i}$ units of output by operating a tree of quality θ_i, where $\chi \in (0,1)$ and $\chi x_H \geq x_L$.[6] When $\omega_j = \chi$, we say that "owner j is shocked," and we assume that each period an owner is shocked with probability $P(\omega_j = \chi) = \lambda \in (0,1)$. All potential buyers are assumed to be unshocked.[7] An owner's productivity status is assumed to be independent of the quality of the tree she operates and of her productivity status in the past; we extend our analysis to persistent productivity shocks in Section 4.

Remark 1 *Although throughout we will think of ω as a productivity shock, one could interpret the differences in ω's as arising from the heterogeneity in agents' valuations of the cashflow, which can be due to liquidity constraints or hedging demands. Of course, these different interpretations will have important implications about how to take the model to the data.*

The market for assets is competitive - in each period, at least two buyers are randomly matched with an owner, and they compete for the owner's tree a la Bertrand.[8] When an owner receives offers from the buyers, she decides which if any offer to accept. If the owner rejects all offers, then she continues to be an owner in the next period and is rematched with a new set of buyers. If the owner accepts an offer, then she sells her tree and enters the pool of potential buyers.[9] A buyer whose offer is rejected continues to be a buyer in the next period, whereas a buyer whose offer is accepted, gets the tree and becomes an owner in the next period.[10]

Trade in our economy may be hindered by the presence of asymmetric information. In particular, we assume that the quality of an owner's tree θ and her productivity status ω are both that owner's private information.

We suppose that the time-t information set of a buyer includes aggregate histories (e.g., aggregate output, aggregate trading volume), but not the trading history of the individual

[6]This condition states that the adverse selection problem is sufficiently severe. Although inessential for our results, it reduces the number of cases that we need to consider.

[7]As long as there are at least two unshocked buyers competing for each owner's tree, these buyers will be setting the asset prices, and therefore it is without loss of generality to simply assume that the buyers are unshocked.

[8]Perfect competition among buyers is not needed for our results, but it simplifies the analysis.

[9]Since there is a continuum of trees and matching is random, the probability that an owner who sells her tree is rematched to bid for that same tree is zero.

[10]We implicitly assume that all agents have sufficiently large endowments in each period.

asset for which he bids.[11] The strategy of each buyer is a mapping from his information set to a probability distribution over offers. An owner's information set includes the quality θ of her asset, her productivity status ω, and the buyers' information set. The strategy of each owner is a mapping from her information set to a probability of acceptance.

We use Perfect Bayesian Equilibrium (PBE) as our equilibrium concept. This has the following implications. First, each owner's acceptance rule must maximize her expected payoff taking as given the buyers' strategies (*Owner Optimality*). Second, any offer in the support of a buyer's strategy must maximize his expected payoff given his beliefs, the owner's and the other buyers' strategies (*Buyer Optimality*). Third, given their information set, buyers' beliefs are updated using Bayes' rule whenever possible (*Belief Consistency*).

3 Equilibrium

In this section, we characterize the set of equilibria of our model. We start by analyzing a benchmark economy in which asset qualities are observable (Section 3.1). In the unique equilibrium of this economy, there are no aggregate fluctuations and all assets are allocated efficiently (Proposition 1). Next, we analyze the model with asymmetric information about asset quality. We start by focusing on equilibria that do not feature aggregate fluctuations (Section 3.2), and we show that multiple equilibria can arise and be ranked in terms of asset prices, output and welfare (Theorem 1). We then consider sentiment equilibria (Section 3.3), and we provide necessary and sufficient conditions under which these equilibria exist and feature belief-driven fluctuations in asset prices, output and welfare (Proposition 4 and Theorem 2).

3.1 Benchmark without information frictions

Here, we consider a useful benchmark economy in which the qualities of the Lucas trees are public information. It turns out that observability of asset qualities suffices to ensure that the asset allocations are efficient. The following proposition characterizes the unique equilibrium of this benchmark. Let $E\{\cdot\}$ denote the expectations operator, then:

Proposition 1 (Observable Quality) *If asset qualities are publicly observable, then the equilibrium is unique, in it all assets are efficiently allocated and, for all t, the price of θ-quality*

[11] The primary role of this assumption is to eliminate signaling considerations which would complicate our analysis considerably. If trading history of individual trees were observable, owners may reject certain offers and engage in costly delay in order to signal their types. We conjecture that our qualitative results extend to a setting where such signaling is possible as long as trading history provides an imperfect signal of asset quality.

assets is $p_\theta^{FB} = (1-\delta)^{-1}x_\theta$, *and the output and welfare are* $Y^{FB} = E\{x_\theta\}$ *and* $W^{FB} = (1-\delta)^{-1} \cdot Y^{FB}$ *respectively.*

For any given (observable) quality, buyers value the trees weakly more than the owners (strictly so if owners are shocked). Thus, in equilibrium, all trees must be reallocated from shocked owners to the buyers, i.e., asset allocation is efficient. As a result, the aggregate output is the output of all trees at their most efficient allocation, and welfare is simply the present discounted value of this output. Finally, because markets are competitive, all trees are priced at the present discounted value of their ouput.

We next study how these results change in the presence of information frictions.

3.2 Constant price equilibrium

We begin our analysis by considering a simple class of stationary equilibria which allow us to clearly illustrate the link between asset prices and market liquidity.

From now on, we will refer to an owner with productivity status ω and an asset of quality θ as a (θ, ω)-type owner. Let p_t denote the (common) asset price that prevails in equilibrium at time t.[12] Then,

Definition 1 *We say that a PBE is a* **constant price equilibrium** *if the equilibrium asset price is the same in every period.*

In what follows, we drop time subscripts and use x (x') to denote a variable in the current (next) period. Consider the problem of a (θ, ω)-type owner, who must decide whether to trade her tree or to hold on to it. Let p^* denote the (constant price) equilibrium price, and let $V(\theta, \omega; p^*)$ denote the equilibrium value of (θ, ω)-type asset owner, which will depend on the equilibrium price. In particular, it must be that:

$$V(\theta, \omega; p^*) = \max \{p^*, E\{\omega x_\theta + \delta V(\theta', \omega'; p^*)|\theta, \omega\}\} \tag{1}$$

If the owner sells her tree today, then she gets the price p^*. If she does not, then she produces the output ωx_θ today plus she gets the expected discounted value from owning the tree tomorrow, where expectations are conditional on the holder knowing her type today.[13] Optimality requires

[12]The price of an asset is defined to be the maximal bid of the buyers for that asset, and it is common to all assets since all assets appear identical to the buyers at any time t.

[13]Although in our baseline model current productivity status is uninformative about the future, we include it in the agents' information sets as we will use this formulation in Section 4.

that the owner's equilibrium value be the maximum of the expected payoffs from either trading the asset or holding on to it.

Adverse selection can arise when the quality mix of traded assets depends on the asset price itself. Suppose that the highest offer made to an owner is p, which may or may not equal the equilibrium price p^*. The owner would accept such an offer if and only if it exceeded her equilibrium value V. In particular, the set of owner types who accept an offer p is:

$$\Gamma(p; p^*) = \{(\theta, \omega) : V(\theta, \omega; p^*) \leq p\}, \tag{2}$$

where we assume that the owner trades whenever she is indifferent.[14] Because V is different for owners of different quality assets, the set $\Gamma(p; p^*)$ depends on p. Note that, because today's offers for an asset are unobserved by the buyers of that asset in the future, the equilibrium value V depends on the equilibrium price p^* and not on offers made off-equilibrium.

Consider the problem of the buyers who are bidding for an owner's tree. Because at least two buyers compete for the owner's tree, in any equilibrium the buyers' expected profits must be zero. Therefore, in equilibrium, the asset price must satisfy:

$$p^* = E\{x_\theta + \delta V(\theta', \widetilde{\omega}'; p^*) | (\theta, \omega) \in \Gamma(p^*; p^*), \widetilde{\omega} = 1\}, \tag{3}$$

where tilde on $\widetilde{\omega}$ indicates that it is the productivity status of the buyer and not the owner. Rationality and belief consistency require that buyers understand the potential adverse selection problem and condition their expectations of asset quality on the set of owner types who accept their offers.[15] Since a buyer who gets the tree in the current period becomes an owner in the next, the equilibrium asset price depends on the expected value from being an asset owner.

Finally, buyer optimality requires that in equilibrium no buyer can profitably deviate by making an offer to the owner that strictly exceeds the equilibrium price, i.e., it must be that:

$$\widehat{p} \geq E\{x_\theta + \delta V(\theta', \widetilde{\omega}'; p^*) | (\theta, \omega) \in \Gamma(\widehat{p}; p^*), \widetilde{\omega} = 1\} \tag{4}$$

for all offers $\widehat{p}$ strictly greater than the equilibrium price p^*.

The following lemma puts further structure on the possible constant price equilibria by showing that in any such equilibria the owners with low quality assets always trade whereas

[14] This assumption is innocuous. Generically, the only type who can be indifferent to trade in equilibrium is the $(L, 1)$-type. In such a case, however, whether this type trades has no effect on asset prices or the efficiency of asset allocation.

[15] If $\Gamma(p) = \emptyset$, we set without loss of generality $p^* = E\{x_\theta + \delta V(\theta', \tilde{\omega}'; p^*) | \tilde{\omega} = 1\}$.

the owners with high quality assets who are unshocked never do.

Lemma 1 *Any constant price equilibrium is characterized by a value function V and asset price p^* satisfying (1)-(4). In any such equilibrium,*

$$V(L,\chi;p^*) = V(L,1;p^*) = p^* \leq V(H,\chi;p^*) < V(H,1;p^*).$$

Thus, the low quality assets always trade, whereas the high quality assets held by unshocked owners never trade.

First, because the flow payoff to a shocked owner is lower than to an unshocked owner, the values can be ranked according to the productivity status, $V(\theta,1;p^*) \geq V(\theta,\chi;p^*)$ for $\theta \in \{L,H\}$. Second, because buyers are unshocked and can guarantee themselves at least a low quality tree, their asset valuation is higher than that of low quality asset owners. Hence, it must be that all owners with low quality trees trade and $V(L,1;p^*) = p^*$. Finally, since all low quality trees trade, the unconditional expected value $E\{V(\theta,1;p^*)\}$ is an upper bound on the payoff that any buyer can attain by purchasing a tree. Thus, the buyers will never be able to attract the $(H,1)$-type owner to trade without making losses in expectation.

From Lemma 1, it follows that there can be two types of constant price equilibria, depending on whether the (H,χ)-type owner trades in equilibrium. We adopt the following definition in order to distinguish among them.

Definition 2 *We say that a constant price equilibrium features* **efficient trade** *if in it both high and low quality assets trade. Otherwise, if only low quality assets trade, we say that it features* **inefficient trade**.

In the efficient trade equilibrium, all shocked owners trade and the trees are efficiently allocated. Instead, in the inefficient trade equilibrium, the allocation is inefficient because the high types who are shocked stay out of the market.

Since we have narrowed down the set of constant price equilibria to two types, it will be convenient to index the equilibrium prices and allocations by the equilibrium type: e.g. we will denote asset the price in the efficient trade (ET) equilibrium by p^{*ET} and in the inefficient trade (IT) equilibrium by p^{*IT}.

The following theorem states our first main result by providing the conditions under which each type of equilibrium exists and under which they coexist. It also shows that when the two equilibria coexist, they can be ranked according to the level of asset prices, output and welfare.

Theorem 1 (Constant Price Equilibrium) *A constant price equilibrium exists. There exist thresholds* $0 < \bar{\pi}_{ET} < \bar{\pi}_{IT} < 1$ *on the proportion of high quality assets such that:*

1. *Efficient trade. There is at most one efficient trade equilibrium, which exists if and only if* $\pi \geq \bar{\pi}_{ET}$*,*

2. *Inefficient trade. There is at most one inefficient trade equilibrium, which exists if and only if* $\pi \leq \bar{\pi}_{IT}$*.*

Thus, the two equilibria coexist when $\pi \in [\bar{\pi}_{ET}, \bar{\pi}_{IT}]$*. Furthermore, the asset prices, output and welfare (in a Pareto sense) are higher in the efficient than in the inefficient trade equilibrium.*

In what follows, we show explicitly how to construct the constant price equilibria (and the corresponding prices, output and welfare), and we provide the intuition for when each type of equilibrium exists and why multiple equilibria arise in our setting. We begin with the construction of the efficient trade equilibrium.

3.2.1 Efficient trade equilibrium

In the efficient trade equilibrium, all owners except for the $(H, 1)$-type trade at price p^{*ET}. Therefore, their values are:

$$V(L, \chi; p^{*ET}) = V(L, 1; p^{*ET}) = V(H, \chi; p^{*ET}) = p^{*ET}, \tag{5}$$

whereas the value of the $(H, 1)$-type owner is:

$$V(H, 1; p^{*ET}) = x_H + \delta \left(\lambda p^{*ET} + (1 - \lambda) V(H, 1; p^{*ET}) \right), \tag{6}$$

i.e., this owner consumes the output this period, and in the next period she is either shocked (w.p. λ) in which case she sells her tree, or she remains unshocked (w.p. $1 - \lambda$) and holds on to it. The equilibrium price is:

$$p^{*ET} = \hat{\pi} V(H, 1; p^{*ET}) + (1 - \hat{\pi}) \left(x_L + \delta p^{*ET} \right). \tag{7}$$

where $\hat{\pi} \equiv \frac{\lambda \pi}{\lambda \pi + 1 - \pi}$ is the probability that the tree is of high quality, conditional on being sold.[16] A buyer who gets the tree today gets the same value as the $(H, 1)$-type if the tree turns out to

[16] In the stationary distribution of owner types, the probability that an owner is an (H, χ)-type is $\lambda \pi$ and the probability that she is a low type is $1 - \pi$.

be of high quality (w.p. $\hat{\pi}$), and he expects to consume its flow payoff and then resell the tree tomorrow if it turns out to be of low quality (w.p. $1-\hat{\pi}$). Importantly, the buyer understands that due to adverse selection the tree is of high quality with probability strictly smaller than π. We can combine (6) and (7) to get an analytical expression for the asset price:

$$p^{*ET} = (1-\delta)^{-1}\left(\hat{\pi}x_H + (1-\hat{\pi})x_L + \delta(1-\hat{\pi})(1-\lambda)\frac{\hat{\pi}(x_H - x_L)}{1-\delta(1-\hat{\pi})(1-\lambda)}\right). \tag{8}$$

In this equilibrium, all owners except the $(H,1)$-type trade their trees. As a result, the asset allocation is efficient and the output and weflare, as in our benchmark economy of Section 3.1, are respectively given by:

$$Y^{ET} = E\{x_\theta\} \quad \text{and} \quad W^{ET} = (1-\delta)^{-1}\cdot Y^{ET}.$$

For existence of such an equilibrium, we must rule out profitable deviations for the owners and the buyers. It is clear that there are no deviations for the buyers, since any such deviation would need to attract the $(H,1)$-type, which is impossible without the buyers making losses in expectation. For the owners, it is sufficient to check that the (H,χ)-type gets a lower payoff if she were to keep the asset for one period rather than sell it:

$$\chi x_H + \delta\left(\lambda p^{*ET} + (1-\lambda)V(H,1;p^{*ET})\right) \leq p^{*ET}. \tag{9}$$

Using the equations (5)-(7), we can re-express this condition as:

$$\hat{\pi}x_H + (1-\hat{\pi})\,x_L - \chi x_H \geq \delta\,(1-\hat{\pi}) \times \underbrace{\frac{(1-\hat{\pi})\,(1-\lambda)\,(x_H - x_L)}{1-\delta\,(1-\hat{\pi})\,(1-\lambda)}}_{\lambda V(H,\chi,p^{*ET})+(1-\lambda)V(H,1,p^{*ET})-p^{*ET}} \tag{10}$$

Thus, the efficient trade equilibrium exists when the (H,χ)-type's static gain from trading today, as given by $\hat{\pi}x_H + (1-\hat{\pi})\,x_L - \chi x_H$, exceeds the dynamic loss that she suffers by giving up her asset, as given by $\delta(1-\hat{\pi})\left(\lambda V(H,\chi;p^{*ET}) + (1-\lambda)V(H,1;p^{*ET}) - p^{*ET}\right)$. This dynamic loss arises because, whereas the owner knows that her asset will be of high quality tomorrow, the buyers believe that the asset will be of low quality with probability $1-\hat{\pi}$, in which case they value the asset at the resale price p^{*ET} that is strictly lower than the high type's expected value $\lambda V(H,\chi;p^{*ET}) + (1-\lambda)V(H,1;p^{*ET})$. The threshold $\bar{\pi}_{ET}$ in Theorem 1 is the value of π at which condition (10) holds with equality, which can be shown to be interior and unique.

Next, we construct the inefficient trade equilibrium.

3.2.2 Inefficient trade equilibrium

In the inefficient trade equilibrium, only owners of low quality assets trade. Therefore, their values are given by:

$$V(L,\chi;p^{*IT}) = V(L,1;p^{*IT}) = p^{*IT}, \tag{11}$$

whereas the values of the owners of the high quality assets are:

$$V(H,\omega;p^{*IT}) = \omega x_H + \delta\left(\lambda V(H,\chi;p^{*IT}) + (1-\lambda)V(H,1;p^{*IT})\right), \tag{12}$$

for $\omega \in \{\chi, 1\}$, as these owners both consume the output of their trees today and expect to do so in the future. The equilibrium price is:

$$p^{*IT} = (1-\delta)^{-1}x_L, \tag{13}$$

since buyers understand that due to adverse selection only low quality trees trade. It is clear then that p^{*IT} is lower than its counterpart in the efficient trade equilibrium.

In this equilibrium, not all gains from trade are realized. Since all (H,χ)-types (mass $\pi\lambda$ of owners) keep their trees and produce with productivity χ rather than 1, the output and welfare are respectively given by:

$$Y^{IT} = E\{x_\theta\} - \pi\lambda(1-\chi)x_H \quad \text{and} \quad W^{IT} = (1-\delta)^{-1}\cdot Y^{IT},$$

which are strictly lower than their counterparts in the efficient trade equilibrium.

For existence of such an equilibrium, we must rule out profitable deviations for the owners and the buyers. It is clear that there are no deviations for the owners, since the high types strictly prefer to keep their trees (recall that $\chi x_H \geq x_L$), whereas the low types prefer to trade. To rule out deviations for the buyers, it suffices to check that the buyers' profits are non-positive at any offer that attracts the (H,χ)-type:

$$\hat{\pi}V(H,1;p^{*IT}) + (1-\hat{\pi})\left(x_L + \delta p^{*IT}\right) \leq V(H,\chi;p^{*IT}), \tag{14}$$

where as before $\hat{\pi} \equiv \frac{\lambda\pi}{\lambda\pi+1-\pi}$ is the probability that the tree is of high quality, conditional on being sold. Using the equations (11)-(13), we can re-express this condition as:

$$\hat{\pi}x_H + (1-\hat{\pi})\,x_L - \chi x_H \leq \delta\,(1-\hat{\pi}) \times \underbrace{\frac{(1-\lambda+\lambda\chi)\,x_H - x_L}{1-\delta}}_{\lambda V(H,\chi,p^{*IT})+(1-\lambda)V(H,1,p^{*IT})-p^{*IT}}. \tag{15}$$

In contrast, the inefficient trade equilibrium exists whenever the (H,χ)-type's static gain from trading today is lower than the dynamic loss that she suffers by giving up her asset. The threshold $\bar{\pi}_{IT}$ in Theorem 1 is the value of π at which condition (15) holds with equality, which can also be shown to be interior and unique.

We have characterized the conditions for the existence of each type of equilibrium. But why do multiple equilibria arise in our setting? We turn to this question next.

3.2.3 Source of multiplicity

From the conditions (10) and (15) for the existence of each type of equilibrium, we can see that what is crucial for the existence of multiple equilibria is that the difference between the expected future value of the asset to the high type and the expected asset price is endogenous to the equilibrium itself:

$$\lambda V(H,\chi;p^*) + (1-\lambda)V(H,1;p^*) - p^* = \begin{cases} \frac{(1-\hat{\pi})(1-\lambda)(x_H-x_L)}{1-\delta(1-\hat{\pi})(1-\lambda)} & \text{if} \quad p^* = p^{*ET} \\ \frac{(1-\lambda+\lambda\chi)x_H - x_L}{1-\delta} & \text{if} \quad p^* = p^{*IT}. \end{cases}$$

Importantly, this difference is strictly smaller when the equilibrium features efficient than inefficient trade: when assets are allocated more efficiently, the asset pries are higher and therefore the gap between the high types' and the market's asset valuation is reduced.

To illustrate that dynamics are essential for our result, the next proposition shows that the parameter region where multiple equilibria arise expands when agents care more about the future, but vanishes as they become arbitrarily impatient.

Proposition 2 *The gap $\bar{\pi}_{IT} - \bar{\pi}_{ET}$ is increasing in δ, and it goes to zero as $\delta \to 0$. Thus, the equilibrium becomes generically unique as re-sale considerations vanish.*

Figure 1 illustrates this result graphically by plotting the thresholds $\bar{\pi}_{ET}$ and $\bar{\pi}_{IT}$ against the discount factor δ. As we can see, the region of multiplicity disappears as δ goes zero. The reason is that, in contrast to Akerlof (1970), our buyers are strategic which suffices to eliminate the possibility of multiple equilibria in a static setting (i.e. when $\delta = 0$).[17] Thus, the possibility of multiple equilibria in our setting hinges on dynamic strategic complementarities that arise because, when trading today, the agents care about the future market conditions.

[17] When $\delta = 0$, agents only care about the current flow payoffs of their assets. Thus, whether the equilibrium features efficient or inefficient trade depends only on how the flow payoff of the pool $\hat{\pi}x_H + (1-\hat{\pi})x_L$ compares to the flow payoff χx_H of the (H,χ)-type if the latter were to hold on to the asset, and therefore the two equilibria generically cannot coexist.

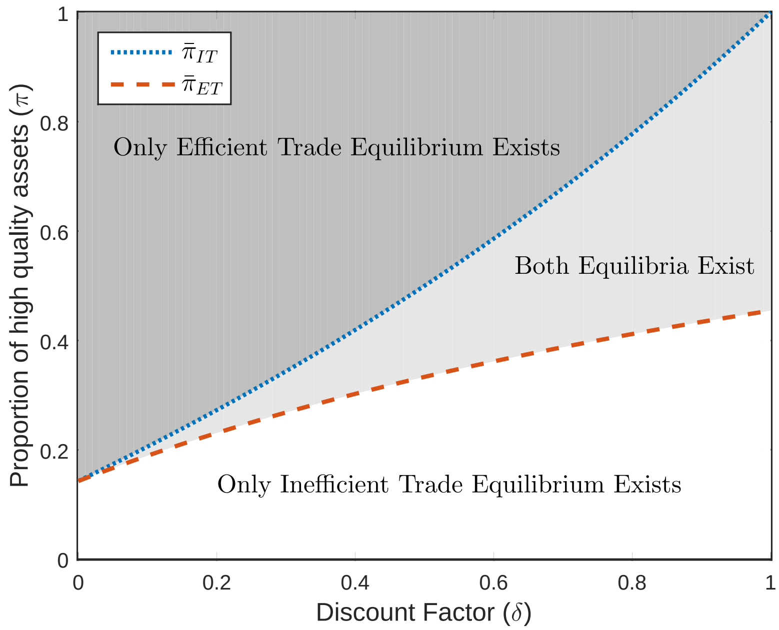

Figure 1: **Equilibrium Set and Role of Dynamics**. Unless stated otherwise, the parameters used are: $\delta = 0.9$, $\lambda = 0.6$, $\chi = 0.5$, $x_H = 1$ and $x_L = 0.45$.

Finally, although Proposition 2 emphasizes that dynamics, as captured by the agents' discount factor, are essential for our multiplicity result, we show in the Appendix that it is also crucial that there be some persistence in asset quality (which we have assumed to be perfect so far). Intuitively, if quality were independent over time, then the asset owner and the buyers would only disagree about the flow payoff of the asset today, which is independent of expected future market conditions. Indeed, we show that allowing for less than perfect quality persistence is essentially equivalent to lowering the discount factor δ.

Are there other types of equilibria?

Thus far, we considered constant price equilibria, in which asset prices and asset allocations do not change over time. But can there also exist equilibria in which asset prices and allocations change either deterministically or stochastically over time? We turn to this question next.

Before we proceed, it is worth noting that we have eliminated a set of constant price equilibria by our indifference-breaking assumption that an asset owner accepts an offer if she is indifferent. It turns out that if we relax this assumption, when the inefficient trade equilibrium exists, there is actually continuum of them, in which the $(L, 1)$-type trades with some probability $\sigma \in [0, 1]$. In all these equilibria, however, the asset prices and the efficiency of asset allocation coincide with those of the inefficient trade equilibrium in Section 3.2.2. Thus, eliminating these equilibria with our indifference-breaking assumption is essentially without loss of generality.

The next proposition shows that we cannot have deterministic cycles.[18] Intuitively, suppose that trade were efficient at t but inefficient at $t+1$ w.p.1. Then, the expected future market conditions must be worse at t than at $t+1$. But, this is not possible since, due to dynamic strategic complementarities, better expected market conditions tomorrow improve the actual market conditions today. By a similar reasoning, we can rule out equilibria in which inefficient trade is followed by efficient trade w.p.1.

Proposition 3 *An equilibrium with deterministic cycles generically does not exist.*

Nevertheless, as we show in the next section, our economy can feature stochastic equilibria, in which fluctuations in asset prices, output and welfare are driven by market sentiments – stochastic sunspots that coordinate agents' beliefs about future market conditions.

3.3 Sentiments and belief-driven volatility

In this section, we study the possibility of stochastic cycles. Consider a sunspot random variable z_t with some probability distribution, and assume that the realization of the random variable is public information. We define a sentiment equilibrium as follows:

Definition 3 *We say a PBE is a* **sentiment equilibrium** *with sunspot z_t if equilibrium asset price depends non-trivially on the realizations of the sunspot.*

Let us begin by considering the simple family of sunspots, where z_t takes values in the set $Z = \{B\text{ad}, G\text{ood}\}$ and follows a symmetric first-order Markov process with persistence parameter $\rho = \mathbb{P}(z_t = z | z_{t-1} = z) \in (0,1)$ for $z \in Z$. Furthermore, let us without loss of generality assume that if consistent with equilibrium, then in the Good state the agents coordinate on efficient trade, whereas in the Bad state they coordinate on inefficient trade.[19] Then, a sentiment equilibrium with sunspot z_t is characterized by a value function V and a

[18] When productivity shocks are correlated over time, then it is indeed possible to construct equilbria with determinstic cycles. See Section 4.

[19] It is straightforward to show that the results in Lemma 1 extend to sunspot equilibria, i.e. it must be that the low quality assets trade at all times whereas the high quality assets held by unshocked owners never do. Thus, the only role of the sunspot is to shift equilibrium play from one in which the (H,χ)-type owner trades to one where she does not.

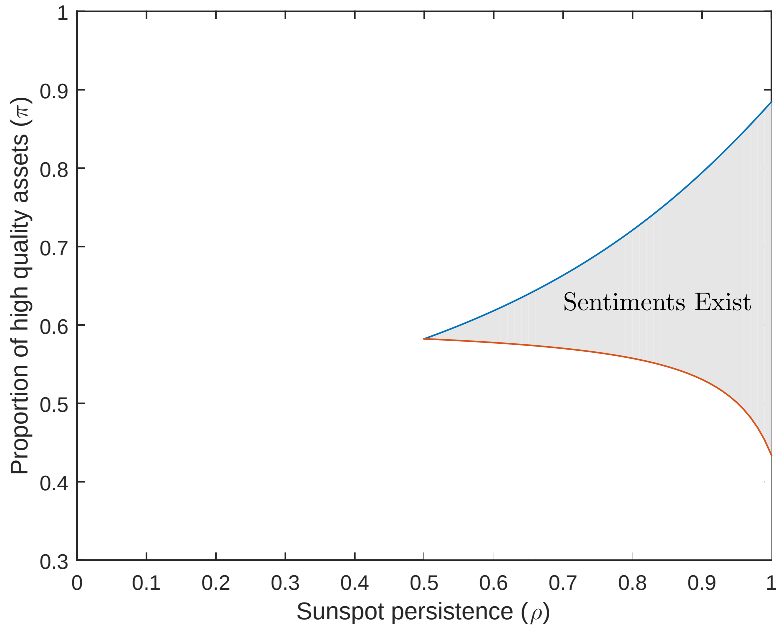

Figure 2: **Sentiment Equilibrium Existence Set**. The parameters used are: $\delta = 0.9$, $\lambda = 0.6$, $\chi = 0.5$, $x_H = 1$ and $x_L = 0.45$. The figure illustrates all the combination of the parameters of π and ρ for which a sentiment equilibrium with a binary-symmetric Markov sunspot process exists.

price function p^* satisfying:

$$V(\theta, \omega, z) = \max\{p^*(z), E\{\omega x_\theta + \delta V(\theta', \omega', z')|(\theta, \omega, z)\}\} \quad \forall(\theta, \omega, z), \tag{16}$$

$$\Gamma(p; z) = \{(\theta, \omega) : V(\theta, \omega, z) \leq p\} \quad \forall(p, z), \tag{17}$$

$$p^*(z) = \mathbb{E}\{x_\theta + \delta V(\theta', \widetilde{\omega}', z')|(\theta, \omega) \in \Gamma(p^*(z); z), z, \tilde{\omega} = 1\} \quad \forall z, \quad \text{and} \tag{18}$$

$$\widehat{p} \geq \mathbb{E}\{x_\theta + \delta V(\theta', \widetilde{\omega}', z')|(\theta, \omega) \in \Gamma(\widehat{p}; z), z, \widetilde{\omega} = 1\} \quad \forall \widehat{p} > p^*(z) \text{ and } \forall z. \tag{19}$$

These are simply the sunspot-contingent analogues of the equations (1)-(4) in Section 3.2. In fact, constant price equilibria are solutions to the above system under the restriction that asset prices satisfy $p^*(z) = p^*(z')$ for all $z, z' \in Z$.

The next proposition provides the necessary and sufficient conditions for the existence of sentiment equilibrium with the above described sunspot process.

Proposition 4 (Sentiments) *Consider a binary-symmetric first-order Markov process z_t with persistence ρ. A sentiment equilibrium with sunspot z_t exists if and only if $\pi \in (\bar{\pi}_{ET}, \bar{\pi}_{IT})$ and $\rho \geq \bar{\rho}$, where $\bar{\rho} < 1$ depends on the parameters of the model.*

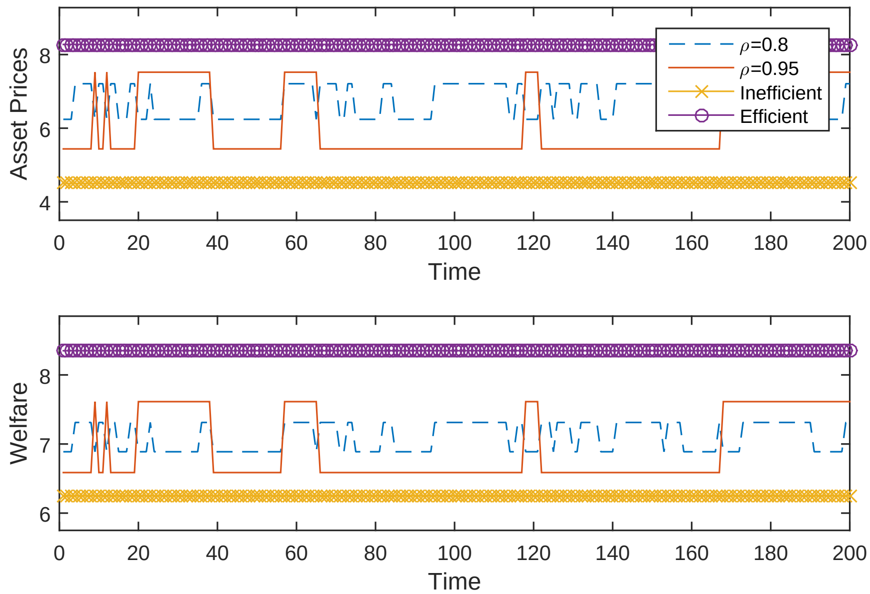

Figure 3: **Asset Prices and Welfare in a Sentiment Equilibrium**. The parameters used are: $\pi = 0.7$, $\delta = 0.9$, $\lambda = 0.6$, $\chi = 0.5$, $x_H = 1$ and $x_L = 0.45$. The solid orange line depicts a simulation with $\rho = 0.95$, whereas the dashed blue line depicts a simulation with $\rho = 0.8$.

This result emphasizes the role of inter-temporal coordination for the existence of multiple equilibria in our setting. The realization of the sunspot not only must signal to the agents what to play today, but it must also be informative about how the equilibrium play will proceed in the future. These two objectives are accomplished precisely by a sunspot process that is sufficiently persistent. Moreover, the amount of persistence that a sunspot needs in order to support sentiments depends on model parameters, as illustrated in Figure 2. Thus, in contrast to static coordination problems, sentiment equilibria cannot be driven by an arbitrary stochastic process, but rather the necessary properties of its evolution are disciplined by the parameters of the model. Though this insight was drawn by analyzing a simple family of sunspot processes, we extend it to more general Markov processes in Theorem 2 in the Appendix. In a nutshell, we show that, for a sentiment equilibrium with a given sunspot to exist, it is necessary and sufficient that $\pi \in (\bar{\pi}_{ET}, \bar{\pi}_{IT})$ and that the equilibrium play (i.e. efficient vs inefficient) generated by the sunspot is persistent enough. When there are more than two states, the relevant persistence is not that of the sunspot itself, but of the equilibrium play induced by that sunspot; as a result, the formal notion of persistence is more nuanced.

Figure 3 depicts the evolution of asset prices and welfare in the economy for a simulation of

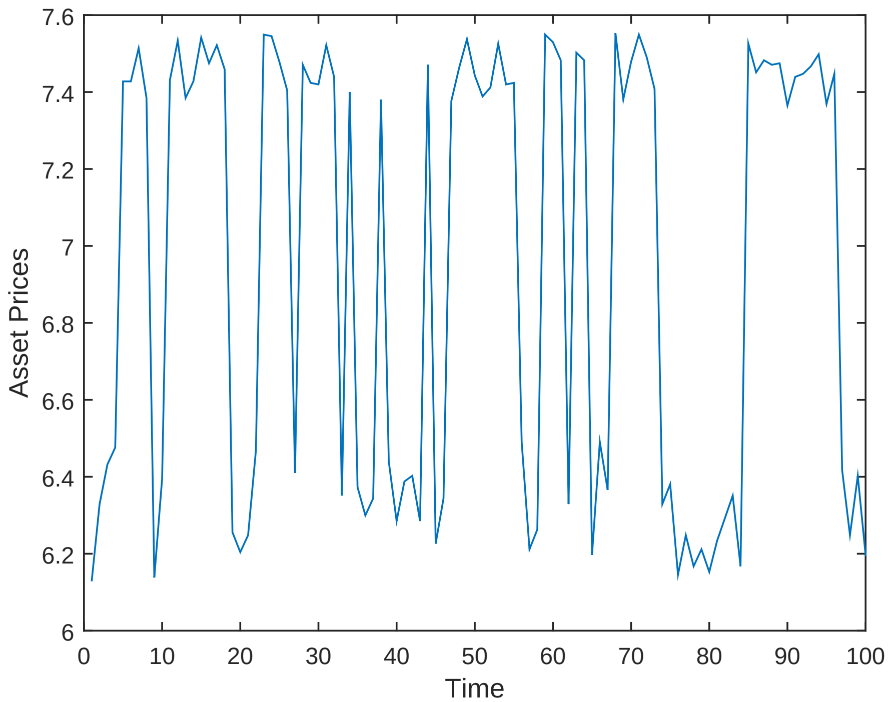

Figure 4: **Richer Sentiments**. The parameters used are: $\pi = 0.7$, $\delta = 0.9$, $\lambda = 0.6$, $\chi = 0.5$, $x_H = 1$ and $x_L = 0.45$.

the simple binary sunspot process. The solid line depicts the case where the sunspot is very persistent ($\rho = 0.95$), whereas the dashed line depics a less persistent process ($\rho = 0.8$). In both cases, the sunspot is sufficiently persistent so that a sentiment equilibrium with that sunspot exists. Note that the more persistent is the sunspot process, the less frequent are asset prices fluctuations and they are of larger size. This is intuitive because asset prices are forward looking and incorporate the expected future transitions of the economy. Importantly, since different states correspond to different asset allocation and output, the fluctuations in asset prices are mirrored by fluctuations in the agents' welfare. For comparison, we also depict the asset prices and welfare in the constant price equilibria, which provide bounds on asset prices and welfare that can be attained in any sentiment equilibrium.

Finally, we illustrate the dynamics of the economy with a richer sunspot process in Figure 4. It depicts the evolution of asset prices in a sentiment equilibrium with a sunspot that takes values in the set $Z = \{1, ..., 100\}$, such that efficient trade is played at time t if and only if $z_t \leq 50$, and the transition probabilities have the following property. Conditional on playing efficient trade at time t, the probability of transitioning to play inefficient trade at $t+1$ declines with z_t;

similarly, conditional on playing inefficient trade at time t, the probability of transitioning to play efficient trade at $t+1$ increases with z_t. The feature of this process that gives rise to richer dynamics is that different realizations of z_t not only change the equilibrium play at t (efficient vs inefficient) but also the agents' expectations about how long the economy will feature efficient vs. inefficient trade in the future. In this example, the persistence of equilibrium play is reflected in the fact that the periods in which the asset prices remain elevated or depressed last sufficiently long on average.

Sentiments as an amplification mechanism

Though the idea of sunspots may seem somewhat esoteric or abstract to some, as discussed in Manuelli and Peck (1992), "the early sunspot literature was motivated by the idea that small shocks to fundamentals are not very different from sunspots." They show that, in an overlapping generations endowment economy with money, small shocks to fundamentals can serve as the coordination device for different monetary equilibria. Furthermore, in the limit, as the undelying shocks have no direct effect on endowments, for every equilibrium of the pure sunspot economy with no shocks to endowments, there is a sequence of equilibria of the economy with risky endowments that converges to it. Our baseline economy can also be extended to allow for aggregate shocks to fundamentals which, even when small, can have large effects by serving as a coordination device for agents' expectations regarding the future market conditions. Of course, as we highlighted in Proposition 4 and Theorem 2, one needs to verify that these fundamentals satisfy the conditions required to coordinate expectations.

To illustrate this point, suppose that the output of the Lucas trees is also a function of some aggregate state $z_t \in \{G, B\}$, which follows a persistent and publicly observable Markov process. Concretely, consider the case where in state $z_t = G$ the payoff or output of a tree of quality θ in the hands of a holder with liquidity or productivity status ω is $(1+\varepsilon) \cdot \omega \cdot x_\theta$, whereas in state $z_t = B$ the output is $(1-\varepsilon) \cdot \omega \cdot x_\theta$, for some $\varepsilon \in [0, 1)$. Note that when $\varepsilon = 0$, we are back to our baseline setup, where the aggregate state is a pure sunspot and has no direct impact on any given tree's output, but can still serve as a coordination device. It is therefore straightforward to see that, for ε small but positive, we have the potential for an amplification of fundamental shocks. The equilibrium features amplification in the sense that the shocks have a negligible direct effect on the cashflow of any given tree. But, as shown in Section 3.3, these news can change market expectations about the future, change the pool of assets that are traded, and thus have a large impact on equilibrium asset prices, output and welfare.

4 Persistent productivity and history dependence

In our main analysis in Section 3, we assumed that the agents' productivity shocks or the gains from trade (as captured by ω's) were uncorrelated over time. This allowed us to illustrate clearly how the expectations about future market conditions can generate dynamic complementarities and lead to belief-driven volatility. When productivity shocks are correlated over time, in addition to future market conditions, past market conditions are also critical in determining the set of equilibria. We turn to this issue next.

Indeed, the work of Janssen and Karamychev (2002) and Janssen and Roy (2002) shows how, even absent re-trade considerations (but with entry of new sellers), past market conditions can be relevant in determining current market conditions. The main idea behind their result is the following. Suppose that the average value of the assets in the market in period t is below the reservation value of the shocked owner of high quality assets (the (H, χ)-type in our setting). Then, in the current period, only low types would trade and the price would be low. Now, in period $t + 1$, even with the entry of a new cohort of sellers, the average quality of the pool must better than at t, because all the high types that did not trade yesterday are still in the market today (assuming implicitly that productivity shocks are fully persistent). This leads to a gradual improvement of the pool over time until the average quality of assets becomes sufficiently high, so that the pooling price is high enough to attract high type sellers as well. Once this happens and all high types exit the market, we start over again with a low average pool quality. Thus, this mechanism gives rise to equilibria with deterministic cycles.[20]

So, how does the introduction of correlation in productivity shocks affect our results? As we show next, it turns out that if shocks are correlated positively over time, then (when it exists) the inefficient trade equilibrium may no longer be characterized by a stationary low price, but rather by deterministic cycles in which asset prices oscillate between low (in which case only the low types trade) and high (in which case also the high types trade); instead, the efficient trade equilibrium (when it exists) will not exhibit any cycles in our model. Thus, our main finding that for intermediate values of π there can be multiple equilibria continues to hold.

Formally, as before let $\lambda = \mathbb{P}(\omega_{j,t} = \chi)$ be the unconditional probability that owner j experiences a productivity shock at time t, but now assume that productivity shocks follow a first-order Markov process with $\rho^\omega = \mathbb{P}(\omega_{j,t} = \chi | \omega_{j,t-1} = \chi)$.[21] Then,

[20]Janssen and Karamychev (2002) also show that their model allows for multiple equilibria, but these equilibria (in a similar way to Akerlof (1970)) are driven by the assumption that agents are price-takers and public offers to all sellers are not allowed. If we modified their equilibrium definition to match the one in our paper, it would become unique.

[21]Our baseline formulation assumes that $\rho^\omega = \lambda$.

Proposition 5 *If $\rho^{\omega} > \lambda$, there exist thresholds $0 < \tilde{\pi}_{ET} < \tilde{\pi}_{ICT} < 1$ such that: (i) efficient constant price equilibrium exist when $\pi \geq \tilde{\pi}_{ET}$, (ii) inefficient constant price or cyclical equilibrium exists when $\pi < \tilde{\pi}_{ICT}$, and (iii) multiple equilibria exist when $\pi \in (\tilde{\pi}_{ET}, \tilde{\pi}_{ICT})$.*

It is worth remarking that when productivity shocks are correlated positively over time, an improvement in past market conditions worsens current market conditions. That is, the more efficient was trade yesterday, the less efficient it will be today, and vice versa. This is clearly different from the role of the re-sale considerations we emphasized in Section 3, where expectations of improved market conditions in the future lead to more efficient trade today, and vice versa.

Finally, things change if productivity shocks are correlated negatively over time, a case that to our knowledge has not been previously studied. Though it may seem less natural at first, negative correlation may arise if gains from trade arise because the asset owner occasionally encounters some other investment opportunity, such as an opportunity to participate in a public procurement auction. It is natural that if such an auction takes place at time t, then it is less likely to take place at $t + 1$, thus generating a negative correlation in gains from trade. We next illustrate that, even in the absence of re-sale considerations, negatively correlated shocks might lead to multiple equilibria.

Proposition 6 *If $\rho^{\omega} < \lambda$, then $\lim_{\delta \to 0} \tilde{\pi}_{ET} < \lim_{\delta \to 0} \tilde{\pi}_{IT}$. Thus, multiple equilibria can exist even as re-sale considerations vanish.*

Intuitively, if trade were more efficient at t, more of the assets would be held by agents who were unshocked at t. But, due to negative correlation in shocks, these are the agents who are more likely to be shocked at $t + 1$, increasing the gains from trade at that date. As a result, negative correlation introduces complementarities between past and current market conditions, which can lead to equilibrium multiplicity even when agents do not care much about future market conditions.

5 Conclusions

We have presented a parsimonious model, which illustrates that when valuing assets we cannot separate sentiments, liquidity and fundamentals. Even in the absence of any changes to underlying asset fundamentals and with asset prices always corresponding to properly discounted cash flows, it is possible to generate volatility in asset prices and market liquidity as a result of changes in agents' expectations about future market conditions.

From a macroeconomic point of view, our model also illustrates that sentiments can generate substantial fluctuations in aggregate (or sectoral) output. Even though measured TFP may fluctuate, it is not the cause of output volatility; both are instead driven by shocks to expectations. These expectations could in turn be driven by pure sunspots or be connected to exogenous changes to fundamentals. In the latter case, sentiments would serve as an amplification mechanism. Thus, an outside observer, who cannot directly measure sentiments, may overestimate the effect of such fundamental shocks. One must therefore be careful when estimating and interpreting measures of productivity shocks.

Finally, we purposefully kept our formulation simple in order to clearly identify the key economic forces at play. Yet, a natural next step is to embed our framework into a richer workhorse macroeconomic model to investigate additional implications of our mechanism and explore its quantitative significance. We leave this avenue for future research.

References

Akerlof, G. (1970). The Market for Lemons: Quality Uncertainty and the Market Mechanism. *Quarterly Journal of Economics 84*, 488–500.

Angeletos, G.-M. and J. La'O (2013). Sentiments. *Econometrica 81*(2), 739–779.

Benhabib, J., F. Dongz, and P. Wangx (2014). Adverse selection and self-ful lling business cycles.

Benhabib, J., P. Wang, and Y. Wen (2015). Sentiments and aggregate demand fluctuations. *Econometrica 83*(2), 549–585.

Bigio, S. (2015). Endogenous liquidity and the business cycle. *The American Economic Review 105*(6), 1883–1927.

Chari, V. V., A. Shourideh, and A. Zetlin-Jones (2010). Adverse selection, reputation and sudden collapses in secondary loan markets. Technical report, National Bureau of Economic Research.

Chiu, J. and T. V. Koeppl (2011).

Daley, B. and B. Green (2016). An information-based theory of time-varying liquidity. *Journal of Finance 71*(2), 809–870.

Donaldson, J. R. and G. Piacentino (2017). Money runs.

Dong, F., J. Miao, and P. Wang (2017). Asset bubbles and monetary policy. Technical report, mimeo, Boston University.

Eisfeldt, A. L. (2004). Endogenous liquidity in asset markets. *The Journal of Finance 59*(1), 1–30.

Fuchs, W., B. Green, and D. Papanikolaou (2016). Adverse selection, slow-moving capital, and misallocation. *Journal of Financial Economics 120*(2), 286–308.

Gorton, G. and G. Ordoñez (2014). Collateral crises. *The American Economic Review 104*(2), 343–378.

Gorton, G. and G. Ordoñez (2016). Good booms, bad booms. Technical report, National Bureau of Economic Research.

Guerrieri, V. and R. Shimer (2014). Dynamic adverse selection: A theory of illiquidity, fire sales, and flight to quality. *The American Economic Review 104*(7), 1875–1908.

Harrison, J. M. and D. M. Kreps (1978). Speculative investor behavior in a stock market with heterogeneous expectations. *The Quarterly Journal of Economics 92*(2), 323–336.

Hassan, T. A. and T. M. Mertens (2011). Market sentiment: A tragedy of the commons. *The American Economic Review 101*(3), 402–405.

Janssen, M. C. and V. A. Karamychev (2002). Cycles and multiple equilibria in the market for durable lemons. *Economic Theory 20*(3), 579–601.

Janssen, M. C. and S. Roy (2002). Dynamic trading in a durable good market with asymmetric information. *International Economic Review 43*(1), 257–282.

Kurlat, P. (2013). Lemons markets and the transmission of aggregate shocks. *The American Economic Review 103*(4), 1463–1489.

Lagos, R. and S. Zhang (2015). Monetary exchange in over-the-counter markets: A theory of speculative bubbles, the fed model, and self-fulfilling liquidity crises. Technical report, National Bureau of Economic Research.

Lagos, R. and S. Zhang (2016). Turnover liquidity and the transmission of monetary policy.

LeRoy, S. F. (2004). Rational exuberance. *Journal of Economic literature 42*(3), 783–804.

Lorenzoni, G. (2009). A theory of demand shocks. *The American economic review 99*(5), 2050–2084.

Mäkinen, T. and F. Palazzo (2017). The double bind of asymmetric information in over-the-counter markets.

Malherbe, F. (2014). Self-fulfilling liquidity dry-ups. *The Journal of Finance 69*(2), 947–970.

Manuelli, R. and J. Peck (1992). Sunspot-like effects of random endowments. *Journal of Economic Dynamics and Control 16*(2), 193–206.

Martin, A. (2005). Endogenous credit cycles.

Martin, A. and J. Ventura (2012). Economic growth with bubbles. *The American Economic Review 102*(6), 3033–3058.

Mas-Colell, A., M. D. Whinston, J. R. Green, et al. (1995). *Microeconomic theory*, Volume 1. Oxford university press New York.

Maurin, V. (2016). Liquidity fluctuations in over the counter markets. In *2016 Meeting Papers*, Number 218. Society for Economic Dynamics.

Plantin, G. (2009). Learning by holding and liquidity. *The Review of Economic Studies 76*(1), 395–412.

Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of political economy 66*(6), 467–482.

Santos, M. S. and M. Woodford (1997). Rational asset pricing bubbles. *Econometrica: Journal of the Econometric Society*, 19–57.

Shiller, R. J. (2005). Definition of irrational exuberance. *Irrational Exuberance*.

Tirole, J. (1985). Asset bubbles and overlapping generations. *Econometrica: Journal of the Econometric Society*, 1071–1100.

Weil, P. (1987). Confidence and the real value of money in an overlapping generations economy. *The Quarterly Journal of Economics 102*(1), 1–22.

Wilson, C. (1980). The nature of equilibrium in markets with adverse selection. *The Bell Journal of Economics*, 108–130.

Appendix A - Proofs for Section 3

Proof of Proposition 1. See text. ■

Proof of Lemma 1. Note that the equilibrium price satisfies:

$$p^* = E\{x_\theta + \delta V(\theta', \widetilde{\omega}'; p^*) | (\theta, \omega) \in \Gamma(p^*; p^*), \widetilde{\omega} = 1\} \geq E\{x_\theta + \delta V(\theta', \widetilde{\omega}'; p^*) | \theta = L, \widetilde{\omega} = 1\} \quad (20)$$

where the right-hand side is equal to the value of the $(L, 1)$-type if she were to hold on to the asset for a period. Thus, it must be that the $(L, 1)$-type always trades and her value is $V(L, 1; p^*) = p^*$. On the other hand, the (L, χ)-type has a weakly lower value than the $(L, 1)$-type since the quality of her asset is the same, but her flow payoff is lower. Hence, in equilibrium she must also trade and her value is $V(L, \chi; p^*) = p^*$. Finally, $V(H, \chi; p^*) \geq p^*$ holds trivially since the holder always has the option to trade at price p^*, and $V(H, 1; p^*) > p^*$ follows from the fact that, since low types always trade, it must be that:

$$p^* = E\{x_\theta + \delta V(\theta', \widetilde{\omega}'; p^*) | (\theta, \omega) \in \Gamma(p^*; p^*), \widetilde{\omega} = 1\} < V(H, 1; p^*), \quad (21)$$

which implies that buyers cannot attract the $(H, 1)$-type without making losses in expectation. Thus, indeed, all low quality asset trade at all times, but the high quality assets helds by unshocked owners never do. ■

Proof of Theorem 1. That there can at most be two types of constant price equilibria follows from Lemma 1, which shows that there are only two possibilities depending on whether the (H, χ)-type trades or not.

Efficient trade. The equations (5), (6), and (7) characterize the equilibrium owner values and asset price in candidate efficient trade equilibria. Since this system is linear, if an efficient trade equilibrium exists, it is unique. Moreover, this equilibrium exists if and only if inequality (9) is satisfied. Combining (5) - (9), we have that the efficient trade equilibrium exists if and only if:

$$(\chi x_H - \hat{\pi} x_H - (1 - \hat{\pi}) x_L) + \delta (1 - \hat{\pi}) \frac{(1 - \lambda)(1 - \hat{\pi})(x_H - x_L)}{1 - \delta (1 - \hat{\pi})(1 - \lambda)} \leq 0, \quad (22)$$

where $\hat{\pi} \equiv \frac{\lambda \pi}{\lambda \pi + 1 - \pi}$. Note that the left-hand side is strictly decreasing in π, positive at $\pi = 0$ and negative at $\pi = 1$. Hence, the threshold $\bar{\pi}_{ET} \in (0, 1)$ exists, is unique, and the efficient trade equilibrium exists if and only if $\pi \geq \bar{\pi}_{ET}$.

Inefficient trade. The equations (11), (12), and (13) characterize the equilibrium owner values and asset price in candidate inefficient trade equilibria. Since this is a system of linear equations, if an inefficient trade equilibrium exists, it is unique. Moreover, this equilibrium exists if and only if inequality (14) is satisfied. Combining (11) - (14), we have that the inefficient trade equilibrium exists if and only if:

$$0 \leq (\chi x_H - \hat{\pi} x_H - (1-\hat{\pi}) x_L) + \delta (1-\hat{\pi}) \frac{(1-\lambda+\lambda\chi) x_H - x_L}{1-\delta}, \tag{23}$$

where $\hat{\pi} \equiv \frac{\lambda\pi}{\lambda\pi+1-\pi}$. Note that the right-hand side is strictly decreasing in π, positive when $\pi = 0$ and negative when $\pi = 1$. Hence, the threshold $\bar{\pi}_{IT} \in (0,1)$ exists, is unique, and the inefficient trade equilibrium exists if and only if $\pi \leq \bar{\pi}_{IT}$.

Existence and Multiplicity. Next, we show that $\bar{\pi}_{ET} < \bar{\pi}_{IT}$, which will establish that an equilibrium exists and that the two equilibria coexist whenever $\pi \in (\bar{\pi}_{ET}, \bar{\pi}_{IT})$. From (22) and (23), we have that $\bar{\pi}_{ET} < \bar{\pi}_{IT}$ if and only if:

$$\frac{(1-\lambda)(1-\hat{\pi})(x_H - x_L)}{1-\delta(1-\hat{\pi})(1-\lambda)}\Big|_{\pi=\bar{\pi}_{ET}} < \frac{(1-\lambda+\lambda\chi) x_H - x_L}{1-\delta}, \tag{24}$$

which holds because, for any $\pi < 1$, we have:

$$\begin{aligned} \frac{(1-\lambda)(1-\hat{\pi})(x_H - x_L)}{1-\delta(1-\hat{\pi})(1-\lambda)} &\leq \frac{(1-\lambda)(x_H - x_L)}{1-\delta(1-\lambda)} \\ &< \frac{(1-\lambda)(x_H - x_L) + \lambda(\chi x_H - x_L)}{1-\delta} \\ &= \frac{(1-\lambda+\lambda\chi) x_H - x_L}{1-\delta}, \end{aligned}$$

where we used our parametric assumption that $\chi x_H \geq x_L$.

Finally, as shown in text, the asset prices, output and welfare are strictly higher in the efficient trade than in the inefficient trade equilibrium. That the two equilibria are Pareto ranked follows from the fact that asset prices are higher in the efficient than in the efficient trade equilibrium and by revealed preference of the high type to trade in the former. ■

Proof of Proposition 2. Consider the expressions defining the thesholds $\bar{\pi}_{IT}$ and $\bar{\pi}_{ET}$:

$$(\chi x_H - \hat{\pi} x_H - (1-\hat{\pi}) x_L) + \delta (1-\hat{\pi}) \frac{(1-\lambda+\lambda\chi) x_H - x_L}{1-\delta}\Big|_{\pi=\bar{\pi}_{IT}} = 0, \tag{25}$$

and

$$(\chi x_H - \hat{\pi} x_H - (1-\hat{\pi}) x_L) + \delta (1-\hat{\pi}) \frac{(1-\lambda)(1-\hat{\pi})(x_H - x_L)}{1-\delta(1-\hat{\pi})(1-\lambda)}\Big|_{\pi=\bar{\pi}_{ET}} = 0, \tag{26}$$

where in both cases the left-hand side is decreasing in π.

First, note that $\lim_{\delta\to 0}\bar{\pi}_{IT} = \lim_{\delta\to 0}\bar{\pi}_{ET} = \frac{\frac{\chi x_H - x_L}{x_H - x_L}}{\frac{\chi x_H - x_L}{x_H - x_L} + \left(1 - \frac{\chi x_H - x_L}{x_H - x_L}\right)\cdot\lambda}$, and thus the equilibrium becomes generically unique as $\delta \to 0$.

Second, note that (i) thresholds $\bar{\pi}_{IT}$ and $\bar{\pi}_{ET}$ coincide as $\delta \to 0$, (ii) $\frac{(1-\lambda)(1-\hat{\pi})(x_H - x_L)}{1-\delta(1-\hat{\pi})(1-\lambda)}|_{\pi=\bar{\pi}_{ET}} < \frac{(1-\lambda+\lambda\chi)x_H - x_L}{1-\delta}$ (see proof of Theorem 1), and (iii) $\frac{(1-\lambda)(1-\hat{\pi})(x_H - x_L)}{1-\delta(1-\hat{\pi})(1-\lambda)}$ is decreasing in π. Therefore, $\bar{\pi}_{IT}$ is increasing faster in δ than $\bar{\pi}_{ET}$, and so the difference $\bar{\pi}_{IT} - \bar{\pi}_{ET}$ is increasing in δ. ■

Proof of Proposition 3. Let p_t^* and $V_t(\theta, \omega, p_t^*)$ denote the equilibrium asset price and value of owner (θ, ω). It is again straightforward to show that at any time t all low types trade w.p.1 whereas the unshocked high types do not trade. Now, suppose that there exists a deterministic equilibrium such that trade is efficient at t (i.e. (H,χ)-type trades) and inefficient at $t+1$ (i.e. (H,χ)-type does not trade). Then, such an equilibrium must satisfy the following conditions:

(1) The equilibrium price and values at time t are:

$$V_t(H,1;p_t^*) = x_H + \delta\left(\lambda V_{t+1}\left(H,\chi;p_{t+1}^*\right) + (1-\lambda)V_{t+1}\left(H,1;p_{t+1}^*\right)\right),$$

$$p_t^* = \hat{\pi}V_t(H,1;p_t^*) + (1-\hat{\pi})\left(x_L + \delta p_{t+1}^*\right),$$

and it suffices to check that the (H,χ)-type does not want to deviate:

$$\chi x_H + \delta\left(\lambda V_{t+1}\left(H,\chi;p_{t+1}^*\right) + (1-\lambda)V_{t+1}\left(H,1;p_{t+1}^*\right)\right) \leq \hat{\pi}V_t(H,1;p_t^*) + (1-\hat{\pi})\left(x_L + \delta p_{t+1}^*\right),$$

$$\Longleftrightarrow$$

$$\hat{\pi}x_H + (1-\hat{\pi})x_L - \chi x_H \geq \delta(1-\hat{\pi})\left(\lambda V_{t+1}\left(H,\chi;p_{t+1}^*\right) + (1-\lambda)V_{t+1}\left(H,1;p_{t+1}^*\right) - p_{t+1}^*\right). \quad (27)$$

(2) The equilibrium price and values at time $t+1$ are:

$$V_{t+1}\left(H,\omega;p_{t+1}^*\right) = \omega x_H + \delta\left(\lambda V_{t+2}\left(H,\chi;p_{t+2}^*\right) + (1-\lambda)V_{t+2}\left(H,\chi;p_{t+2}^*\right)\right) \text{ for } \omega \in (\chi, 1),$$

$$p_{t+1}^* = x_L + \delta p_{t+2}^*,$$

and it suffices to check that buyers do not want to deviate:

$$\chi x_H + \delta\left(\lambda V_{t+2}\left(H,\chi;p_{t+2}^*\right) + (1-\lambda)V_{t+2}\left(H,1;p_{t+2}^*\right)\right) \geq \hat{\pi}V_{t+1}\left(H,1;p_{t+1}^*\right) + (1-\hat{\pi})\left(x_L + \delta p_{t+2}^*\right),$$

$$\Longleftrightarrow$$

$$\hat{\pi}x_H + (1-\hat{\pi})x_L - \chi x_H \leq \delta(1-\hat{\pi})\left(\lambda V_{t+2}\left(H,\chi;;p_{t+2}^*\right) + (1-\lambda)V_{t+2}\left(H,\chi;;p_{t+2}^*\right) - p_{t+2}^*\right). \quad (28)$$

But, because trade is inefficient at $t+1$, it must be that $\lambda V_{t+1}\left(H,\chi;p_{t+1}^{*}\right)+(1-\lambda)\,V_{t+1}\left(H,1;p_{t+1}^{*}\right)-p_{t+1}^{*}\geq\lambda V_{t+2}\left(H,\chi;;p_{t+2}^{*}\right)+(1-\lambda)\,V_{t+2}\left(H,\chi;;p_{t+2}^{*}\right)-p_{t+2}^{*}$, with strict inequality if trade is efficient at any time after $t+1$. Therefore, generically the inequalities (27) and (28) cannot be satisfied at the same time, a contradiction. By analogous reasoning we can rule out inefficient trade at t followed by efficient trade at $t+1$. ∎

Consider a first-order Markov process z_t that takes values in some finite set $Z=\{z_1,...,z_N\}$ with $N\geq 2$ elements and transition matrix Q; we assume that the process does not have any absorbing states or trivial states that are never visited. Note that our results generalize to higher order Markov processes since these can be transformed into first-order ones. Let Z^* denote the subset of states in which the agents coordinate on playing efficient trade. Let I_Z denote the $N\times N$ identity matrix and 1_Z be the $N\times 1$ vector of ones. Also, let I_{Z^*} be the matrix which coincides with I_Z except that it has zeros on the diagonal entries that correspond to the states $z\notin Z^*$. Finally, define $\Delta_{(Q,Z,Z^*)}(z)\equiv\lambda V(H,\chi,z)+(1-\lambda)V(H,1,z)-p^*(z)$, which can be expressed in terms of primitives as:

$$\Delta_{(Q,Z,Z^*)}=M_{(Q,Z,Z^*)}\cdot v_{(Z,Z^*)}$$

where

$$M_{(Q,Z,Z^*)}=\left[I_Z-\left(I_{Z^*}\cdot(1-\lambda)\cdot(1-\hat{\pi})+I_Z-I_{Z^*}\right)\cdot\delta\cdot Q\right]^{-1},$$

and

$$v_{(Z,Z^*)}=I_{Z^*}\cdot(1-\lambda)\cdot(1-\hat{\pi})\cdot(x_H-x_L)\cdot 1_Z+(I_Z-I_{Z^*})\cdot((\lambda\chi+1-\lambda)\cdot x_H-x_L)\cdot 1_Z.$$

The following theorem provides the necessary and sufficient conditions on the transition matrix Q and the parameters of the model for the existence of a sentiment equilibrium. The result stated in Proposition 4 is a special case.

Theorem 2 (Sentiments) *Consider a first-order Markov process z_t with values in some finite set Z with at least two elements and with transition matrix Q. A sentiment equilibrium with sunspot z_t exists if and only if there is a non-empty set $Z^*\subsetneq Z$ such that:*

1. *There are no profitable deviations for the owners, which holds if and only if*

$$\hat{\pi}x_H+(1-\hat{\pi})\,x_L-\chi x_H\geq\delta\cdot(1-\hat{\pi})\cdot\max_{z_j\in Z^*}\left(Q\cdot\Delta_{(Q,Z,Z^*)}\right)(j)\,.$$

2. There are no profitable deviations for the buyers, which holds if and only if:

$$\hat{\pi}x_H + (1-\hat{\pi})\, x_L - \chi x_H \leq \delta \cdot (1-\hat{\pi}) \cdot \min_{z_j \notin Z^*} \left(Q \cdot \Delta_{(Q,Z,Z^*)}\right)(j)\,.$$

In particular, a sentiment equilibrium exists only if $\pi \in (\bar{\pi}_{ET}, \bar{\pi}_{IT})$.

Proof of Theorem 2. Consider a sunspot process as stated in the theorem. For $\omega \in \{\chi, 1\}$, let $V(H,\omega) \equiv (V(H,\omega,z))_{z\in Z}$ denote the vector of high type values and $p^* \equiv (p^*(z))_{z\in Z}$ denote the vector of equilibrium prices across states. A sentiment equilibrium with that sunspot exists if and only if there exists a non-empty $Z^* \subsetneq Z$ such that:

1. There are no profitable deviations for the owners if and only if:

$$I_{Z^*} \cdot (\chi x_H \cdot 1_Z + \delta Q (\lambda V(H,\chi) + (1-\lambda) V(H,1))) \leq I_{Z^*} \cdot p^*,$$

   i.e. the (H,χ)-type owner would rather trade than keep her asset when the state is in Z^*.

2. There are no profitable deviations for the buyers if and only if:

$$(I_Z - I_{Z^*}) \cdot (\hat{\pi} V(H,1) + (1-\hat{\pi})(x_L \cdot 1_Z + \delta Q p^*)) \leq (I_Z - I_{Z^*}) \cdot V(H,\chi)\,,$$

   i.e. the buyers cannot make positive profits by attracting the (H,χ)-type to trade when the state is outside Z^*.

The equilibrium values and prices can be computed using the equations (16) and (18):

$$V(H,\chi) = I_{Z^*} \cdot p^* + (I_Z - I_{Z^*}) \cdot (\chi x_H \cdot 1_Z + \delta Q (\lambda V(H,\chi) + (1-\lambda) V(H,1)))\,,$$

$$V(H,1) = x_H \cdot 1_Z + \delta Q (\lambda V(H,\chi) + (1-\lambda) V(H,1))\,,$$

$$p^* = I_{Z^*} \cdot (\hat{\pi} V(H,1) + (1-\hat{\pi})(x_L \cdot 1_Z + \delta Q p^*)) + (I_Z - I_{Z^*}) \cdot (x_L \cdot 1_Z + \delta Q p^*)\,.$$

Define $\Delta_{(Q,Z,Z^*)} \equiv \lambda V(H,\chi) + (1-\lambda) V(H,1) - p^*$, then using the equilibrium equations, we can express the first existence condition as follows:

$$I_{Z^*} \cdot \delta (1-\hat{\pi})\, Q \Delta_{(Q,Z,Z^*)} \leq I_{Z^*} \cdot (\hat{\pi} x_H + (1-\hat{\pi})\, x_L - \chi x_H) \cdot 1_Z,$$

i.e. the elements of the vector $\delta(1-\hat{\pi})\, Q\Delta_{(Q,Z,Z^*)}$ correspoding to states $z \in Z^*$ must be lower than the same elements of the vector $(\hat{\pi} x_H + (1-\hat{\pi})\, x_L - \chi x_H) \cdot 1_Z$. The second existence condition can in turn be expressed as:

$$(I_Z - I_{Z^*}) \cdot \delta\,(1-\hat{\pi})\,Q\Delta_{(Q,Z,Z^*)} \geq (I_Z - I_{Z^*}) \cdot (\hat{\pi}x_H + (1-\hat{\pi})\,x_L - \chi x_H) \cdot 1_Z,$$

i.e. the elements of the vector $\delta\,(1-\hat{\pi})\,Q\Delta_{(Q,Z,Z^*)}$ correspoding to states $z \notin Z^*$ must be greater than the same elements of the vector $(\hat{\pi}x_H + (1-\hat{\pi})\,x_L - \chi x_H) \cdot 1_Z$. These are precisely the existence conditions stated in Theorem 2. Finally, we can solve for $\Delta_{(Q,Z,Z^*)}$ as:

$$\Delta_{(Q,Z,Z^*)} = M_{(Q,Z,Z^*)} \cdot v_{(Z,Z^*)}$$

where $M_{(Q,Z,Z^*)}$ and $v_{(Z,Z^*)}$ are given by:

$$M_{(Q,Z,Z^*)} = \left[I_Z - (I_{Z^*} \cdot (1-\lambda) \cdot (1-\hat{\pi}) + I_Z - I_{Z^*}) \cdot \delta \cdot Q\right]^{-1},$$

and

$$v_{(Z,Z^*)} = I_{Z^*} \cdot (1-\lambda) \cdot (1-\hat{\pi}) \cdot (x_H - x_L) \cdot 1_Z + (I_Z - I_{Z^*}) \cdot ((\lambda\chi + 1 - \lambda) \cdot x_H - x_L) \cdot 1_Z.$$

Next, because $(1-\lambda+\lambda\chi)\,x_H - x_L > (1-\lambda)\,(1-\hat{\pi})\,(x_H - x_L)$, we have that:

$$\min_{z_j \in Z^*} \left(Q\Delta_{(Q,Z,Z^*)}\right)(j) > (1-\delta\,(1-\lambda)\,(1-\hat{\pi}))^{-1} \cdot (1-\lambda)\,(1-\hat{\pi})\,(x_H - x_L)$$

and

$$\max_{z_j \notin Z^*} \left(Q\Delta_{(Q,Z,Z^*)}\right)(j) < (1-\delta)^{-1} \cdot ((1-\lambda+\lambda\chi)\,x_H - x_L)\,.$$

Hence, for a sentiment equilibrium to exist, it is necessary that $\pi \in (\bar{\pi}_{ET}, \bar{\pi}_{IT})$ (see the conditions (10) and (15)).

To establish Proposition 4, let $Z = \{G, B\}$, $Z^* = \{G\}$ and $Q = \begin{pmatrix} \rho & 1-\rho \\ 1-\rho & \rho \end{pmatrix}$. Then, the vector $\Delta_{(Q,Z,Z^*)}$ is given by:

$$\begin{bmatrix} \Delta_{(Q,Z,Z^*)}(1) \\ \Delta_{(Q,Z,Z^*)}(2) \end{bmatrix} = \begin{bmatrix} 1-\delta\,(1-\lambda)\,(1-\hat{\pi})\,\rho & -\delta\,(1-\lambda)\,(1-\hat{\pi})\,(1-\rho) \\ -\delta\,(1-\rho) & 1-\delta\rho \end{bmatrix}^{-1} \cdot \begin{bmatrix} (1-\lambda)(1-\hat{\pi})(x_H - x_L) \\ (\lambda\chi + 1 - \lambda)x_H - x_L \end{bmatrix},$$

and the existence conditions become:

$$\hat{\pi}x_H + (1-\hat{\pi})\,x_L - \chi x_H \geq \delta\,(1-\hat{\pi})\left(\rho\Delta_{(Q,Z,Z^*)}(1) + (1-\rho)\Delta_{(Q,Z,Z^*)}(2)\right),$$

and

$$\hat{\pi} x_H + (1 - \hat{\pi}) x_L - \chi x_H \leq \delta (1 - \hat{\pi}) \left((1-\rho)\Delta_{(Q,Z,Z^*)}(1) + \rho \Delta_{(Q,Z,Z^*)}(2)\right).$$

By inspection, it is clear that generically these conditions hold if and only if $\pi \in (\bar{\pi}_{ET}, \bar{\pi}_{IT})$ and ρ is sufficiently large. ■

Appendix B - Proofs for Section 4

Proof of Proposition 5. Using the same arguments as in the construction of the efficient trade equilibrium in Section 3.2, we can show that the efficient trade equilibrium exists if and only if:

$$\chi x_H - \hat{\pi} x_H - (1 - \hat{\pi}) x_L + \delta (1 - \hat{\rho}^\omega)(1 - \hat{\pi}) \cdot \frac{\left(\frac{1-\rho^\omega}{1-\hat{\rho}^\omega} - \hat{\pi}\right)(x_H - x_L)}{1 - \delta (1 - \hat{\rho}^\omega)(1 - \hat{\pi})} \leq 0, \tag{29}$$

where $\hat{\rho}^\omega = \frac{1-\rho^\omega}{1-\lambda}\lambda$ and $\hat{\pi} = \frac{\pi \hat{\rho}^\omega}{\pi \hat{\rho}^\omega + 1 - \pi}$. Note that this inequality becomes the same as (10) when $\rho^\omega \to \lambda$. The left-hand side is strictly decreasing in π, positive at $\pi = 0$ and negative at $\pi = 1$. Hence, the threshold $\tilde{\pi}_{ET} \in (0, 1)$ that sets this inequality to an equality exists, is unique, and the efficient trade equilibrium exists if and only if $\pi \geq \tilde{\pi}_{ET}$.

Analogously, we can show that the inefficient trade equilibrium exists if and only if:

$$0 \leq \chi x_H - \hat{\pi} x_H - (1 - \hat{\pi}) x_L + \delta (1 - \hat{\pi}) \frac{\left(\chi + \frac{1-\rho^\omega}{1-\delta(\rho^\omega - \hat{\rho}^\omega)}(1 - \chi)\right) x_H - x_L}{1 - \delta}, \tag{30}$$

where $\hat{\rho}^\omega = \frac{1-\rho^\omega}{1-\lambda}\lambda$ and $\hat{\pi} = \frac{\pi\lambda}{\pi\lambda + 1 - \pi}$. Note that this inequality becomes the same as (15) when $\rho^\omega \to \lambda$. The right-hand side is strictly decreasing in π, positive at $\pi = 0$ and negative at $\pi = 1$. Hence, the threshold $\tilde{\pi}_{IT} \in (0, 1)$ that sets this inequality to an equality exists, is unique, and the inefficient trade equilibrium exists if and only if $\pi \leq \tilde{\pi}_{IT}$.

Define threshold $\tilde{\pi}_{ITC}$ to be the value of π that sets inequality (30) to equality, but where $\hat{\pi} = \frac{\pi \hat{\rho}^\omega}{\pi \hat{\rho}^\omega + 1 - \pi}$. We will show shortly that a cyclical equilibrium of period $T > 1$ exists when $\pi \in (\tilde{\pi}_{IT}, \tilde{\pi}_{ITC})$, where the interval is non-empty since $\rho^\omega > \lambda$ implies $\tilde{\pi}_{IT} < \tilde{\pi}_{ITC}$. Next, we establish that $\tilde{\pi}_{ET} < \tilde{\pi}_{ITC}$, which proves our result that when $\pi \in (\tilde{\pi}_{ET}, \tilde{\pi}_{ITC})$, the efficient trade equilibrium coexists with either the inefficient or the cyclical trade equilibrium. But the latter inequality holds if and only if:

$$\frac{\left(1-\hat{\rho}^{\omega}\right)\left(\frac{1-\rho^{\omega}}{1-\hat{\rho}^{\omega}}-\hat{\pi}\right)\left(x_H-x_L\right)}{1-\delta\left(1-\hat{\rho}^{\omega}\right)\left(1-\hat{\pi}\right)}\Big|_{\pi=\tilde{\pi}_{ET}}<\frac{\left(\chi+\frac{1-\rho^{\omega}}{1-\delta\left(\rho^{\omega}-\hat{\rho}^{\omega}\right)}\left(1-\chi\right)\right)x_H-x_L}{1-\delta}, \tag{31}$$

which follows from the fact that for any $\pi<1$,

$$\begin{aligned}\frac{\left(1-\hat{\rho}^{\omega}\right)\left(\frac{1-\rho^{\omega}}{1-\hat{\rho}^{\omega}}-\hat{\pi}\right)\left(x_H-x_L\right)}{1-\delta\left(1-\hat{\rho}^{\omega}\right)\left(1-\hat{\pi}\right)}\Big|_{\pi=\pi_{ET}} &\leq \frac{\left(1-\rho^{\omega}\right)\left(x_H-x_L\right)}{1-\delta\left(1-\hat{\rho}^{\omega}\right)} \\ &< \frac{\left(1-\rho^{\omega}\right)\left(x_H-x_L\right)+\rho^{\omega}\left(\chi x_H-x_L\right)}{1-\delta} \\ &= \frac{\left(1-\rho^{\omega}+\rho^{\omega}\chi\right)x_H-x_L}{1-\delta} \\ &\leq \frac{\left(\chi+\frac{1-\rho^{\omega}}{1-\delta\left(\rho^{\omega}-\hat{\rho}^{\omega}\right)}\left(1-\chi\right)\right)x_H-x_L}{1-\delta},\end{aligned}$$

where we used that $\chi x_H \geq x_L$ and $\rho^{\omega}>\hat{\rho}^{\omega}$.

We now show that when $\pi\in\left(\tilde{\pi}_{IT},\tilde{\pi}_{ITC}\right)$, then there exists a cyclical equilibrium of length $T>1$. Thus, consider a candidate equilibrium with cycle length T. Let $\tau\in\{1,...,T\}$ denote the time that has passed since the (H,χ)-types traded the last time. The stationary distribution of pool quality is then given by:

$$\hat{\pi}_{\tau}=\frac{\pi\hat{\rho}^{\omega}\cdot\frac{1-\left(\rho^{\omega}-\hat{\rho}^{\omega}\right)^{\tau}}{1-\left(\rho^{\omega}-\hat{\rho}^{\omega}\right)}}{\pi\hat{\rho}^{\omega}\cdot\frac{1-\left(\rho^{\omega}-\hat{\rho}^{\omega}\right)^{\tau}}{1-\left(\rho^{\omega}-\hat{\rho}^{\omega}\right)}+1-\pi}, \tag{32}$$

where $\hat{\pi}_{\tau}$ is strictly increasing in τ with $\hat{\pi}_1=\frac{\pi\cdot\hat{\rho}^{\omega}}{\pi\cdot\hat{\rho}^{\omega}+1-\pi}<\frac{\pi\cdot\lambda}{\pi\cdot\lambda+1-\pi}=\lim_{\tau\to\infty}\hat{\pi}$.

Let V_{τ} and p_{τ} denote the equilibrium owner values and asset prices, as they depend on τ. Then,

$$p_{\tau}^{*}=\begin{cases}x_L+\delta\cdot p_{\tau+1}^{*} & \text{if } \tau<T\\ \hat{\pi}_T\cdot V_T\left(H,1\right)+\left(1-\hat{\pi}_T\right)\cdot\left(x_L+\delta\cdot p_1^{*}\right) & \text{if } \tau=T\end{cases}, \tag{33}$$

and the values are $V_{\tau}\left(L,\chi\right)=V_{\tau}\left(L,1\right)=p_{\tau}^{*}$,

$$V_{\tau}\left(H,1\right)=x_H+\delta\cdot\left(\hat{\rho}^{\omega}\cdot V_{\tau+1}\left(H,\chi\right)+\left(1-\hat{\rho}^{\omega}\right)\cdot V_{\tau+1}\left(H,1\right)\right), \tag{34}$$

$$V_{\tau}\left(H,\chi\right)=\begin{cases}\chi\cdot x_H+\delta\cdot\left(\rho^{\omega}\cdot V_{\tau+1}\left(H,\chi\right)+\left(1-\rho^{\omega}\right)\cdot V_{\tau+1}\left(H,1\right)\right) & \text{if } \tau<T\\ p_T^{*} & \text{if } \tau=T\end{cases}. \tag{35}$$

To show that such an equilibrium exists, we must check that neither the buyers nor the owners want to deviate, i.e., the buyers cannot attract the (H, χ)-type in periods $\tau \neq T$:

$$V_{\tau}(H,\chi) \geq \hat{\pi}_{\tau} \cdot V_{\tau}(H,1) + (1-\hat{\pi}_{\tau}) \cdot \left(x_L + \delta \cdot p^{*}_{\tau+1}\right), \tag{36}$$

and the (H, χ)-type prefers to trade rather than keep her asset in period T:

$$p^{*}_{T} \geq \chi \cdot x_H + \delta \cdot \left(\rho^{\omega} \cdot V_1(H,\chi) + (1-\rho^{\omega}) \cdot V_1(H,1)\right). \tag{37}$$

Notice that the efficient trade equilibrium is a special case of a cyclical equilibrium, in which the cycle legth is $T = 1$, whereas the inefficient trade equilibrium has cycle $T = \infty$. Therefore, when $\pi \in (\tilde{\pi}_{IT}, \tilde{\pi}_{ICT})$, neither $T = 1$ nor $T = \infty$ can be an equilibrium. In the former case, the (H, χ)-type owner wants to deviate and keep her asset. In the latter case, the buyers want to deviate and attract the (H, χ)-type to trade. Next, we show that there exists a $0 < T < \infty$ such that neither the buyers nor the owners want to deviate, thus establishing the result.

These no-deviation conditions can be expressed compactly as follows:

$$\hat{\pi}_{\tau} \cdot V_{\tau}(H,1) + (1-\hat{\pi}_{\tau}) \cdot \left(x_L + \delta \cdot p^{*}_{\tau+1}\right) \begin{cases} \leq \chi \cdot x_H + \delta \cdot \left(\rho^{\omega} \cdot V_{\tau+1}(H,\chi) + (1-\rho^{\omega}) \cdot V_{\tau+1}(H,1)\right) & \text{if } \tau < T \\ \geq \chi \cdot x_H + \delta \cdot \left(\rho^{\omega} \cdot V_1(H,\chi) + (1-\rho^{\omega}) \cdot V_1(H,1)\right) & \text{if } \tau = T \end{cases}. \tag{38}$$

In search of a contradiction, suppose that for all T:

$$\hat{\pi}_{\tau} \cdot V_{\tau}(H,1) + (1-\hat{\pi}_{\tau}) \cdot \left(x_L + \delta \cdot p^{*}_{\tau+1}\right) \leq \chi \cdot x_H + \delta \cdot \left(\rho^{\omega} \cdot V_{\tau+1}(H,\chi) + (1-\rho^{\omega}) \cdot V_{\tau+1}(H,1)\right). \tag{39}$$

Fix τ and note that $\lim_{T\to\infty} V_{\tau+1}(H,\omega) = V\left(H,\omega,p^{*IT}\right)$ and $\lim_{T\to\infty} p^{*}_{\tau+1} = p^{IT}$. But then as T grows large, (39) becomes the same as (30), which defines threshold $\tilde{\pi}_{IT}$, except that the pool quality $\hat{\pi} = \frac{\pi\lambda}{\pi\lambda+1-\pi}$ is replaced with $\hat{\pi}_{\tau}$. Since $\pi > \tilde{\pi}_{IT}$, (30) is violated. Because $\hat{\pi}_{\tau} \to \hat{\pi}$, there exists a finite τ such that (39) is violated as well. ■

Proof of Proposition 6. If $\rho^{\omega} < \lambda$, using the definitions of thresholds $\tilde{\pi}^{ET}$ and $\tilde{\pi}^{IT}$, we have

$$\lim_{\delta\to 0} \tilde{\pi}^{ET} = \frac{\frac{\chi x_H - x_L}{x_H - x_L}}{\frac{\chi x_H - x_L}{x_H - x_L} + \left(1 - \frac{\chi x_H - x_L}{x_H - x_L}\right) \cdot \hat{\rho}^{\omega}} < \frac{\frac{\chi x_H - x_L}{x_H - x_L}}{\frac{\chi x_H - x_L}{x_H - x_L} + \left(1 - \frac{\chi x_H - x_L}{x_H - x_L}\right) \cdot \lambda} = \lim_{\delta\to 0} \tilde{\pi}^{IT}.$$

■

Appendix C - Asset quality shocks

We now extend our baseline setup to the case in which there are shocks to asset quality. We denote the quality of asset i at time t by $\theta_{i,t}$. As in our baseline setup, the unconditional probability of an asset being high quality is $P(\theta_{i,t} = H) = \pi$; but we now assume that $P(\theta_{i,t+1} = H|\theta_{i,t} = H) = \rho^\theta \in [\pi, 1]$. The quality shocks are assumed to be independent across assets, so the fraction of good quality assets at any point in time is also given by π.

Using the same arguments as in the construction of the efficient trade equilibrium in Section 3.2, we can show that the efficient trade equilibrium exists if and only if:

$$\chi x_H - \hat{\pi} x_H - (1-\hat{\pi})x_L + \hat{\delta}(1-\hat{\pi})\frac{(1-\lambda)(1-\hat{\pi})(x_H - x_L)}{1-\hat{\delta}(1-\lambda)(1-\hat{\pi})} \leq 0, \tag{40}$$

where $\hat{\delta} \equiv \delta\frac{\rho^\theta - \pi}{1-\pi}$ and $\hat{\pi} = \frac{\pi\lambda}{\pi\lambda+1-\pi}$. This condition is the same as equation (10) that determines the existence of the efficient trade equilibrium in our baseline model, with the only exception of $\hat{\delta}$ replacing the discount factor δ. Therefore, the efficient trade equilibrium exists under the same conditions as in the baseline economy, but with the discount factor adjusted to $\hat{\delta}$. Otherwise, the equilibrium is generically unique.

Analogously, we can show that the inefficient trade equilibrium exists if and only if:

$$0 \leq \chi x_H - \hat{\pi} x_H - (1-\hat{\pi} x_L) + \hat{\delta}\,(1-\hat{\pi})\,\frac{(1-\lambda+\lambda\chi)\,x_H - x_L}{1-\hat{\delta}}, \tag{41}$$

which is the same as equation (15) that determines the existence of the inefficient trade equilibrium in our baseline model, with the only exception of $\hat{\delta}$ replacing the discount factor δ. Therefore, the inefficient trade equilibrium exists under the same conditions as in the baseline economy, but with the discount factor adjusted to $\hat{\delta}$.

Therefore, we have that $0 < \tilde{\pi}_{ET} < \tilde{\pi}_{IT} < 1$ if and only if $\rho^\theta > \pi$, $\tilde{\pi}_{IT} - \tilde{\pi}_{ET}$ is increasing in ρ^θ, and it goes to zero as $\rho^\theta \to \pi$.