Information and Competition with Symmetry*

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Abstract

This paper investigates the strategic foundations for rational expectations equilibrium. In the model, risk-averse traders with two signal—private information and endowment shocks—submit demand schedules to trade a risky asset. Traders are divided into groups. Within groups, traders share common signals; signals are different across different groups. Either traders become price takers or the price becomes fully revealing, but not both, as the number of competitors per group goes to infinity. As the number of groups goes to infinity, neither price taking nor fully revealing prices are obtained. Measuring competition by the quantity traded as a fraction of that traded by a price-taker, we show that optimal exercise of market power has opposite implications for competition and price informativeness.

Keywords: Rational expectations equilibrium, private information, endowment shocks, price informativeness, strategic trading, competition, noise trading.

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The perfect competition assumption, widely used in many areas of economics, assumes agents are so small that they take prices as given. By applying this assumption to information asymmetry, rational expectations equilibrium features both price-taking behavior and fully revealing prices. For example, Grossman and Stiglitz (1980) and Hellwig (1980) assume perfect competition and show the price becomes fully revealing as noise trading vanishes. This is problematic because traders must ignore their effect of trading on prices. It raises the questions of whether perfect competition is an adequate approximation when traders have private information.

Here we investigate the strategic foundations for rational expectations equilibrium. We ask whether the Bayesian Nash equilibrium of a model with a finite number of traders converges to rational expectations equilibrium as the market becomes large. We study a one-period model in which traders submit demand schedules to buy or sell shares of a risky asset. Traders have two private signals, one about the asset's value and the other about their endowments. Random endowments motivate risk-averse traders to trade for hedging motives. A large market is typically modeled as consisting of infinitely many traders with different private information. In this paper, we divide traders into groups so that each trader competes not only with the other groups, who have different signals, but also with his own group members, who who share the same signals. We assume random variables are normally distributed, and traders have exponential utility with the same risk aversion, endowment shocks with the same variance, and private information with the same precision. These symmetry assumptions make the model analytically tractable. Each group's aggregative risk aversion, each group's aggregate endowment shock, and the precision of each group's information are held constant as the number of members per group increases.

We find that a strategic equilibrium does not approach a rational expectations equilibrium as the number of groups or the number of competitors per group goes to infinity. Perfect competition and fully revealing prices cannot be achieved simultaneously. The economic intuition for this result is developed in the following six steps.

First, we measure competition by the quantity a trader trades as a fraction of the quantity he would trade as a price taker, and we measure price informativeness by the fraction of other traders' information a trader extracts from prices. In characterizing equilibrium, only three parameters matter: the number of groups, the number of competitors per group, and our measure of adverse selection, the ratio of private information to endowment shocks.

Second, price informativeness is solely a function of adverse selection; it does not depend on the number of groups or number of competitors per group. As infinitely many competitors share the same information, the market becomes perfectly competitive. We show that this limit corresponds to the model of Diamond and Verrecchia (1981). In equilibrium, the price remains partially revealing. Intuitively, more competition makes traders trade more aggressively on their private information. This does not affect price informativeness because traders trade equally more aggressively on their endowment shocks.

Third, the market does not become perfectly competitive as infinitely many groups compete with different information. Since each group has unique private information, traders remain "large" because new information continues to move the price even when the group becomes "small" relative to the market. Perfect competition requires, but is not guaranteed by, infinitely many competitors sharing the same information.

Fourth, as adverse selection increases, the price becomes more informative but the market becomes less competitive. The price becomes informative through the process by which each trader moves the price towards his own valuation. The more the price incorporates private information of a trader, the smaller the quantity he trades compared with a price taker. Thus, importantly, optimal exercise of market power has opposite implications for price informativeness and competition.

Fifth, the existence of equilibrium requires adverse selection to be sufficiently small. Following the spirit of trembling hand perfect equilibrium, we add a vanishingly small amount of exogenous noise trading to a setup in which endowment shocks alone are too small for equilibrium to exist. This guarantees that there is always a well-defined equilibrium in the limit, even though the expected losses of noise traders are zero. In such an equilibrium, the market is infinitely noncompetitive in the sense that trade vanishes.

Lastly, without endowment shocks, the price becomes fully revealing while the market remains infinitely noncompetitive as the number of competitors per group goes to infinity. With a finite number of traders, the price is partially revealing. Each trader incorporates a maximum of one half of his private information into prices. As the number of competitors increases to infinity, each trader continues to move the price and the price becomes fully revealing. Even when they become "small" in terms of private information, they remain "large" in terms of their effect on the price. Importantly, the price becomes fully revealing *because* the market remains noncompetitive. Almost-perfect competitors trade large quantities like price takers, and they choose to do so because little of their private information gets incorporated into prices.

Combining the six results, we confirm that either traders become price takers or the price becomes fully revealing, but not both, as the number of competitors who share the same information goes to infinity. The key intuition is the opposite implications for competition and price informativeness that result from strategic traders' optimal exercise of market power.

Such an interaction between price informativeness and competition is absent in the model of Grossman and Stiglitz (1980), who assume price-taking. The paradox in their model is that no trader has an incentive to acquire costly information because the price becomes fully revealing as noise trading vanishes. In our model, the price is always partially revealing with a finite number of traders; the paradox disappears.

Our no-trade and noisy-price result with vanishing noise trading is different from the framework of Milgrom and Stokey (1982), who always find a fully revealing price with no trade. Moreover, even though we allow initial allocations to be Pareto inefficient, there may still be no trade despite substantial gains from trade. Whether there are better trading mechanisms for internalizing gains from trade is left for future study.

Developing the strategic foundations for rational expectations equilibrium has been the topic of many papers. Wilson (1977), Milgrom (1981), Pesendorfer and Swinkels (1997), and Kremer (2002) study whether the price becomes fully revealing as the market becomes large in a model where buyers are strategic but sellers are not. Kyle (1989) assumes that informed traders are strategic, but trading is sustained by exogenous noise traders rather than endowment shocks. In a model in which traders can buy or sell one unit of an indivisible good, Reny and Perry (2006) show that rational expectations equilibrium is obtained as the market becomes large. Vives (2011) examines a model of strategic supply-function competition that is similar to our demand-schedule competition.

Our paper is different from the closely related literature in three ways. First, we use exponential utility while Vives (2011), Rostek and Weretka (2012, 2015) and Bergemann, Heumann and Morris (2015) use quadratic storage costs. This is qualitatively different from exponential utility. With exponential utility, but not with quadratic storage costs, a more informative price makes the market less competitive by decreasing the riskiness of the asset. Second, in our model, each trader has two signals, and this prevents traders from inferring the average of other traders signals perfectly. In a model of correlated private values, Vives (2011) assumes each trader has one signal about his own private value; symmetry makes the price fully revealing. Rostek and Weretka (2012, 2015) extend this approach to allow heterogeneous correlations among traders' private values. Bergemann, Heumann and Morris (2015) study a more general informational environment that allows confounding between common and private components of a signal. They note such confounding is an important element in the informational environment. In our model, there is confounding between the two signals. Lastly, by dividing traders into groups, we clearly show how the effect of the number of groups differs from that of the number of competitors per group.

The plan for this paper is as follows. Section 1 describes the setup of the model and defines an equilibrium. Section 2 characterizes an equilibrium and provides the conditions for existence and uniqueness of an equilibrium. Section 3 analyzes comparative statics when there is no noise trading and shows that price informativeness is independent of the number of competitors who share the same private information. Section 4 introduces an equilibrium with vanishing noise trading and shows that the market is always infinitely noncompetitive in such an equilibrium. Section 5 concludes.

1 Setup

There is one round of trading in which traders exchange a single risky asset against a safe asset whose return is normalized to one. The exogenous liquidation value of the risky asset v is distributed $\mathrm{N}\left(0,\, \sigma_V^2\right)$ with $\sigma_V^2 > 0$. There are N symmetric groups of traders. Each group consists of M identical informed speculators. Altogether, there are MN traders indexed $(m,\, n)$. Each informed trader has exponential utility with constant risk aversion parameter $M\rho$; thus, $\rho > 0$ measures the aggregate absolute risk aversion of each group.

Before trading, each trader $(m,\, n)$ in group n obtains two identical signals. First, all traders in group n obtain an identical private signal about v given by

$$i_n = \tau_I^{1/2}\left(\frac{v}{\sigma_V}\right) + e_n, \qquad \text{where} \qquad e_n \sim \mathrm{N}\left(0,\, 1\right). \tag{1}$$

Each signal provides imperfect information about the liquidation value v. Since $\operatorname{var}\{i_n\} = 1 + \tau_I$, the precision parameter τ_I is a signal-to-noise ratio in which the signal $\tau_I^{1/2}\left(\frac{v}{\sigma_V}\right)$ has variance τ_I and the noise e_n has variance equal to one. The error terms in the signals $e_1, \ldots, e_N$ are distributed independently. This implies that private information is different across different groups $n = 1, \ldots, N$ even though it is the same for the M members of group N.

Second, each trader in group n receives an identical random endowment of $s_{m,n} := \frac{1}{M} s_n$ shares of the risky asset. The quantity s_n is distributed independently across groups n with s_n satisfying

$$s_n \sim \mathrm{N}\left(\bar{s}_n, \sigma_S^2\right), \qquad \text{where} \qquad \sum_{n=1}^{N} \bar{s}_n = 0. \tag{2}$$

Each trader observes the realization of his own endowment s_n, knows his own mean $\bar{s}_n$, and knows that the aggregate mean endowment is zero. Each trader infers the identical endowment of other members of his own group n but does not observe the endowment of any other group.

The model is set up so that the group's aggregate absolute risk aversion (ρ), aggregate endowment shock (σ_S^2), and quality of private information (τ_I) do not change when the number of traders within each group (M) changes. As M increases, more and more traders with the same private information compete with one another. At the same time, each trader becomes smaller in the sense that his risk bearing capacity ($(M\rho)^{-1}$) is less. Varying the parameter M changes in the competitiveness with which trading on private information about payoffs and endowments takes place without changing other aspects of the economy. We refer to M as the number of competitors.

Noise traders demand a random quantity z which is distributed $\mathrm{N}\left(0, \sigma_Z^2\right)$. Noise traders do not optimize anything; their trading is exogenous. Since the model does not require exogenous noise traders, the model allows $\sigma_Z^2 = 0$. In fact, most of our analysis assumes that σ_Z^2 is exactly zero or approaches zero, in which case the expected loss of noise traders is zero. Assume all random variables are jointly normally distributed so that v; $e_1, \ldots, e_N$; $s_1, \ldots, s_N$; and z are all independently distributed.

Except for different mean endowments $\bar{s}_n$, the model is symmetric in that it looks the same from the perspective of every informed trader. Before the realization of private information and endowment shocks, all traders have identical risk aversion, identical beliefs about the signal precision, and identical beliefs about the distribution of endowment shocks about their means. If we were to think of M as representing a continuum of perfect competitors, our model without noise trading would collapse to the model of Diamond and Verrecchia (1981).

Trading. After observing his own endowment shocks s_n and private signal i_n, each trader (m, n) submits a demand schedule $X_{m,n}(p \mid i_n, s_n)$. This notation means that $X_{m,n}$ is a function of the price p, and the function is measurable with respect to s_n and i_n.

Let X denote the $M \times N$ matrix of submitted demand functions whose (m, n)th element corresponds to $X_{m,n}$. An auctioneer aggregates all MN functions to calculate a market clearing price, denoted $p(X)$, which satisfies the market clearing condition

$$\sum_{n=1}^{N} \sum_{m=1}^{M} X_{m,n}(p) + z = 0. \tag{3}$$

If there is no market clearing price, then there is no trade ($x_{m,n} = 0$ for all (m, n)). If there are many market clearing prices, then the auctioneer chooses the smallest price which minimizes trading volume. Given the matrix of submitted demand schedules, trader (m, n) realizes wealth

$$w_{m,n}(X) := v \frac{s_n}{M} + (v - p(X)) \; X_{m,n}(p(X)) \tag{4}$$

and achieves expected utility

$$u_{m,n}(X) := \mathrm{E}\left\{- \exp\left(-M \rho w_{m,n}(X)\right)\right\}. \tag{5}$$

The model is described by seven exogenous parameters: $M, N, \tau_I, \rho, \sigma_V, \sigma_Z$ and σ_S. There are two dimensions of measurement: dollars and shares. The parameters σ_Z and σ_S have dimensions of shares, the parameter σ_V has dimensions of dollars-per-share, and ρ has dimensions of per-dollar. In what follows, we use ρ and σ_V as units to scale variables in dollars-per-share by σ_V and variables in shares by $(\rho\sigma_V)^{-1}$. Therefore it is crucial to assume $\rho > 0$ and $\sigma_V > 0$.

The equilibrium concept is a Bayesian Nash equilibrium. An *equilibrium* is a matrix of demand schedules X such that (1) a market clearing price $p(X)$ is always well defined, and (2) for all $m = 1, \ldots, M$ and $n = 1, \ldots, N$, trader (m, n) chooses his demand schedule $X_{m,n}$ to maximize his expected utility $u_{m,n}(X)$, taking as given the demand schedules of the other $MN - 1$ traders.

Define a *symmetric linear equilibrium* as an equilibrium in which all traders choose the same linear demand schedule

$$\rho\sigma_V X_{m,n}(p \mid i_n, s_n) = \pi_C - \pi_S \rho\sigma_V s_n + \pi_I i_n - \pi_P \frac{p}{\sigma_V}, \tag{6}$$

where the four endogenous parameters π_C, π_S, π_I, and π_P define the same linear function $X_{m,n}$ for all $m = 1, \ldots, M$ and $n = 1, \ldots, N$. Multiplying $X_{m,n}(p \mid i_n, s_n)$ and s_n by $\rho\sigma_V$ and dividing p by σ_V scales the four parameters π_C, π_S, π_I, and π_P to make them dimensionless. This scaling convention is used throughout this paper.

If $MN\pi_P = 0$, then every trader submits a totally inelastic demand schedule, and the resulting aggregate demand is either identically zero or some random quantity which is non-zero with probability one. Market clearing requires this aggregate demand to be identically zero; this further requires noise trading to be zero with probability one ($\sigma_Z^2 = 0$) and each trader's demand to be identically zero ($X_{m,n} \equiv 0$, for all m, n). In such a no-trade equilibrium, the market clearing price is not uniquely determined since any price can support the allocation. Such an equilibrium exists whenever $\sigma_Z^2 = 0$. We call this a *trivial no-trade equilibrium* and exclude it from the following analysis.

Our goal is to characterize existence and uniqueness of symmetric linear equilibria. Discussing asymmetric equilibria or equilibria with non-linear strategies takes us beyond the scope of this paper.

2 Equilibrium

To understand equilibrium, it is intuitively necessary to model how learning from prices and exercising market power simultaneously affect the demand schedules traders choose. To accomplish this, we introduce a "price informativeness" parameter and a "market noncompetitiveness" parameter and then discuss how the values of these two parameters trade off against each other. This discussion leads to the counterintuitive result that a more informative price goes together with a less competitive market.

The equilibrium solution proceeds in five steps using the no-regret pricing approach. A trader (1) observes his residual supply schedule, (2) learns about other traders' private information from the intercept of this schedule, (3) finds the optimal quantity on his residual supply schedule, (4) and implements this optimal quantity by submitting a demand schedule, which (5) is the same as the demand schedules conjectured for other traders.

Residual Supply Schedule. Trader (m, n) conjectures and takes as given symmetric linear demand schedules for the other traders, described by the four endogenous parameters π_C, π_I, π_S and π_P as in (6). Ruling out trivial no-trade equilibria as discussed above ($MN\pi_P \neq 0$), the market clearing condition (3) implies that trader (m, n) has a well-defined residual supply schedule given by

$$\frac{p}{\sigma_V} = \frac{p_{m,n}}{\sigma_V} + \frac{1}{(MN-1)\,\pi_P}\rho\sigma_V x_{m,n}. \tag{7}$$

The intercept $p_{m,n}$ is given by

$$\frac{p_{m,n}}{\sigma_V} = \frac{\sum_{(m',n')\neq(m,n)}(\pi_C + \pi_I i_{n'} - \pi_S \rho\sigma_V s_{n'}) + \rho\sigma_V z}{(MN-1)\,\pi_P}. \tag{8}$$

Trader (m, n), by submitting a demand schedule, is able to condition the quantity demanded on the market clearing price. This implies that the trader can choose the optimal demand as if he already observes the market clearing price p as well as the price that would prevail if he did not trade $p_{m,n}$.

In units of dollars per share, (7) can be written

$$p = p_{m,n} + \lambda x_{m,n}, \qquad \text{with} \qquad \lambda = \frac{\rho\sigma_V^2}{(MN-1)\pi_P}, \tag{9}$$

where λ is the price impact parameter defined in Kyle (1985) and Kyle (1989). Price impact λ measures how much the per-unit price of the risky asset changes in response to the informed trader's buying one more share.

Since the model is symmetric about zero, this implies that the constant π_C in the demand schedule can be shown to be zero in equilibrium, regardless of the values of other parameters. For the purpose of exposition, we assume $\pi_C = 0$ without loss of generality.

Learning from the Price. Trader (m, n) infers other traders' private information from the intercept $p_{m,n}$ in (8). Let τ^* denote the dimensionless ratio of the prior variance

(σ_V^2) to the posterior variance (var $\{v \mid p_{m,n},\, i_n,\, s_n\}$) of the liquidation value given by

$$\tau^* = \frac{\sigma_V^2}{\mathrm{var}\left\{v \mid p_{m,n},\, i_n,\, s_n\right\}}. \tag{10}$$

Symmetry implies that τ^* is the same across all traders. Since the posterior variance is at least as accurate as the prior variance, the inequality $\tau^* \geq 1$ holds by definition. Trader $(m,\, n)$'s learning from the price is described by the following lemma. All proofs are in the Appendix.

Lemma 1 (Learning From Prices.)**.** *Assume* $(N-1)\,\tau_I \pi_I \neq 0$. *Then* τ^* *can be written*

$$\tau^* = 1 + \tau_I + (N-1)\,\tau_I \varphi, \tag{11}$$

where φ *is given by*

$$\varphi = \left(1 + \left(\frac{\pi_S \rho \sigma_V \sigma_S}{\pi_I}\right)^2 + \left(\frac{1}{N-1}\right)\left(\frac{\rho \sigma_V \sigma_Z}{M \pi_I}\right)^2\right)^{-1}. \tag{12}$$

Trader $(m,\, n)$*'s conditional expectation of the fundamental value is given by*

$$\mathrm{E}\left\{\frac{v}{\sigma_V} \,\middle|\, p_{m,n},\, i_n,\, s_n\right\} = \frac{\sqrt{\tau_I}}{\tau^*}\left(i_n + \frac{(N-1)\,\varphi \pi_P}{\pi_I}\frac{p_{m,n}}{\sigma_V} - \left(\frac{M-1}{M}\right)\varphi\left(i_n - \frac{\pi_P}{\pi_I}\frac{p_{m,n}}{\sigma_V} - \frac{\pi_S}{\pi_I}\rho\sigma_V s_n\right)\right). \tag{13}$$

The dimensionless endogenous parameter φ given by (12) is both intuitively and analytically important in this paper. From (10), the parameter $\varphi \in [0,\, 1]$ intuitively measures how much information about v trader $(m,\, n)$ extracts from prices as a fraction of the total precision of other traders signals; we refer to φ as "price informativeness." If $\varphi = 0$, then no information is extracted. If $\varphi = 1$, then the maximum amount of information is extracted. When traders are identical ($N = 1$), traders have no private information ($\tau_I = 0$), or traders do not trade on the private information they have ($\pi_I = 0$), then there is no learning from the price. We set $\varphi = 0$ in these cases.

When there is more than one trader in each group ($M > 1$), the intercept of trader $(m,\, n)$'s residual supply schedule $p_{m,n}$ is already a function of his own private information i_n and endowment shock s_n because of the demand schedules submitted by the other $M - 1$ traders in the group. In the valuation of the risky asset (13), trader $(m,\, n)$

uses i_n and s_n to eliminate the effect of trading by his own group members to extract the private information of other groups accurately.

Optimal Quantity Traded. All random variables are jointly normally distributed, and trading strategies are linear. Thus, the trading strategy which maximizes exponential utility in (5) is the same as the trading strategy which solves the quadratic maximization problem

$$\max_{x_{m,n}} \left\{ \mathrm{E}\left\{ w_{m,n}(x_{m,n}) \mid p_{m,n}, s_n, i_n \right\} - \frac{M\rho}{2} \mathrm{var}\left\{ w_{m,n}\left(x_{m,n}\right) \mid p_{m,n}, s_n, i_n \right\} \right\}. \tag{14}$$

Using (7) and (10), the first-order condition is

$$\mathrm{E}\left\{ \frac{v}{\sigma_V} \middle| p_{m,n}, s_n, i_n \right\} - \frac{p_{m,n}}{\sigma_V} - \frac{M}{\tau^*}\rho\sigma_V s_{m,n} - \left(\frac{2}{(MN-1)\pi_P} + \frac{M}{\tau^*} \right) \rho\sigma_V x_{m,n} = 0. \tag{15}$$

Define the "market noncompetitiveness" parameter χ by[1]

$$\chi := \frac{\tau^*}{(MN-1)\pi_P M} = \frac{\tau^*}{M\rho\sigma_V^2}\lambda. \tag{16}$$

Then the second-order condition can be written as

$$\chi > -\frac{1}{2}. \tag{17}$$

If $\chi < -1/2$ holds, the trader can obtain infinite utility by buying or selling an infinite quantity, and hence an equilibrium does not exist.

To provide the intuition for χ, define a trader's "target inventory" by

$$s_{m,n}^{TI} := \frac{\mathrm{E}\left\{ v \mid p_{m,n}, i_n, s_n \right\} - p_{m,n}}{M\rho\sigma_V^2/\tau^*}. \tag{18}$$

The target inventory $s_{m,n}^{TI}$ is the quantity a trader would demand to hold if he traded like a perfect competitor, ignoring price impact resulting from a nonzero slope λ of

[1]The second inequality in (16) gives an alternative way of writing χ using the definition of λ from (9). In the one-period model of Kyle (1985), the informed trader is a monopolist. This corresponds to $\chi \to \infty$, with the informed trader moving the price to a point half-way between his value and his expected price if he does not trade. In Kyle (1989), the "information incidence" parameter ζ measures the change in the equilibrium price as a response to the change in the trader's valuation. When $M = 1$, this is similar to parameter $\chi/(1+2\chi)$, which is always positive but less than one-half.

the residual demand schedule. If the trader achieves the target inventory, he would choose not to trade. Substituting (18) into the first-order condition (15) yields the optimal quantity traded

$$x_{m,n} = \frac{1}{1+2\chi}\left(s_{m,n}^{TI} - s_{m,n}\right). \tag{19}$$

The fraction $1/(1+2\chi)$ measures the optimal quantity a trader chooses to trade as a fraction of how much he would trade if he were a price taker. If $\chi = 0$, the market is perfectly competitive; if $\chi \to \infty$, , the market is infinitely noncompetitive, and traders trade a vanishingly small fraction of the quantity needed to achieve their target inventories. We show in Theorem 1 that χ is nonnegative in equilibrium.[2]

The endogenous parameter χ is a scaled ratio of market impact to risk aversion. When χ is small, risk aversion restricts trading more than market impact, and thus traders trade quantities closer to target levels. When χ is large, market impact restricts trading more than risk aversion, and traders trade only a small fraction of the quantity needed to reach target inventories.

Market noncompetitiveness χ also determines the extent to which a trader moves the price towards his valuation of the risky asset:

$$p - p_{m,n} = \frac{1}{2}\left(1 - \frac{1}{1+2\chi}\right)\left(\mathrm{E}\left\{v \mid p_{m,n}, i_n, s_n\right\} - \frac{M\rho\sigma_V^2}{\tau^*}s_{m,n} - p_{m,n}\right). \tag{20}$$

Trader (m, n) optimally chooses how much to move the price from $p_{m,n}$, the prevailing price if trader (m, n) did not trade, towards his own valuation of the risky asset adjusted for his endowment shock $(\mathrm{E}\left\{v \mid p_{m,n}, i_n, s_n\right\} - (M\rho\sigma_V^2/\tau^*)s_{m,n})$. If the market is very competitive (χ close to zero), the trader does not move the price much (p close to $p_{m,n}$). When the trader exercises great monopoly power (χ close to infinity), a trader will move the the price more, to a level almost halfway between the prevailing price and his valuation of the asset.

Comparing the quantity factor $\frac{1}{1+2\chi}$ from (20) with the price factor $\frac{1}{2}\left(1 - \frac{1}{1+2\chi}\right)$ from (19) shows that market noncompetitiveness χ has opposite effects on quantity and price. When noncompetitiveness χ increases, a trader chooses to restrict his trad-

[2]This paper supersedes an earlier version Kyle and Lee (2016) which considers a more general model in which traders have heterogeneous prior beliefs on the precision of their signals. In the special case when all traders are relatively underconfident and $M \geq 2$, χ can satisfy $-\frac{1}{2} < \chi <, 0$ in equilibrium. This paper's common prior setup implies $\chi \geq 0$ in equilibrium. Kyle, Obizhaeva and Wang (2016) consider a model with disagreements but no endowment shock and no noise trading.

ing more, but the price incorporates a larger fraction of his information. A trader chooses to trade a competitive quantity only when his private information does not get incorporated into the price.

As an index of noncompetitiveness, χ may be broadly applied outside our symmetric model to compare how much illiquidity affects the trading of different traders in different markets. To evaluate and refine trading strategies, asset managers often calculate the ratio of market impact costs to expected trading profits (alpha). The parameter χ measures exactly this ratio. It is a trader's dollar cost of reaching target inventory as a fraction of a "paper-trading" profit (dollar profit in the absence of price impact). To apply χ to asset management, think of the reciprocal of risk aversion as measuring assets under management. Market noncompetitiveness χ is high when assets under management are high, market impact is high, or learning significantly increases the precision.[3]

This effect of learning on market noncompetitiveness is absent in the quadratic storage cost model used by Vives (2011), Rostek and Weretka (2012, 2015) and Bergemann, Heumann and Morris (2015). We can define a hypothetical quadratic storage cost parameter γ by $\gamma := \rho\sigma_V^2/\tau^*$. If τ^* were a constant, then the quadratic storage cost model and the exponential utility model would be the same. In fact, τ^* defined by (10) increases in exogenous parameters τ_I and N and endogeneous parameter φ. Therefore, when the amount of learning varies due to changes in τ_I, N, or φ, the two models obtain different results. According to (16), if τ^* were a constant, then χ would be simply proportional to λ. In this sense, with quadratic storage costs, a more informative price does not directly reduce competition.

Trader (m, n) can implement his optimal quantity $x_{m,n}$ by substituting his valuation of the risky asset (13) into the optimal quantity traded (19) to obtain the best response strategy $X_{m,n}$ to the symmetric strategies of other traders.

Lemma 2 (Best Response)**.** *Assume $\chi > -\frac{1}{2}$. If the other $MN-1$ traders chose strategies defined by the parameters π_I, π_P, and π_S, then trader (m, n)'s best response is the optimal*

[3] Risk tolerance $1/(M\rho)$ intuitively corresponds to assets under management. For a small mean μ and small variance σ^2, the competitive demand function for a log-utility investor with wealth W is approximately $W\mu/\sigma^2$. When this is compared to the CARA-normal competitive demand $\rho^{-1}\mu/\sigma^2$, it is easy to see that $1/\rho$ maps directly into wealth W.

demand schedule $X_{m,n}(p \mid i_n, s_n)$ *given by*

$$\left(1+\chi+\frac{\varphi\tau_I^{1/2}}{M^2\pi_I}\right)M\rho\sigma_V X_{m,n}(p \mid i_n, s_n) = \tau_I^{1/2} i_n - \left(\tau^* - \frac{(N-1)\varphi\tau_I^{1/2}\pi_P}{\pi_I}\right)\frac{p}{\sigma_V} - \rho\sigma_V s_n - \frac{(M-1)\varphi\tau_I^{1/2}}{M}\left(i_n - \frac{\pi_P}{\pi_I}\frac{p}{\sigma_V} - \frac{\pi_S}{\pi_I}\rho\sigma_V s_n\right), \tag{21}$$

where τ^*, φ *and* χ *are given by* (11), (12) *and* (16) *respectively.*

Characterization of Equilibrium. A linear symmetric equilibrium is found by equating a trader's best response (21) to the strategy the trader conjectures that others are playing. An equilibrium can be fully characterized using the exogenous parameters and the single endogenous informativeness parameter φ.

Theorem 1 (Characterization of Symmetric Linear Equilibrium)**.** *Suppose* $\rho > 0$, $\sigma_V > 0$, *and* $MN > 2$. *If* $(N-1)\tau_I > 0$, *then the set of symmetric linear equilibria, excluding trivial no-trade equilibria, is characterized by the set of all endogenous parameters* φ *such that (1)* φ *solves*

$$\frac{1-\varphi}{\varphi} = \frac{(\rho\sigma_V\sigma_S)^2}{\tau_I} + \frac{(\rho\sigma_V\sigma_Z)^2/(N-1)}{\tau_I\left(\frac{MN-2}{MN-1} - \left(\frac{MN+N-2}{MN-1}\right)\varphi\right)^2}, \tag{22}$$

and (2) φ *satisfies the second-order condition*

$$\varphi < \frac{MN-2}{MN+N-2}. \tag{23}$$

If $(N-1)\tau_I = 0$, *an equilibrium is characterized by* $\varphi = 0$.

With τ^* *given by* (11), *the equilibrium demand schedule of trader* (m, n) *is given by*

$$\rho\sigma_V X_{m,n}(p \mid i_n, s_n) = \frac{(MN-2-(MN+N-2)\varphi)}{M(MN-1)}\left(\tau_I^{1/2} i_n - \rho\sigma_V s_n - \frac{\tau^*}{1+(N-1)\varphi}\frac{p}{\sigma_V}\right), \tag{24}$$

the market clearing price is given by

$$\frac{p}{\sigma_V} = \left(\frac{1+(N-1)\varphi}{N\tau^*}\right)\left(\tau_I^{1/2}\sum_{i=1}^{N} i_n - \sum_{n=1}^{N}\rho\sigma_V s_n + \frac{\rho\sigma_V z}{\frac{MN-2}{MN-1} - \left(\frac{MN+N-2}{MN-1}\right)\varphi}\right), \tag{25}$$

and market noncompetitiveness χ is given by

$$\chi = \frac{1 + (N-1)\,\varphi}{MN - 2 - (MN + N - 2)\,\varphi}. \tag{26}$$

Equation (22) defines the endogenous parameter φ as the solution to an equation whose other parameters are all exogenous. Once φ is determined, χ is expressed as a simple function of φ, M and N in (26). Price informativeness (φ) and market noncompetitiveness (χ) together capture the important aspects equilibrium quantities and prices. We discuss how exogenous parameters determine equilibrium φ and χ in detail in the next section.

In (22), there are two dimensionless products of dimensional quantities: $\rho\sigma_V\sigma_S$ and $\rho\sigma_V\sigma_Z$. Changing units of measurement has no real effect on (22), (24), (25), and (26). This implies ρ and σ_V can be interpreted as scaling variable with $\rho = \sigma_V = 1$ assumed without loss of generality. Thus, if the four dimensional exogenous variables ρ, σ_V, σ_S, and σ_Z, change in such a way that the two dimensionless products $\rho\sigma_V\sigma_S$ and $\rho\sigma_V\sigma_Z$ do not change, then φ does not change, and the properties of the equilibrium do not change in many respects.[4]

Existence and Uniqueness. Theorem 1 can be used to characterize existence and uniqueness in terms of exogenous parameters only.

Corollary 1 (Existence and Uniqueness)**.** *Assume $\rho > 0$ and $\sigma_V > 0$. Then there exists a symmetric linear equilibrium, excluding trivial no-trade equilibria, if and only if $MN > 2$ and at least one of the following three conditions holds:*

$$(N-1)\,\tau_I = 0, \tag{27}$$

$$\sigma_Z^2 > 0, \tag{28}$$

$$\frac{\tau_I}{(\rho\sigma_V\sigma_S)^2} < M - \frac{2}{N}. \tag{29}$$

If a symmetric linear equilibrium exists, it is unique.

[4]This property is shared by many finance models. Fundamental model properties depend on the ratio of the risks to be borne—measured by $\sigma_V\sigma_Z$ and $\sigma_V\sigma_S$—to dollar risk-bearing capacity ρ^{-1}. For example, $\rho\sigma_V\sigma_Z$ or $\rho\sigma_V\sigma_S$ can become small either because risk bearing capacity increases (ρ becomes small) or because the risks to be borne $\sigma_V\sigma_Z$ and $\sigma_V\sigma_S$ become small. Either way, the effect on equilibrium is the same.

A symmetric linear equilibrium exists when either (1) there is no information asymmetry, in which case $\varphi = 0$ is defined by continuity (in condition (27)); or (2) there is any amount of noise trading (in condition (28)); or (3) endowment shocks are large enough, relative to the precision of private signals, to prevent the market from shutting down due to too much adverse selection (in condition (29)). These conditions are equivalent to the second-order condition (23), which in turn results from (1) maintaining concavity of the objective function in (14) and (2) ruling out trivial no-trade equilibria ($MN\pi_P \neq 0$). Uniqueness follows from (22) as its left hand side is strictly decreasing in φ and the right hand side is weakly increasing in φ.

While the endowment shock needs to be sufficiently large for an equilibrium to exist, an arbitrarily small amount of exogenous noise trading guarantees existence of equilibrium. Unlike optimizing traders who receive endowment shocks, noise traders are willing to incur whatever trading costs are necessary to sustain an equilibrium with trade.

3 Competition and Price Informativeness

In this section we discuss how price informativeness and competition depend on exogenous parameters when there is some information asymmetry ($(N-1)\tau_I > 0$) but no noise trading ($\sigma_Z = 0$). While there remain six exogenous parameters (M, N, ρ, σ_V, σ_S and τ_I), equations (22), (23), and (26) show that characterizing φ and χ only requires the three exogenous parameters M, N, and "adverse selection" θ, defined by

$$\theta := \frac{\tau_I}{(\rho\sigma_V\sigma_S)^2}. \tag{30}$$

This parameter $\theta \in (0, \infty]$ is the ratio of private information to endowment shocks. A higher precision of private information (τ_I) increases adverse selection; a higher variance of endowment shocks (σ_S^2) reduces adverse selection.

Corollary 2. *Assume $(N-1)\tau_I > 0$ and $\sigma_Z^2 = 0$. Then a symmetric linear equilibrium exists if and only if the second order condition*

$$\theta < M - \frac{2}{N} \tag{31}$$

holds. If equilibrium exists, price informativeness φ is given by

$$\varphi = \frac{\theta}{1+\theta}, \tag{32}$$

and market noncompetitiveness χ is given by

$$\chi = \frac{1+N\theta}{MN-2-N\theta}. \tag{33}$$

If equilibrium exists for particular values of M, N and θ, then it exists for larger M, larger N, and smaller θ. We discuss next the comparative statics results for the three exogenous parameters M, N and θ.

The Number of Competitors (M). Increasing the number of competitors M implies increasing competition. Not surprisingly, (33) implies that market noncompetitiveness χ monotonically decreases as M increases. As $M \to \infty$, equilibrium outcomes approach perfect competition since $\chi \to 0$, and the market becomes perfectly competitive in the sense that all traders, acting as price takers, achieve their target inventories in one round of trading (see (19)); this equilibrium corresponds to the model of Diamond and Verrecchia (1981). The number of competitors (M) indeed captures the notion of competitiveness of the market.

More surprisingly, (32) implies that price informativeness φ is not affected when the number of competitors M changes. Price informativeness φ is determined solely by adverse selection θ. It does not depend on how many traders share the same private information or how many different groups have different private information. This is counter to the conventional wisdom that more competition makes prices more informative by inducing traders to shade their bids less and thus trade more aggressively.

While traders trade more aggressively on private information as M increases, this does not lead to more informative price because traders also trade equally more aggressively on their endowment shocks. The proportion by which traders shade their trading, as a function of M, is the same for private information and endowment shocks because optimal exercise of market power is governed by the same incentives for both private information and endowment shocks.

The Number of Groups (N). Setting the model up with N groups of M traders distinguishes the effects of changing the number of groups from changing the number of members of each group.

Like increasing the number of competitors M, (33) implies that noncompetitiveness χ decreases as the number of groups N increases. Unlike the result for the number of competitors M, as the number of groups with different private information (N) goes to infinity, equilibrium outcomes do not approach perfect competition. Taking the limit as N approaches infinity in (33) yields

$$\chi \to \frac{\theta}{M - \theta} > 0 \qquad \text{as} \qquad N \to \infty. \tag{34}$$

The economic intuition for why the market remains imperfectly competitive, even when the total number of traders goes to infinity, is that each group's uniquely different private information is known to only a finite number of traders M. Traders maintain incentives to trade less aggressively because they have market power over their private information. With $M = 1$, the equilibrium is like monopolistic competition. Like the corresponding result for changing M, changing N does not change price informativeness φ.

Adverse Selection (θ). Adverse selection (θ) affects both price informativeness and competition. Equation (32) implies that price informativeness (φ) depends only on θ and increases in θ. More adverse selection increases the informativeness of prices. Equation (33) implies that market noncompetitiveness (χ) depends on M, N and θ. When M and N are fixed and θ varies, χ increases in θ, and thus increases in φ. This implies an inverse relationship between price informativeness and competition: As adverse selection θ increases, a more informative price (larger φ) is associated with less competition (larger χ).

This inverse relationship between price informativeness and competition may seem counterintuitive. Imperfect competition is mainly driven by information asymmetry. As the price becomes more informative, each trader knows more about others' information, and there is less information asymmetry. One might think that therefore the market should be more competitive. Our earlier discussion on the opposing effects of competition on the price and the quantity, as shown in (19) and (20), provides economic intuition for this result.

We can also examine the positive relationship between φ and χ in the best response strategy (21). Suppose all traders have the same conjectures about other traders' strategies, given by π_I, π_P, π_S and the resulting φ and χ defined by (12) and (16). By aggregating across traders, we find the best-response value of market noncompetitiveness, denoted $BR(\chi)$. Since traders share the same conjectures, the resulting best response strategies are the same as well. To reduce the dimensionality of the best responses, we impose a restriction that the ratio π_P/π_I is a fixed point conditional on φ. The assumption $\chi \geq 0$ ensures the existence of a best response.

Corollary 3. *Suppose each trader* (m, n)*, for all* m *and* n*, takes as given other players' strategies* π_I*,* π_P *and* π_S *that satisfy*

$$\frac{\pi_P}{\pi_I} = \frac{1+\tau_I+(N-1)\,\tau_I\varphi}{(1+(N-1)\,\varphi)\,\tau_I^{1/2}} \qquad \text{and} \qquad \chi \geq 0, \tag{35}$$

where φ *and* χ *are given as* (12) *and* (16)*, respectively. Then the market noncompetitiveness that results from the best response strategy is given by*

$$BR(\chi) = \left(\frac{1+\tau_I+(N-1)\,\tau_I BR(\varphi)}{1+\tau_I+(N-1)\,\tau_I\varphi}\right)\left(\frac{1+\chi}{MN-1}+\frac{\varphi\,(1+\chi)}{M-(M-1)\,\varphi}+\frac{\varphi\chi}{M-(M-1)\,\varphi}\right). \tag{36}$$

The noncompetitiveness that results from the best response strategies is given by the product of the two factors of the right-hand side in (36). The first factor is a coefficient that is independent of χ. This coefficient is greater than one if the price informativeness that results from the best response strategies ($BR(\varphi)$) is higher than the conjectured price informativeness (φ). If the conjectured price informativeness is a fixed point ($\varphi = BR(\varphi)$), which is the case when the conjectured strategies are already in equilibrium, the coefficient reduces to one.

There are three terms in the second factor on the right-hand side of (36). The first term is the pure effect of competition, which corresponds to the case when there is no private information ($(N-1)\,\tau_I = 0$). When all traders conjecture $\chi = 0$, the best response strategy yields $\frac{1}{MN-1}$ because all traders are risk averse and owning the asset is risky. As the conjectured χ increases, the best response χ increases. The condition that there are strictly more than two traders ($MN > 2$) guarantees that the slope is strictly less than one. The second and the third terms reflect the effect of price informativeness on competition. The distinction between the two terms comes from the fact that, in a double auction, all traders are demanders as well as suppliers of liquidity. As a liquidity supplier, a trader is concerned about suffering the winner's curse. When other traders want to buy an asset from the trader, it is likely that the valuation of the asset is higher because other traders may have different information from him. As a liquidity demander, the trader needs to compensate other traders for the winner's curse that they may suffer as a result of his own private information. This compensation disappears if the conjectured χ is zero because trading becomes costless. The two winner's curses—as a liquidity supplier and as a liquidity demander—become more severe as the price becomes more informative. This feeds back into the best response strategy and results in the inverse relationship between price informativeness and competition.

Collecting all terms, (36) shows how and why price informativeness and competition are inversely related in equilibrium. This also explains the equilibrium existence condition. When price informativeness is a fixed point ($\varphi = BR(\varphi)$), the slope of χ is the sum of all three coefficients in the second line. The sum needs to be strictly less than one for an equilibrium to exist. This is equivalent to the equilibrium second-order condition (23).

4 Equilibrium with Vanishing Noise Trading

When there is no noise trading ($\sigma_Z = 0$), equilibrium does not exist unless adverse selection θ is sufficiently small ($\theta < M - \frac{2}{N}$). This nonexistence result suggests that some issue needs to be addressed in our model. While informal intuition may suggest that there is no trade when equilibrium fails to exist—and indeed, a trivial no-trade equilibrium does exist is this case—nonexistence does not formally imply no trade. Instead, nonexistence represents failure of the model to make a prediction even though no trade is a possible equilibrium outcome.

We address this modeling issue by defining an equilibrium without noise trading to be the result of taking a limit as noise trading vanishes ($\sigma_Z \to 0$). We interpret vanishing noise as adding small perturbations to the trading environment. While this is in the spirit of trembling hand perfect equilibrium, it involves perturbations of the players' actions rather than the exogenous perturbations which we propose. According to Corollary 1, any arbitrarily small amount of exogenous noise trading allows an equilibrium to exist (because (28) is satisfied), despite the presence of an arbitrarily large amount of adverse selection (which makes (29) fail to be satisfied). Confirming intuition, there is indeed no trade in such a limit. Contrary to intuition, the price remains noisy even in the limit when both endowment shocks and noise trading vanish. This result contrasts with the fully revealing prices obtained by Milgrom and Stokey (1982) and the price taking assumed by Grossman and Stiglitz (1980).

The next theorem formalizes this result.

Theorem 2. *Let $\mathcal{E}$ denote an economy described by $\{M, N, \theta, \sigma_Z\}$ such that $MN > 2$, $(N-1)\tau_I > 0$, and $\sigma_Z = 0$. Let $(\mathcal{E}_k)_{k=0}^{\infty} = (\mathcal{E}_0, \mathcal{E}_1, \ldots)$ denote an arbitrary sequence of economies such that $\mathcal{E}_k$ has exactly the same exogenous parameters as $\mathcal{E}$ for all k except that $\sigma_Z^2(k) > 0$ and $\sigma_Z^2(k) \to 0$ as $k \to \infty$. Then there exists a unique equilibrium in $\mathcal{E}_k$ for all k. Moreover, price informativeness φ_k in economy $\mathcal{E}_k$ satisfies*

$$\varphi_k \to \varphi := \min\left\{\frac{\theta}{1+\theta}, \frac{MN-2}{MN-2+N}\right\} \qquad as \qquad k \to \infty, \tag{37}$$

and market noncompetitiveness χ_k satisfies

$$\chi_k^{-1} \to \chi^{-1} := \max\left\{\frac{MN-2-N\theta}{1+N\theta}, 0\right\} \qquad as \qquad k \to \infty. \tag{38}$$

Comparing Theorem 2 with Corollary 2 shows that adding vanishing noise trading does not have any effect when an equilibrium already exists without vanishing noise ($\theta < M - \frac{2}{N}$). When an equilibrium does not exist without noise trading, adding vanishing noise trading keeps the price sufficiently noisy to support an equilibrium that is infinitely noncompetitive. The price is noisy enough so that the second-order condition is satisfied for all k. This result is unique to noise trading; a similar result is not obtained from vanishing endowment shocks. Although the limit as noise trading vanishes has no trade, this limit is not a trivial no-trade equilibrium because the noisy price is well-defined in the limit.

Noisy Price With Vanishing Noise. When an equilibrium does not exist without vanishing noise ($\theta \geq M - \frac{2}{N}$), price informativeness is independent of adverse selection θ and is determined by M and N. The economic intuition for this result is that in such an equilibrium, the market becomes infinitely noncompetitive (since $\chi \to \infty$ from (38)). This implies that each trader incorporates half of his private information into prices (since $\frac{1}{2}\left(1 - \frac{1}{1+2\chi}\right) \to \frac{1}{2}$ in (20)).

If all traders with the same private information collude (equivalent to $M = 1$), this implies that price informativeness φ satisfies $\varphi \approx 1/2$ for large enough N (or, more precisely, $\varphi = \frac{1}{2}\left(1 - \frac{1}{N-1}\right)$). This corresponds to the monopolistic competition limit in Kyle (1989) as the number of different informed traders (N) approaches infinity. As the number of non-colluding, oligopolistic competitors within each group increases to $M = 2, 3, \ldots$, the informativeness of prices φ increases to approximately 2/3, 3/4, 4/5,..., for large enough N. While each trader still incorporates half of his marginal private information into prices, the price becomes more informative when the overlapping private information is shared among many traders M.

The outcome of this oligopolistic competition among M competitors resembles quantity competition in a Cournot equilibrium in which each firm tries to maximize its profit by supplying only the half of its residual demand. As the number of firms increases in quantity Cournot competition, each firm becomes a smaller fraction of the market, and the total quantity produced increases to fractions 2/3, 3/4,..., of the quantity with perfect competition. Of course, the important distinction here is that even when traders become "small" in terms of their private information, they remain "large" in the market because their trading costs are constrained by vanishingly low market liquidity, not by risk aversion.

Price Impact with Vanishing Noise. The equilibrium with vanishing noise replaces a trivial no-trade equilibrium—which has no well-defined price—with a non-trivial no-trade equilibrium in which the price is well-defined as a limit. Mathematically, the difference between taking a limit and not taking a limit shows up in the denominator of the σ_Z-term in the φ-equation (22). When a limit is taken, both the numerator and the denominator of the σ_Z-term in (22) converge to zero, but the ratio has a well-defined limit given by

$$\frac{(\rho\sigma_V\sigma_Z)^2 / (N-1)}{\tau_I\left(\frac{MN-2}{MN-1} - \left(\frac{MN+N-2}{MN-1}\right)\varphi\right)^2} \to \max\left\{\frac{1}{M-\frac{2}{N}} - \frac{1}{\theta}, 0\right\} \qquad \text{as} \qquad \sigma_Z \to 0. \tag{39}$$

If $\theta > M - 2/N$, implying an equilibrium would not exist without noise trading, this limit is strictly positive. While noise trading vanishes, the price noise created by vanishing noise remains strictly positive. The intuition for this result is that the price impact of noise trading goes to infinity as $\sigma_Z \to 0$.

According to (25), the price impact of noise trading λ_Z—the per-share price change in response to per-share noise trading—is defined by

$$\lambda_Z := \left(\frac{1+(N-1)\,\varphi}{N\tau^*}\right)\left(\frac{\rho\sigma_V^2}{\frac{MN-2}{MN-1}-\left(\frac{MN+N-2}{MN-1}\right)\varphi}\right). \tag{40}$$

When $\theta \geq M - 2/N$, the price impact of vanishing noise trading goes to infinity (see (37).)

The limit result (39) implies that even though the expected losses of noise traders $E\{z \cdot \lambda_Z z\}$ vanish as noise trading vanishes, because

$$\mathrm{E}\{z \cdot \lambda_Z z\} = \lambda_Z \sigma_Z^2 \to 0 \qquad \text{as} \qquad \sigma_Z \to 0, \tag{41}$$

the variance of the price created by noise traders (var $\{\lambda_Z z\}$) does not vanish, because

$$\lambda_Z^2 \sigma_Z^2 \to \max\left\{\frac{(MN-1)^2\,(N-1)\,\tau_I \sigma_V^2}{(MN-2+N)^2\,\tau^*}\left(\frac{1}{M-\frac{2}{N}}-\frac{1}{\theta}\right), 0\right\} \qquad \text{as} \qquad \sigma_Z \to 0. \tag{42}$$

Mathematically, this limit is finite because price impact λ goes to infinity at the same rate as σ_Z goes to zero. Since the price noise created by vanishing noise trading does not vanish, the price is sufficiently uninformative to allow an equilibrium to exist at the limit as noise trading vanishes.

No-Trade Theorem. In the vanishing-noise equilibrium, the market is infinitely non-competitive ($\chi \to \infty$), and thus there is no trade (since $\frac{1}{1+2\chi} \to 0$ in (19)). For $\sigma_Z > 0$, the second-order condition (23) holds as a strict inequality, but the second-order condition holds as an exact equality in the limit $\sigma_Z \to 0$. Even though the price is well-defined in the limit, the at-the-limit-strategies $X_{m,n}(p) \equiv 0$ themselves define a trivial no-trade equilibrium in which there is no well-defined price.

Recall that initial endowments can have unequal, deterministic components s_n in (2). Thus, it is common knowledge that there are ex-ante gains from trade when initial allocations are different. Even though there may be large potential gains from trade to equalize inventories across traders, traders will not participate in any trade at all. When adverse selection (θ) is sufficiently severe (i.e., $\theta \geq M - \frac{2}{N}$), there is no trade even when there is a small endowment shock or when many traders have the same private information.

The economic intuition for this market breakdown is that the market is so illiquid that any attempt to trade away from inefficient endowments would move the price infinitely against the traders. While this is similar to the lemons problem described by Akerlof (1970), the nature of information asymmetry is different because Akerlof assumes that only the sellers have private information. We show that even when each trader has private information in a symmetric fashion, the market may fail to realize ex-ante gains from trade.

Our no-trade theorem can be contrasted with other no-trade results which obtain fully revealing prices in the absence of noise trading. While Milgrom and Stokey (1982) allow more general preferences and distributions of random variables, they do not specify a mechanism for determining prices.[5] Moreover, they assume the initial allocation of resources is Pareto optimal. In contrast, our model allows inefficient initial allocations and also requires the market-clearing equilibrium price to be defined by the well-defined mechanism of aggregating demand schedules. While we obtain no trade, the price in such an equilibrium cannot be fully revealing for any finite M. The price cannot be more informative than (37).

The equilibrium in demand schedules is attractive because all traders are treated symmetrically and limit orders are protected. These are properties of well-functioning markets which organized exchanges and their regulators strive to implement. From the perspective of welfare economics, the main weakness of the equilibrium in demand schedules is that modest adverse selection can make trade break down even when there are large gains from trade due to large non-stochastic initial endowments.

Whether there are better trading mechanisms for internalizing gains from trade is an interesting issue. Liu and Wang (2016) examine a dealer-market model in which dealers make profits by buying at the bid and selling at the offer while customers are not allowed to trade with one another at the the same price (such as the midpoint of the bid-ask spread). The monopolistic spread profits earned by dealers may allow trade to occur when it would not occur in our equilibrium in demand schedules. Duffie and Zhu (Forthcoming) study a workup process that allows traders to trade at fixed prices which do not necessarily clear the market. Glode and Opp (2016) study the welfare effects of trading with intermediation chains. Malamud and Rostek (2016) study the welfare effects of decentralized exchanges when traders have heterogeneous risk aversion.

[5] Tirole (1982) considers both static and dynamic settings. Dow, Madrigal and da Costa Werlang (1990) emphasize market completeness and common knowledge.

Comparative Statics: Large Number of Traders. Defining an equilibrium using a limit when noise trading vanishes makes it possible to describe comparative statics properties of the equilibrium when there is no noise trading and equilibrium would otherwise not exist because there is too much adverse selection ($\theta > M - \frac{2}{N}$).

For finite adverse selection ($\theta < \infty$), the effects of changing the number of traders per group M or the number of groups N follow from Theorem 2. As M increases, the second order condition ($\theta < M - \frac{2}{N}$) becomes more relaxed and must eventually hold for sufficiently large M. Equation (38) implies that the equilibrium becomes competitive in the sense that $\chi \to 0$ as $M \to \infty$. Equation (37) implies that when M is large enough for the second order condition to be satisfied, price informativeness φ is determined by θ alone and the price remains noisy in the sense that $\varphi \to \theta/(1+\theta)$ as $M \to \infty$.

As N increases, the second order condition ($\theta < M - \frac{2}{N}$) needs not be satisfied. While increasing N does relax the second order condition, its effect is much more limited. If (and only if) θ is sufficiently large ($\theta \geq M$), the market remains infinitely noncompetitive as $N \to \infty$ (see (38).) The price becomes more informative as the number of groups increases but remains noisy with $\varphi \to M/(M+1)$ as $N \to \infty$ (see (37)).

If endowment shocks are zero ($\sigma_S = 0$), then adverse selection is infinite ($\theta = \infty$) and the second order condition is never satisfied. For all M, equilibrium remains infinitely noncompetitive and there is no trade. In the limit $M \to \infty$, the price becomes fully revealing as half of each trader's private information is incorporated into the price. Fully revealing prices and price taking are mutually exclusive with strategic trading.

This result is the opposite of competitive rational expectations models such as Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981), where in the limit as noise trading vanishes, (1) traders remain price takers and (2) prices become fully revealing. These limiting results might be justified with the following informal logic: Competitive rational expectations models are an approximation to what would happen in an oligopolistic model with an arbitrarily large number of traders. As noise trading vanishes, the noise from prices will disappear. Thus, it is reasonable to believe that price noise will disappear and price taking behavior will apply in a noncompetitive model with a large number of traders observing the same private information.

Our model articulates a flaw in the preceding argument. In a model with private information, the competitiveness of the market cannot be assumed exogenously because both market competitiveness and price informativeness are determined by strategic trading motives. The resulting inverse relationship between competition and price informativeness is so important that the market may remain noncompetitive even as infinitely many traders compete over the same information. Moreover, the model of Grossman and Stiglitz (1980) entails a well-known paradox that no trader has an incentive to acquire costly information because the price is fully revealing as noise trading vanishes. This paradox disappears in our model because the price is always partially revealing with a finite number of traders.

Order of Limits. Our symmetric model is in general simpler than asymmetric models in which traders differ in the quality of their information. For example, Grossman and Stiglitz (1980) describe an asymmetric model whose equilibrium is more analytically complicated due to having both informed and uninformed traders. Our model with $N = 2, \sigma_S^2 = 0$, and $\sigma_Z^2 > 0$ resembles theirs as $M \to \infty$, with the informed and uninformed replaced by two groups of symmetrically informed traders with independent private signals.

The approach of competitve models such as Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981) implicitly takes the limit $M \to \infty$ first. Taking the limit as $M \to \infty$ in (22) and (23) for $\sigma_Z > 0$ yields

$$(1-\varphi)^3 = \frac{(\rho\sigma_V\sigma_Z)^2}{(N-1)\,\tau_I}\varphi, \qquad \text{where} \qquad \varphi<1. \tag{43}$$

This equilibrium corresponds to a symmetric version of the equilibrium of Hellwig (1980). Since $\varphi < 1$, (26) implies $\chi \to 0$ for any positive amount of noise trading σ_Z so that the market is perfectly competitive. Next, taking the limit $\sigma_Z \to 0$ in (43) makes the price fully revealing since $\varphi \to 1$. By first assuming M to be infinity, their analysis completely assumes away the delicate interaction between price informativeness and market competition.

The fact that different results are obtained when limits are taken in different orders implies that different results are also possible when a double limit is taken with both $\sigma_Z \to 0$ and $M \to \infty$. Suppose very small target values χ and $1-\varphi$, denoted $\hat{\chi}$ and $1-\hat{\varphi}$, are given. Now choose $M = \frac{1}{\hat{\chi}(1-\hat{\varphi})}$ and $\sigma_Z^2 = \frac{(N-1)\tau_I}{\rho^2\sigma_V^2}\frac{(1-\hat{\varphi})^3}{\hat{\varphi}}$. Then it can be shown from Theorem 1 ((22) and (26)) that both perfect competition and fully revealing prices are obtained because $\frac{\chi}{\hat{\chi}} \to 1$ and $\frac{1-\varphi}{1-\hat{\varphi}} \to 1$ as both $\hat{\chi} \to 0$ and $\hat{\varphi} \to 1$ simultaneously.

Intuitively, prices can be made more informative by making noise trading σ_Z small, and making the market simultaneously more competitive requires M to grow large faster to compensate for the effect from increased price informativeness ($M = \frac{1}{\hat{\chi}(1-\hat{\varphi})}$). This implies as the price becomes more informative, increasingly more traders must share and compete over the same information in order for prices to be fully revealing. The fact that such a strong condition is necessary to replicate the results of competitive models shows that perfect competition is not an adequate approximation for markets with a large number of traders and, thus, strategic trading motives must be taken into account when traders have private information.

5 Conclusion

By examining the comparative statics properties of an equilibrium in demand schedules, we have found that a strategic equilibrium does not approach a rational expectations equilibrium as the market becomes large. Perfect competition and fully revealing prices cannot be achieved simultaneously. The result that traders choose not to trade when endowment shocks are sufficiently small indicates that an equilibrium in demand schedules does not efficiently internalize gains from trade. It is an interesting question for future research whether other trading mechanisms can achieve greater gains from trade than the single-price double auction analyzed in this paper.

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A Proofs

Proof of Lemma 1. As discussed, we assume $\pi_C = 0$ without loss of generality. Since $\pi_I \neq 0$ and $(N-1)\tau_I \neq 0$, there is information asymmetry and the equilibrium price is potentially informative about the signals of traders in other groups. Trader (m, n) can learn from the intercept of the residual supply schedule $p_{m,n}$ in (8).

$$(MN-1)\pi_P \frac{p_{m,n}}{\sigma_V} = \sum_{(m',n')\neq(m,n)} (\pi_I i_{n'} - \pi_S \rho\sigma_V s_{n'}) + \rho\sigma_V z. \tag{44}$$

When $M \neq 1$, the value of $p_{m,n}$ already includes private information i_n and endowment s_n because of trading of other traders in his group. By subtracting the overlapping information and endowments, we get

$$\begin{aligned}(MN-1)\pi_P \frac{p_{m,n}}{\sigma_V} &- (M-1)\pi_I i_n + (M-1)\pi_S \rho\sigma_V s_n \\ &= M\sum_{n'\neq n} \pi_I i_{n'} - M\sum_{n'\neq n} \pi_S \rho\sigma_V s_{n'} + \rho\sigma_V z.\end{aligned} \tag{45}$$

Dividing both sides by $M(N-1)\pi_I$ yields

$$\begin{aligned}\frac{(MN-1)}{M(N-1)\pi_I/\pi_P}\frac{p_{m,n}}{\sigma_V} &- \frac{(M-1)}{M(N-1)} i_n + \frac{(M-1)}{M(N-1)\pi_I/\pi_S}\rho\sigma_V s_n \\ &= \frac{\sum_{n'\neq n} i_{n'}}{N-1} - \frac{\sum_{n'\neq n}\rho\sigma_V s_{n'}}{(N-1)\pi_I/\pi_S} + \frac{\rho\sigma_V z}{M(N-1)\pi_I}.\end{aligned} \tag{46}$$

Since v, e_n, and $\frac{\sum_{n'\neq n} e_{n'}}{N-1} - \frac{\pi_S}{\pi_I}\frac{\sum_{n'\neq n}\rho\sigma_V s_{n'}}{N-1} + \frac{\rho\sigma_V z}{M(N-1)\pi_I}$ are independently distributed, it is straightforward to show that

$$\begin{aligned}\tau^* &= \sigma_V^2 \cdot \operatorname{var}^{-1}\left\{v \middle| i_n, \frac{(MN-1)}{M(N-1)\pi_I/\pi_P}\frac{p_{m,n}}{\sigma_V} - \frac{(M-1)}{M(N-1)} i_n + \frac{(M-1)}{M(N-1)\pi_I/\pi_S}\rho\sigma_V s_n\right\} \\ &= 1 + \tau_I + \tau_I \operatorname{var}^{-1}\left\{\frac{\sum_{n'\neq n} e_{n'}}{N-1} - \frac{\pi_S}{\pi_I}\frac{\sum_{n'\neq n}\rho\sigma_V s_{n'}}{N-1} + \frac{\rho\sigma_V z}{M(N-1)\pi_I}\right\}.\end{aligned} \tag{47}$$

Writing τ^* in the form of (10) implies

$$\frac{1}{\varphi} = 1 + \left(\frac{\rho\sigma_V\sigma_S}{\pi_I/\pi_S}\right)^2 + \frac{1}{N-1}\left(\frac{\rho\sigma_V\sigma_Z}{M\pi_I}\right)^2. \tag{48}$$

The conditional expectation is a weighted average of two signals.

$$\begin{aligned}\mathrm{E}\left\{\frac{v}{\sigma_V}\,\middle|\, i_n, \frac{(MN-1)}{M(N-1)\pi_I/\pi_P}\frac{p_{m,n}}{\sigma_V} - \frac{(M-1)}{M(N-1)} i_n + \frac{(M-1)}{M(N-1)\pi_I/\pi_S}\rho\sigma_V s_n\right\} \\ = \frac{\sqrt{\tau_I}}{\tau^*} i_n + \frac{\varphi\sqrt{\tau_I}}{\tau^*}\left(\frac{(MN-1)}{M\pi_I/\pi_P}\frac{p_{m,n}}{\sigma_V} - \frac{(M-1)}{M} i_n + \frac{(M-1)}{M\pi_I/\pi_S}\rho\sigma_V s_n\right) \\ = \frac{\sqrt{\tau_I}}{\tau^*}\left(i_n + \frac{(N-1)\varphi\pi_P}{\pi_I}\frac{p_{m,n}}{\sigma_V} - \left(i_n - \frac{\pi_P}{\pi_I}\frac{p_{m,n}}{\sigma_V} - \frac{\pi_S}{\pi_I}\rho\sigma_V s_n\right)\left(\frac{M-1}{M}\right)\varphi\right).\end{aligned} \tag{49}$$

If $(N-1)\tau_I\pi_I = 0$, there is no learning from the price, and the conditional expectation is simply

$$\mathrm{E}\left\{\frac{v_n}{\sigma_V}\,\middle|\, i_n, s_n, p_{m,n}\right\} = \frac{\sqrt{\tau_I}}{\tau^*} i_n, \tag{50}$$

which is consistent with (49) when $\varphi = 0$ is substituted. □

Proof of Lemma 2. Substituting the conditional expectation (13) into the optimal demand ((19)) allows the optimal quantity demanded $x_{m,n}$ to be written as

$$\begin{aligned}(1+2\chi) M\rho\sigma_V x_{m,n} = \tau_I^{1/2} i_n - \left(\tau^* - (N-1)\varphi\tau_I^{1/2}\frac{\pi_P}{\pi_I}\right)\frac{p_{m,n}}{\sigma_V} - \rho\sigma_V s_n \\ - \frac{(M-1)\varphi\tau_I^{1/2}}{M}\left(i_n - \frac{\pi_P}{\pi_I}\frac{p_{m,n}}{\sigma_V} - \frac{\pi_S}{\pi_I}\rho\sigma_V s_n\right).\end{aligned} \tag{51}$$

Substituting the residual supply curve (7) allows the optimal quantity demanded $x_{m,n}$ in (51) to be implemented with a demand schedule $X_{m,n}$ given by

$$\begin{aligned}\left(1 + \chi + \frac{\varphi\tau_I^{1/2}}{M^2\pi_I}\right) M\rho\sigma_V X_{m,n}(p \mid i_n, s_n) = \tau_I^{1/2} i_n - \left(\tau^* - (N-1)\varphi\tau_I^{1/2}\frac{\pi_P}{\pi_I}\right)\frac{p}{\sigma_V} - \rho\sigma_V s_n \\ - \frac{(M-1)\varphi\tau_I^{1/2}}{M}\left(i_n - \frac{\pi_P}{\pi_I}\frac{p}{\sigma_V} - \frac{\pi_S}{\pi_I}\rho\sigma_V s_n\right).\end{aligned} \tag{52}$$

In moving from the optimal quantity on the residual supply schedule (51) to the demand schedule (52), we have shown that a trader in a symmetric linear equilibrium can implement a strategy which picks the best point on the linear supply schedule by submitting a demand schedule which is a function of price and is measurable with respect to his private information and endowment shock. $\square$

Proof of Theorem 1. Collecting terms in the demand schedule (21) yields

$$\left(1+\chi+\frac{\varphi\tau_I^{1/2}}{M^2\pi_I}\right)M\rho\sigma_V X_{m,n}\left(p\mid i_n,s_n\right)=\left(1-\frac{(M-1)\,\varphi}{M}\right)\tau_I^{1/2}i_n$$
$$-\left(\tau^*-\frac{\varphi\tau_I^{1/2}\pi_P}{\pi_I}\left(N-\frac{1}{M}\right)\right)\frac{p}{\sigma_V}-\left(1-\frac{(M-1)\,\varphi\tau_I^{1/2}\pi_S}{M\pi_I}\right)\rho\sigma_V s_n. \tag{53}$$

Now equating the coefficients in the best response demand schedule produces the following three equations:

$$\frac{\pi_P}{\pi_I}=\frac{\tau^*-\varphi\tau_I^{1/2}\left(N-\frac{1}{M}\right)\frac{\pi_P}{\pi_I}}{\left(1-\frac{(M-1)\varphi}{M}\right)\tau_I^{1/2}}, \tag{54}$$

$$\frac{\pi_S}{\pi_I}=\frac{1-\frac{(M-1)\varphi\tau_I^{1/2}}{M}\frac{\pi_S}{\pi_I}}{\left(1-\frac{(M-1)\varphi}{M}\right)\tau_I^{1/2}}, \tag{55}$$

$$\pi_I=\frac{\left(1-\frac{(M-1)\varphi}{M}\right)\tau_I^{1/2}}{\left(1+\chi+\frac{\varphi\tau_I^{1/2}}{M^2\pi_I}\right)M}. \tag{56}$$

Combining with (16), these equations can be solved as functions of M, N, τ_I, and φ:

$$\pi_I=\frac{\left(MN-2-(MN+N-2)\,\varphi\right)\tau_I^{1/2}}{M\,(MN-1)}, \tag{57}$$

$$\pi_S=\frac{1}{\tau_I^{1/2}}\pi_I=\frac{\left(MN-2-(MN+N-2)\,\varphi\right)}{M\,(MN-1)}, \tag{58}$$

$$\pi_P=\frac{\tau^*}{\left(1+(N-1)\,\varphi\right)\tau_I^{1/2}}\pi_I=\frac{\left(MN-2-(MN+N-2)\,\varphi\right)\tau^*}{M\,(MN-1)\left(1+(N-1)\,\varphi\right)}, \tag{59}$$

and

$$\chi=\frac{1+(N-1)\,\varphi}{MN-2-(MN+N-2)\,\varphi}. \tag{60}$$

Substituting π_I, π_P, π_S, and χ into (21) yields (24). Substituting (24) into the market clearing condition (3) and rearranging yields (25).

To determine the value of φ, first consider the case when $(N-1)\,\tau\neq 0$. substituting

(58) and (57) into (12) yields

$$\frac{1-\varphi}{\varphi} = \frac{(\rho\sigma_V\sigma_S)^2}{\tau_I} + \frac{(\rho\sigma_V\sigma_Z)^2/(N-1)}{\tau_I\left(\frac{MN-2}{MN-1} - \left(\frac{MN+N-2}{MN-1}\right)\varphi\right)^2}. \tag{61}$$

To exclude trivial no-trade equilibrium, we require $(MN-1)\pi_P \neq 0$, that is,

$$MN - 2 - (MN + N - 2)\varphi \neq 0. \tag{62}$$

The second-order condition ($\chi > -\frac{1}{2}$) combined with no trivial no-trade equilibrium implies that

$$\varphi < \left(\frac{MN-2}{MN+N-2}\right) \qquad \text{or} \qquad \varphi > \left(\frac{M}{M-1}\right). \tag{63}$$

The second inequality is ruled out because $\varphi \leq 1$. If $(N-1)\tau = 0$, then $\varphi = 0$. This satisfies the second-order condition because $MN > 2$. □

Proof of Corollary 1. Proving this corollary is accomplished by analyzing the two equations (22) and (23) determining the endogenous parameter φ. First we prove *if* part. If $MN > 2$ and $(N-1)\tau_I = 0$, $\varphi = 0$ and this satisfies the second order condition (23). If $MN > 2$ and $(N-1)\tau_I \neq 0$, φ is determined by (22). Rewrite (22) as

$$L(\varphi) = R(\varphi), \tag{64}$$

where we define $L(\varphi)$ and $R(\varphi)$ by

$$L(\varphi) := \frac{1-\varphi}{\varphi} - \frac{(\rho\sigma_V\sigma_S)^2}{\tau_I} \tag{65}$$

and

$$R(\varphi) := \frac{(\rho\sigma_V\sigma_Z)^2/(N-1)}{\tau_I\left(\frac{MN-2}{MN-1} - \left(\frac{MN+N-2}{MN-1}\right)\varphi\right)^2}. \tag{66}$$

If $\sigma_Z = 0$, (22) reduces to $L(\varphi) = 0$ with a unique solution. The solution satisfies 23 if and only if (29) holds. If $\sigma_Z > 0$, $R(\varphi)$ is strictly and monotonically increasing for all φ in $\left[0, \left(\frac{MN-2}{MN+N-2}\right)\right)$, with $R(0) < \infty$ and $R(\varphi) \to \infty$ as $\varphi \to \left(\frac{MN-2}{MN+N-2}\right)$. $L(\varphi)$ is strictly decreasing everywhere, with $L(\varphi) \to \infty$ as $\varphi \to 0$ and $L\left(\frac{MN-2}{MN+N-2}\right) < \infty$. Therefore there always exists a unique solution to (64) satisfying $0 < \varphi < \frac{MN-2}{MN+N-2}$.

Now we prove *only if* part. It suffices to show that there exists no equilibrium when $MN \leq 2$. If $MN \leq 2$, (23) implies $\varphi < 0$, which cannot be satisfied because $0 \leq \varphi \leq 1$. □

Proof of Corollary 2. Equation (32) is a direct implication of Corollary 1 and Theorem 1 when $\sigma_Z^2 = 0$ is substituted into (22). This completes the proof.

Proof of Corollary 3. From (51), it follows that

$$BR\left(\frac{\pi_P}{\pi_I}\right) = \frac{\tau^* - \frac{(N-1)\varphi\tau_I^{1/2}\pi_P}{\pi_I} - \frac{(M-1)\varphi\tau_I^{1/2}}{M}\frac{\pi_P}{\pi_I}}{\tau_I^{1/2} - \frac{(M-1)\varphi\tau_I^{1/2}}{M}}, \tag{67}$$

which simplifies to

$$BR\left(\frac{\sqrt{\tau_I}\pi_P}{\tau^*\pi_I}\right) = \frac{1}{1-\left(1-\frac{1}{M}\right)\varphi} - \frac{\left(N-\frac{1}{M}\right)\varphi}{1-\left(1-\frac{1}{M}\right)\varphi}\left(\frac{\sqrt{\tau_I}\pi_P}{\tau^*\pi_I}\right) \tag{68}$$

This implies that $\frac{\pi_P}{\pi_I}$ that satisfies (35) solves the fixed point problem in the sense $\frac{\pi_P}{\pi_I}$ would be an equilibrium value if φ were an equilibrium value.

Again from (51), it follows that

$$BR\left(\pi_P\right) = \frac{\tau^* - (N-1)\varphi\tau_I^{1/2}\frac{\pi_P}{\pi_I} - \frac{(M-1)\varphi\tau_I^{1/2}}{M}\frac{\pi_P}{\pi_I}}{\left(1+\chi+\frac{\varphi\tau_I^{1/2}}{M^2\pi_I}\right)M} \tag{69}$$

By substituting (35) and (16) into (69), we get

$$BR\left(\pi_P\right) = \frac{\tau^*\left(1-\frac{\left(N-\frac{1}{M}\right)\varphi}{1+(N-1)\varphi}\right)}{\left(1+\chi+\chi\frac{\left(N-\frac{1}{M}\right)\varphi}{(1+(N-1)\varphi)}\right)M}. \tag{70}$$

The resulting market noncompetitiveness $BR\left(\chi\right)$ follows substituting (70) into the definition of χ (16):

$$BR\left(\chi\right) = \frac{BR\left(\tau^*\right)}{M\left(MN-1\right)BR\left(\pi_P\right)}, \tag{71}$$

which yields (36). □

Proof of Theorem 2. For given k, economy $\mathcal{E}_k$ has an equilibrium because exogenous noise trading satisfies $\sigma_Z^2(k) > 0$. In equilibrium, φ is determined as a solution to (22), or, equivalently, as a solution to (64). Therefore for all k, there exists unique φ_k which solves

$$\frac{(\rho\sigma_V\sigma_Z(k))^2}{(N-1)} = R(\varphi_k)\cdot\left(\frac{MN-2}{MN-1} - \left(\frac{MN+N-2}{MN-1}\right)\varphi_k\right)^2, \tag{72}$$

and satisfies the second-order condition (23). As k approaches infinity, the left hand side of (72) approaches zero. Then possible solutions to (72) solve either $R(\varphi_k) \to 0$ or

$$\varphi_k \to \frac{MN-2}{MN+N-2}. \tag{73}$$

Consider a candidate solution $\varphi_{\text{candidate}}$ that solves $R(\varphi_{\text{candidate}}) = 0$. Since an equilibrium does not exist in economy $\mathcal{E}$ and more specifically (29) does not hold, $\varphi_{\text{candidate}}$ does not satisfy the second-order condition (23). Therefore, we obtain (37). Substituting (37) into (26) yields (38). $\square$